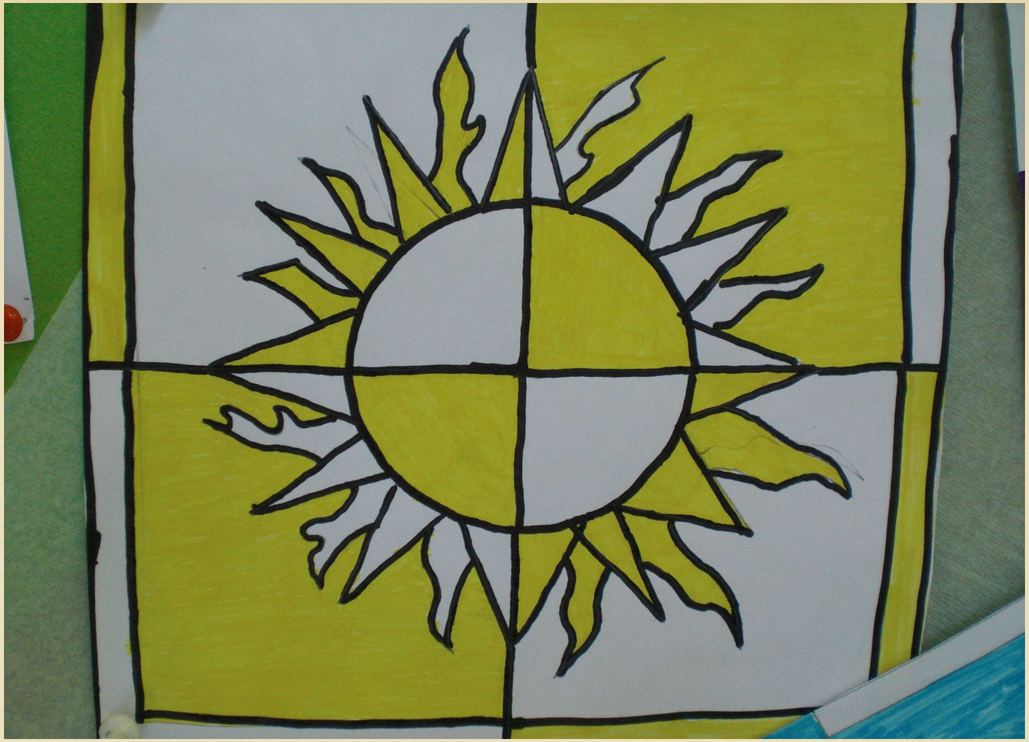
Limiting parameters



for Kaluza – Klein Stars

Gergely Gábor Barnaföldi, P. Lévai, B. Lukács

MTA Wigner RCP RMI

YouResAstro 2012

3rd-6th September 2012 Budapest

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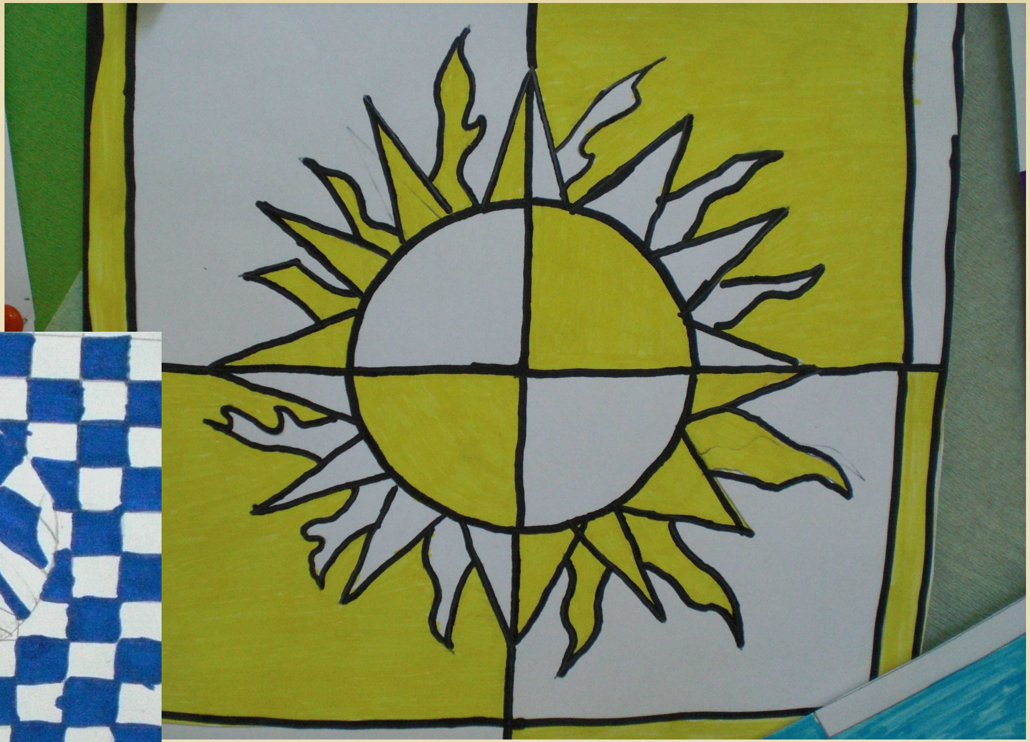

