A Babcock-Leighton flux transport dynamo with solar-like differential rotation

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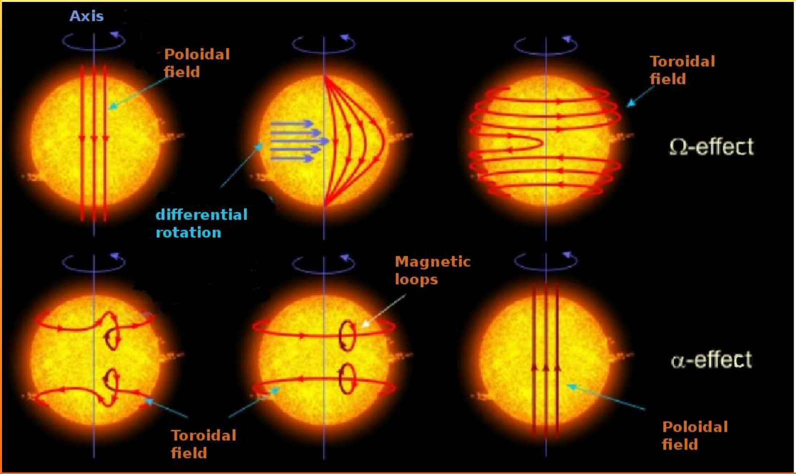
Dynamo models of the solar cycle

- Sun's magnetic field is generated by a magnetic dynamo ⇒ Sun's large-scale magnetic field ⇒ root of all phenomena
- A successful model for the solar dynamo must explain several observations:
  - ❖ the 11-year period of the sunspot cycle (polarity reversal)
  - ❖ equatorward migration of the sunspots (generating the toroidal field) ∼ butterfly-diagram
  - ❖ observed phase lag between poloidal and toroidal components
  - ❖ Hale's polarity law and the 22-year magnetic cycle
  - ❖ Joy's law for the observed tilt of sunspot groups
- the solar dynamo is one of unresolved problems of astrophysics

Dynamo models of the solar cycle



Babcock-Leighton flux transport dynamo

- I investigate a Babcock-Leighton flux transport dynamo.
- The tilt of sunspot pairs ⇒ Joy's Law ⇒ generates a north-south (poloidal) component from an initial east-west (toroidal) magnetic field.
- An equivalent viewpoint is that the twist imparted by the Coriolis force on the rising flux ropes inducing a mean poloidal field.
- Dynamo models relying on this poloidal field regeneration mechanism ⇒ Babcock-Leighton dynamos
- Regenerated in spatially distinct locations
- Buoyant rise of toroidal flux ropes ⇒ transport mechanism for the toroidal field.
- Meridional circulation ⇒ primary poloidal flux transport agent.
- A poleward meridional flow is indeed observed at the solar surface

Matematical formulation

- The starting point ⇒ hydromagnetic induction equation ⇒ the evolution of the large-scale magnetic field B

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B} - \eta \nabla \times \mathbf{B}),$$

  η is the magnetic diffusivity. Under the assumption of axisymmetry, using spherical polar coordinates (r, θ, φ), and working in the kinematic regime, so U is given

- $$\mathbf{B} = B_\varphi(r, \theta, t) e_\varphi + \nabla \times [A(r, \theta, t) e_\varphi]$$
$$U = u(r, \theta) + r \sin\theta \Omega(r, \theta) e_\varphi,$$

  where $B_\varphi(r, \theta, t)$ and $A(r, \theta, t)e_\varphi$ correspond to the toroidal and poloidal components of the magnetic field and the meridional circulation $u(r, \theta)$ and differential rotation $\Omega(r, \theta)$ to the poloidal and toroidal parts of the total flow field U.

The boundary conditions

- The northern hemisphere ($0 \leqslant \theta \leqslant \pi/2$)
- The radius $r = 0.6 \cdot R_\odot$ to $r = R_\odot$
  - ❖ If $\theta = 0$, then $A = 0$ and $B = 0$.
  - ❖ If $\theta = \pi/2$, then $B = 0$ and $\frac{\partial A}{\partial \theta} = 0$.
  - ❖ If $r = 0.6 R_\odot$, then $A = 0$ and $B = 0$.
  - ❖ If $r = R_\odot$, then $B = 0$ and

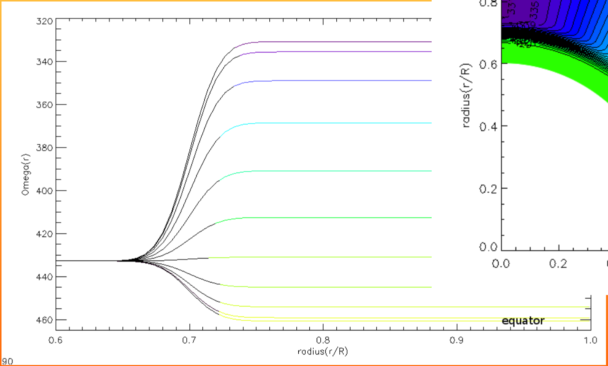
$$\frac{\partial A}{\partial r} = -\sum \frac{(n+1)a_n(t)}{R^{n+2}} P_n^l(\cos\theta)$$

