Black hole tidal charge constrained by strong gravitational lensing

- Null geodesics in spherically symmetric, static space-times
- Gravitational lensing
- Tidal charged brane black holes
- The first relativistic Einstein ring
- Constraints on the tidal charge from strong lensing

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Null geodesics in spherically symmetric, static space-times

* null geodesic motions in the equatorial plane of a spherically symmetric, static metric

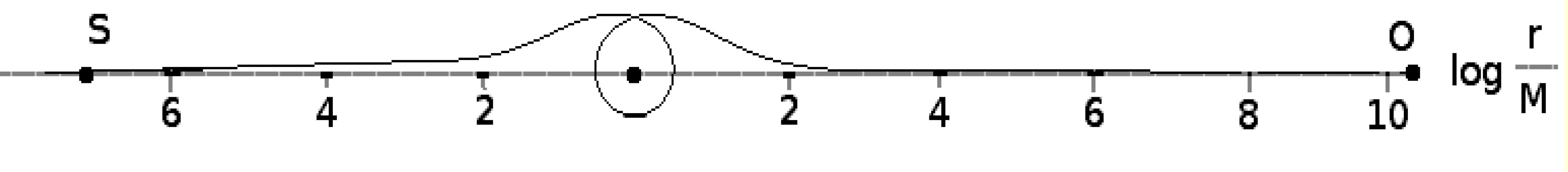
$$ds^2 = g_{tt}dt^2 + g_{rr}dr^2 + r^2 d\theta^2 + r^2 sin^2\theta \, d\varphi^2$$

* the equation of shape characterises the trajectory r(φ):

$$\frac{dr}{d\varphi} = \pm\left[\frac{g_{\varphi\varphi}}{g_{rr}}\left(\frac{E^2}{L^2}\frac{g_{\varphi\varphi}}{-g_{tt}} - 1\right)\right]^{1/2}$$

$$L = g_{\varphi\varphi}\frac{d\varphi}{dp}$$
$$E = g_{tt}\frac{dt}{dp}\,.$$

* coordinate line φ = 0: optical axis, source: S, lens: L, observer: O
* **source at (φ = 0, r = D_{LS}), lens at (0, 0), observer at (φ = π, r = D_L)**



* **special solutions** of the shape equation
* a null geodesic curve along which the photons travel from the source to the observer, while turning around the lens once
* **the impact parameter L/E is free, we have to fine-tune L/E to find a curve which fits on both the source and the observer**

Gravitational lensing

* observable consequences of light deflection
  - multiple image formation, angular separations of the images
  - brightness of the images may differ from the one of the source (magnification).
  - the differences in the arrival times of light signals corresponding to the same moment of emission

* different concepts of weak and strong gravitational lensing
* astronomers:
  - weak lensing: slight elliptical distortion of galaxy images by galaxy clusters acting as lenses (any ordered orientation observed is statistically related to the mass distribution between the source and observer)
  - strong lensing: multiple images, Einstein rings or arcs
* theoreticians:
  - weak lensing: the small angle approximation holds for the deflection angle
  - **strong lensing: the deflection angle is close to an integer multiple of π**