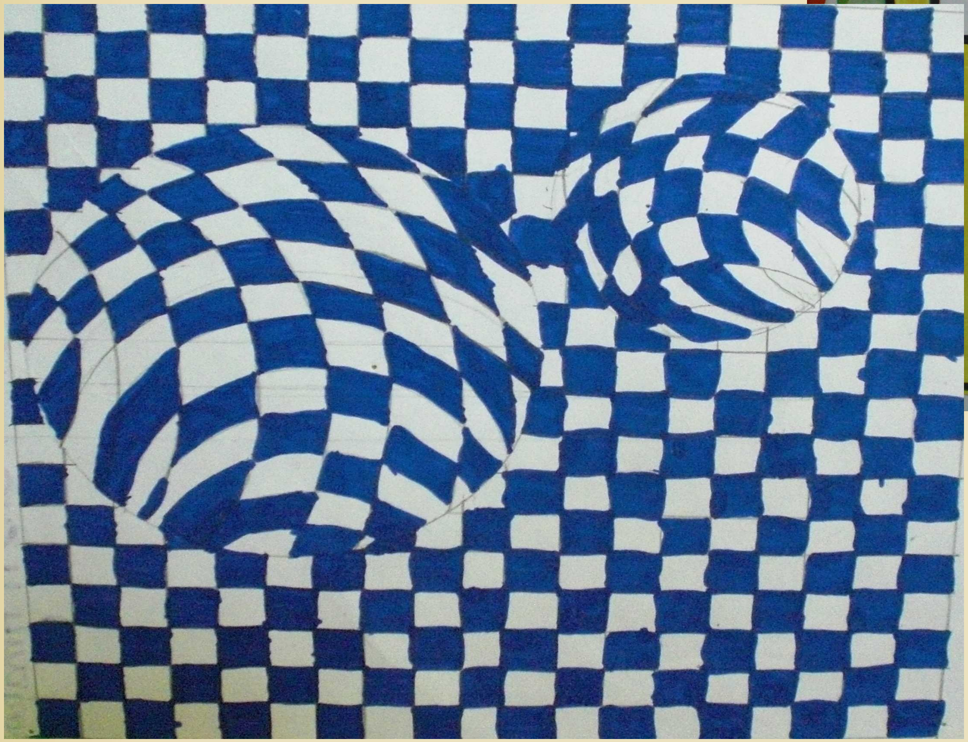

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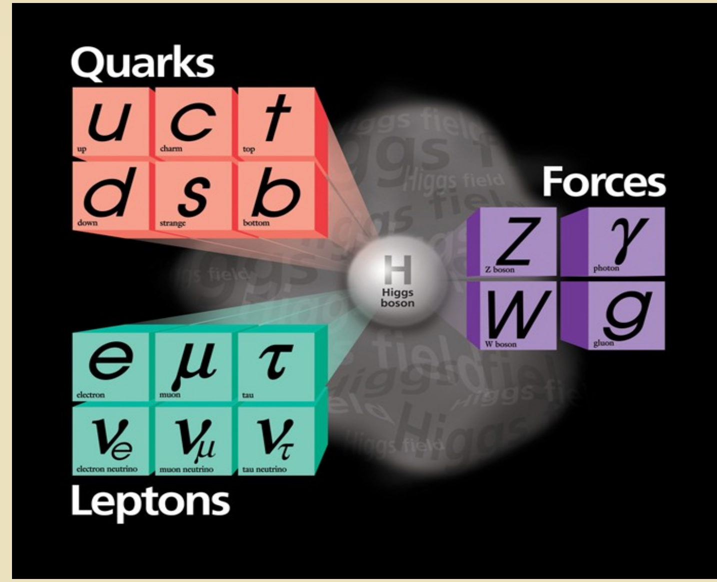
O U T L I N E