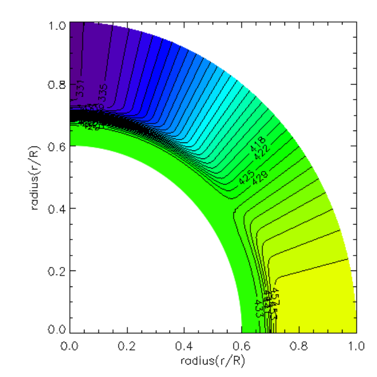
$$a_n(t) = \frac{(2n+1)R^{n+1}}{n(n+1)} \int_0^{\pi/2} A(r = R, \theta, t) P_n^l(\cos\theta) \sin\theta \, d\theta$$

The flow fields – differential rotation

$$\Omega(r,\theta) = \Omega_s + \frac{1}{2}\left[1 + erf\left(2\frac{r-r_c}{d_1}\right)\right]\{\Omega_s(\theta) - \Omega_c\}$$

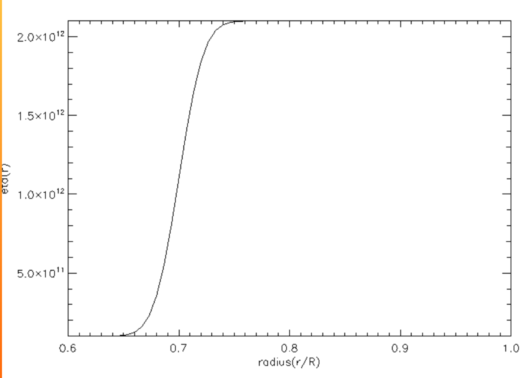
$$\Omega_s = \Omega_{Eq} + a_2 \cos^2\theta + a_4 \cos^4\theta$$





The diffusivity profile

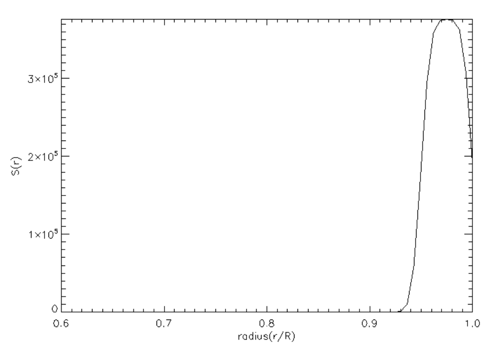
$$\eta(r) = \eta_c + \frac{\eta_T}{2}\left[1 + erf\left(2\frac{r - r_c}{d_1}\right)\right]$$



The poloidal source term

$$S(r,\theta;B_\varphi) = \frac{S_0}{2}[1 + erf(\frac{r-r_2}{d_2})] \times [1 - erf(\frac{r-r_3}{d_3})]$$

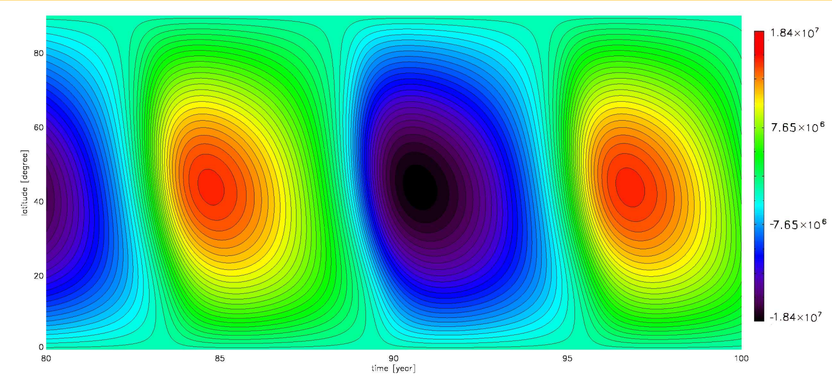
$$[1 + (\frac{B_\varphi(r_c,\theta,t)}{B_0})^2]^{-1} \times \sin\theta\cos\theta B_\varphi(r_c,\theta,t)$$



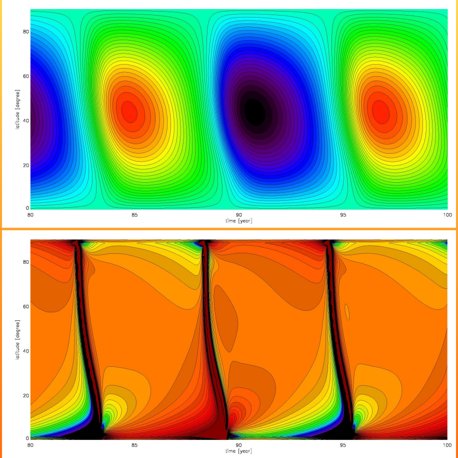
Results

- ✔ polarity of both toroidal and poloidal field reversal
- ✔ from negative toroidal field is regenerated negative poloidal field, and from the negative poloidal field is regenerated positive toroidal field
- ✔ observed phase lag between poloidal and toroidal components
  - ❖ if solar minimum $\Rightarrow$ toroidal field is minimal and poloidal field is maximal
  - ❖ if solar maximum $\Rightarrow$ toroidal field is maximal and poloidal field is minimal
- ✔ the solar cycle is 6-year without meridional circulation, like Dikpati

The toroidal field



The toroidal and poloidal fields



Forthcoming plans

- meridional circulation
- new mathematical method
- new poloidal source term
- new diffusivity profile
- parallel programming
- ...

References

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Thank you for your attention!