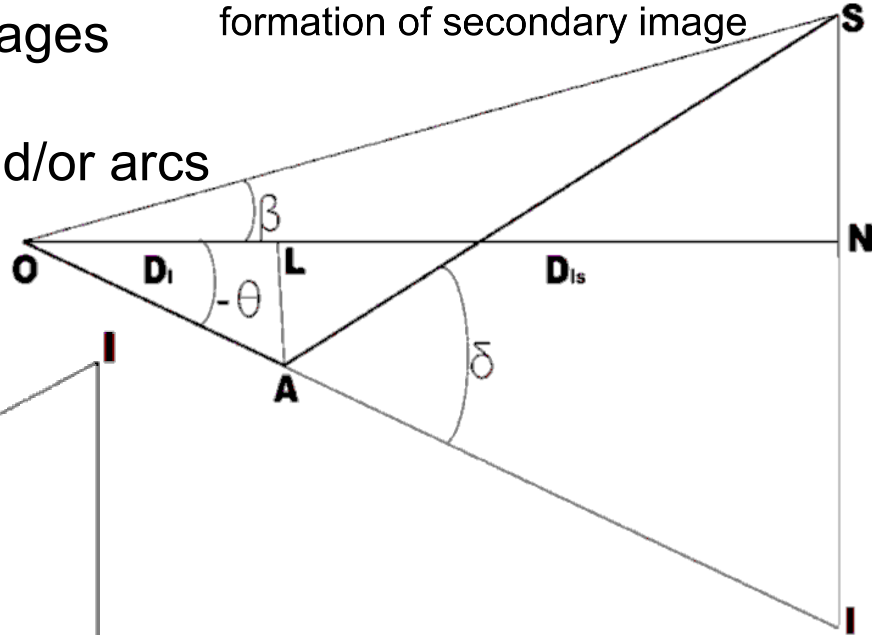
    a. **even multiples: relativistic images and relativistic Einstein ring**

    b. odd multiples: retro-lensing

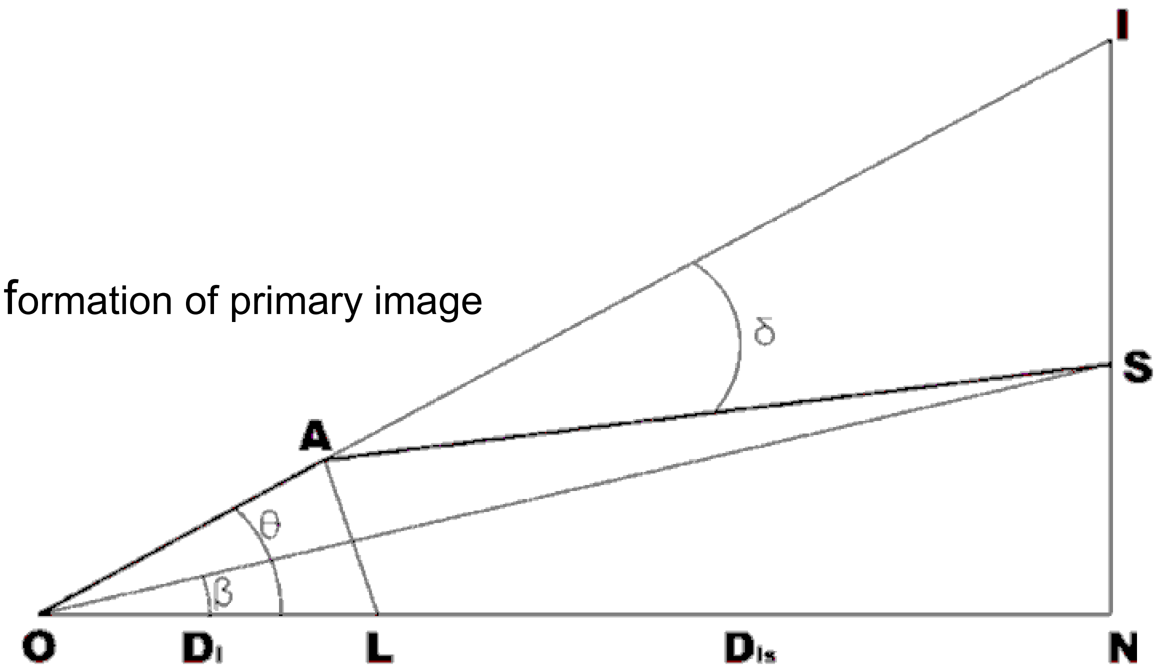
* off-axis source: primary and secondary images