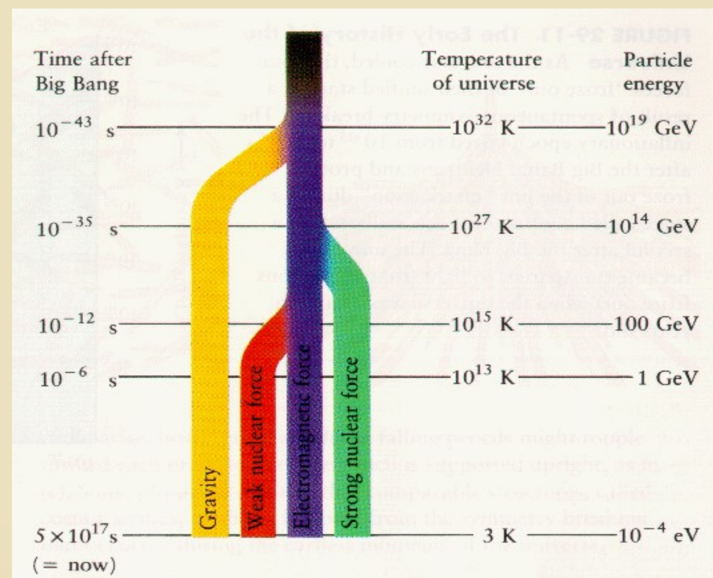
- Motivation for Kaluza–Klein theory & stars
- A special solution in $1+3+1_C$ D space-time
- Compact Star in Kaluza–Klein World
- M vs. R via varying R_C for $1+3+1_C$ compact star
- Limiting parameters & possible experimental tests

Motivation for Kaluza–Klein Stars

- Standard matter by Standard Model, (Higgs with 5σ)
- Grand Unified Theory would be nice, but problem:

    Gravity and QFT are not fit well into the same picture.





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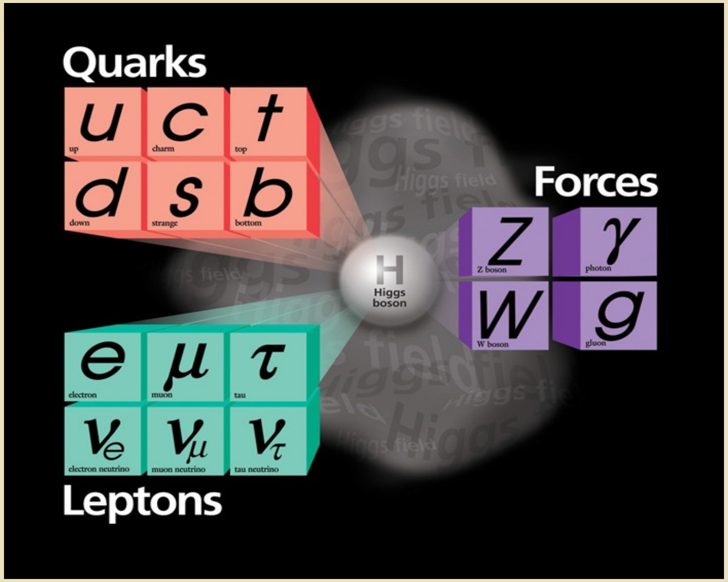
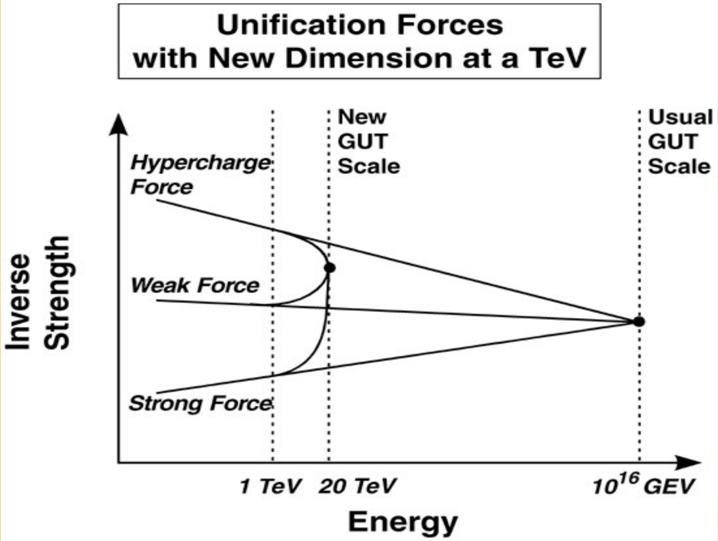