* **collinear configuration**: Einstein rings and/or arcs

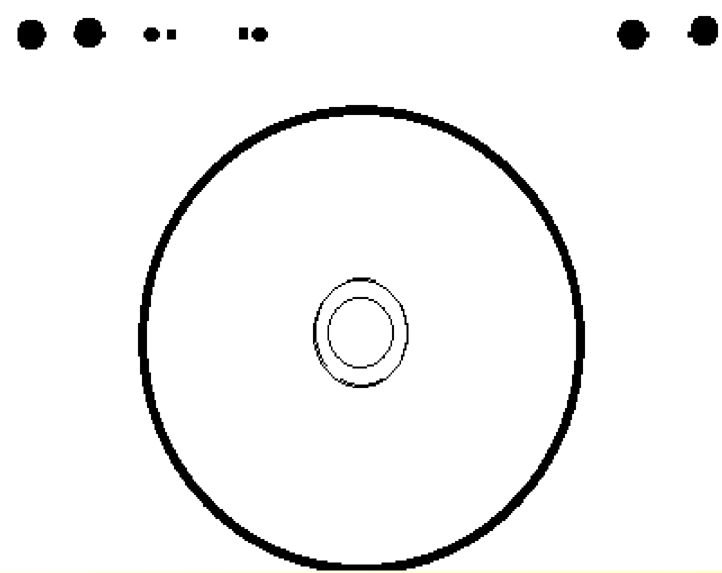
formation of secondary image

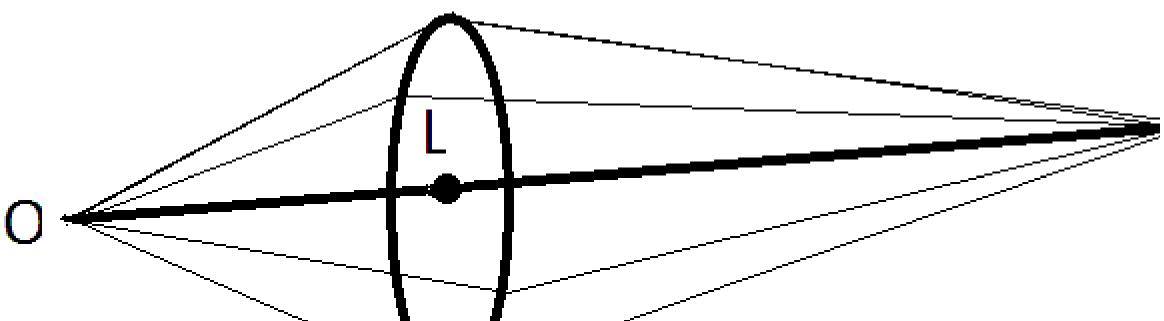


formation of primary image



what the observer sees





formation of Einstein ring

Tidal charged brane black holes

* brane theory: one kind of alternative theory of gravitation
* bulk: 5D space-time in which the brane (our 4D observable universe) is embedded
* interactions act only inside the brane, except gravity
* effective Einstein equation on the brane:

$$G_{ab} = k_4^2 T_{ab} + k_5^4 S_{ab} - E_{ab} \qquad k_4^2 = 8\pi G \qquad k_5^2 = 8\pi G_5$$

- local energy-momentum correction: E_{ab}
- non-local gravitational effect from the bulk: S_{ab}

* **black hole solution** of the effective Einstein equation on a brane

$$g_{tt} = -\frac{1}{g_{rr}} = -1 + \frac{2M}{r} - \frac{q}{r^2}$$

* **tidal charge** q is an imprint of the Weyl curvature of the bulk
* one or two horizons:

$q < 0$: $r_h = M + (M^2 - q)^{1/2}$

$0 < q < M^2$: $r_h = M \pm (M^2 - q)^{1/2}$

$q = M^2$: $r_h = M$

* naked singularity, if $M^2 < q$

Constraints on the tidal charge of the exterior metrics of astrophysical objects:

- from models of quasiperiodic oscillations, for neutron star binaries: $|q| < 10^7 \text{ m}^2$

from the junction condition of uniform density stars with its exteriors represented by the tidal charged metric: $-9.730 \times 10^8 \text{ m}^2 < q < 0$

from the perihelion precession of the Earth, for the Sun: $|q| \leq 6 \times 10^3 \text{ m}^2$

- from **weak lensing**, for the Sun: $|q| \leq 2.966 \times 10^9 \text{ m}^2$
- from **strong lensing**, for the galactic SMBH: $q \in [-1.815,\ 0.524] \times 10^{20} \text{m}^2$

The first relativistic Einstein ring

* specific form of the shape equation, valid in the tidal charged space-time

$$\frac{dR}{d\varphi} = \pm \left[\left(\frac{EM}{L} \right)^2 R^4 - R^2 + 2R - \frac{q}{M^2} \right]^{1/2}$$

normalized radial coordinate R=r/M

* numerical construction of (1-loop) null geodesics (fine-tuning of L/ME)

* **first relativistic Einstein ring:** the image created by 1-loop orbits

* **first relativistic Einstein angle Θ_E**: angle between the optical axis SO and the tangent of the geodesic curve at the point O

* for fixed D_{LS} , D_L, M, Q ---> Θ_E

in general spherically symmetric space-time;

$$\Theta_E = \arccos \left[\frac{\left(\frac{dR}{d\varphi} \right)^2 g_{RR}}{\left(\frac{dR}{d\varphi} \right)^2 g_{RR} + g_{\varphi\varphi}} \right]^{1/2}$$

in the **tidal charged space-time**

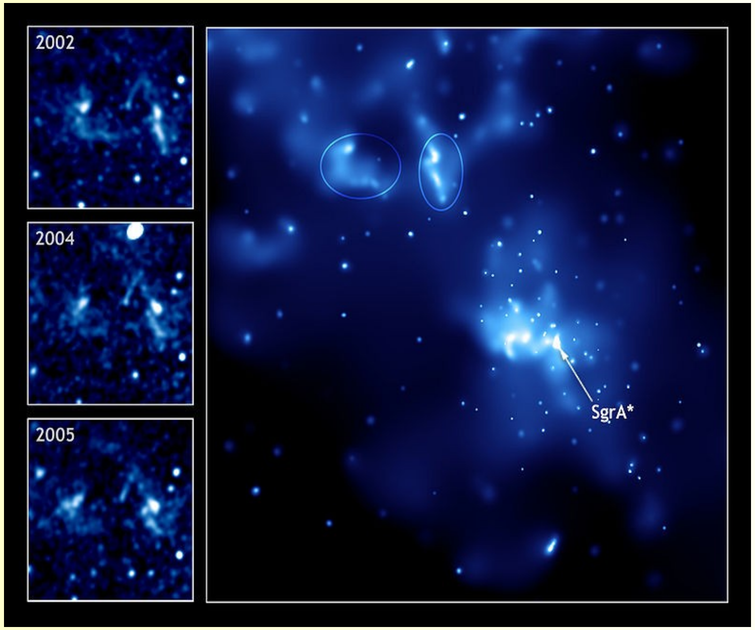
$$\Theta_E = \arccos \left\{ 1 - \left(\frac{L}{ME} \right)^2 \left[\left(\frac{M}{D_L} \right)^2 - 2 \left(\frac{M}{D_L} \right)^3 + \frac{q}{M^2} \left(\frac{M}{D_L} \right)^4 \right] \right\}$$

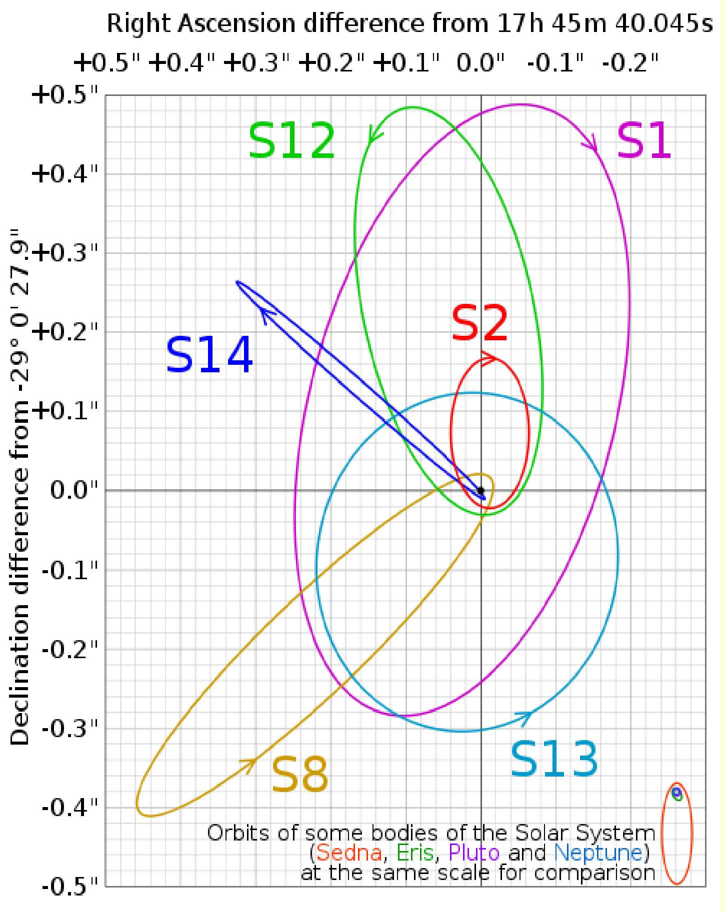
Constraints on the tidal charge from strong lensing

* **lens**: the **galactic SMBH** assumed **tidal charged**

* both the mass and tidal charge are responsible for the lensing characteristics (images, magnifications, arrival times)

* Supermassive Black Hole in radio source Sgr A*, $\mathcal{M}_{BH}=4.31 \times 10^6\ \mathrm{M}_{\odot}$
  distance: 8330 parsec
* the orbits of "S stars" depend on the spin
* observations in NIR and radio,
  absorbing gas and dust
* hard to resolve due to source confusion





* Θ_E can vary in the margin of error of some astrometric instrument, allowing tidal charge q in [q_{min}, q_{max}]

[$\Theta_{E, Schw}$ - $\Delta\Theta$, $\Theta_{E, Schw}$ + $\Delta\Theta$] <====> [q_{min}, q_{max}]

$$\Theta_E(q_{min}, D_{LS}) = \Theta_E(0, D_{LS}) + \Delta\Theta ,$$
$$\Theta_E(q_{max}, D_{LS}) = \Theta_E(0, D_{LS}) - \Delta\Theta .$$

* GRAVITY will be able to use the four telescopes of the VLT as an interferometer in the near-infrared band, astrometric precision: $\Delta\Theta = 12 \times 10^{-6}$ as .

Table 1 Column 1.: the angular radius of the first relativistic Einstein ring, columns 2.-3.: the orbital parameters of the orbits with a polar angle change 3π, column 2.: the minimal distance $R_{min} = r_{min}/M$, larger than the horizon in each case, column 3.: the parameter L/ME, column 4.: the normalized radius of the horizon, column 5.: the tidal charge, column 6.: the normalized tidal charge.

Θ_E [μas]	r_{min}/M	L/ME	r_H/M	q [10^{20} m^2]	q/M^2
38	4.924	7.706	3.345	-1.815141	-4.5
34	4.265	6.797	2.870	-1.008411	-2.5
30	3.745	6.102	2.487	-0.491834	-1.2
26	3.090	5.202	2.000	0.000000	0.0
22	2.564	4.455	1.622	0.148242	0.3
18	1.990	3.823	1.108	0.268619	0.7
14	1.446	3.207	n. s.	0.524374	1.3

* **constraint on the tidal charge of the SMBH** from Θ_E can be read from the table:

$$q \in [-1.815, 0.524] \times 10^{20} \text{m}^2$$

* **constraints on q/M²**

object	Solar System	neuton star binary	neuton star	SMBH
$\lvert q/M^2 \rvert_{max}$	0.003	2.339	227.647	4.485

- the neutron star binary and the SMBH strong lensing constraints are comparable
- the neutron stars constraints are the weakest
- the Solar System constraints (from perihelon precession of the Earth) are the strongest

* SMBHs could have a much larger tidal charge, than the Sun or neutron stars