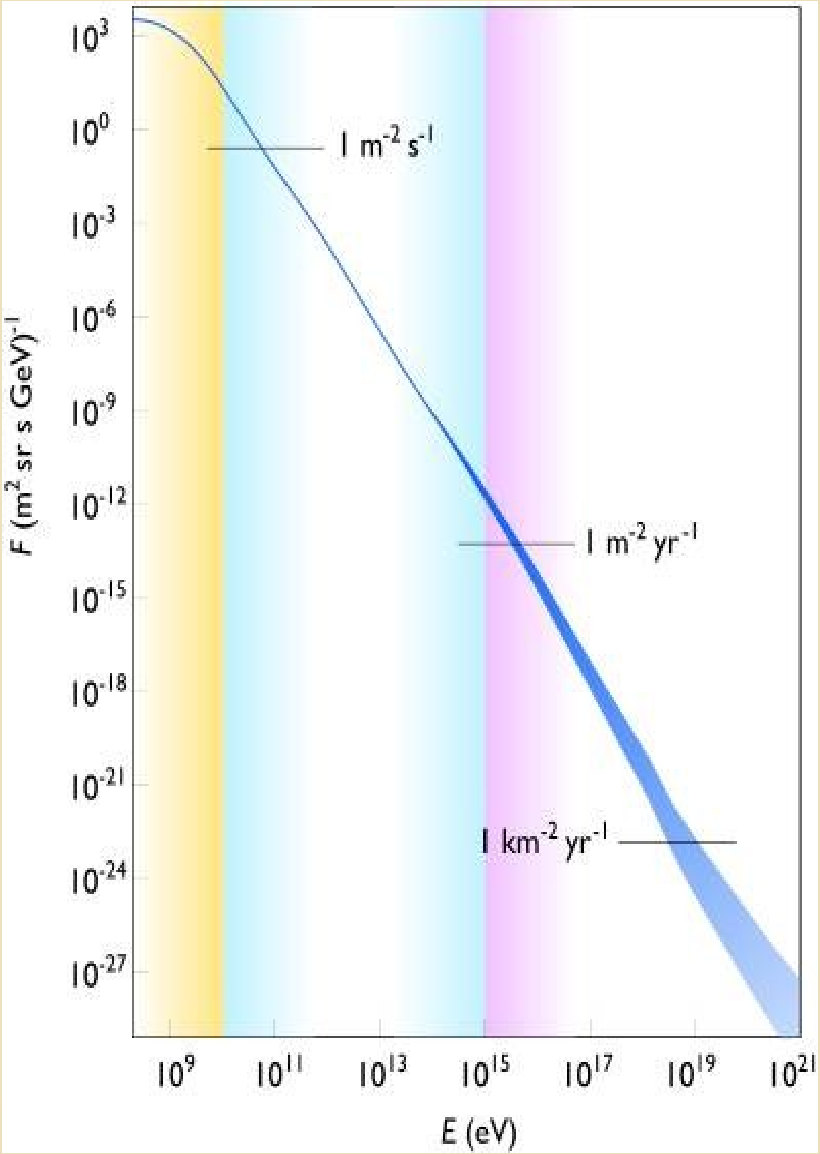


Motivation for Kaluza–Klein Stars

- Standard matter by Standard Model, (Higgs with 5σ)
- Grand Unified Theory would be nice, but problem:

  Gravity and QFT are not fit well into the same picture.

- Possible way: geometization of the elementary forces, introducing extra dimensions.
- It is just below the edge of the energies we can measure in the laboratory (e.g. LHC 14 TeV).



Historical Background for KK

Unifications of gravity with *et al.* rooted back to ~100 yrs:

E Cartan (1923) after Einstein, his generalization, gives up the symmetry of the g^{ik} , means 16 free parameter (10 for gravity, 6 for Maxwell tensor). This generalization is trivial, but leads to non-physical consequences.

Th Kaluza (1921) & O Klein (1926) kept the symmetry of g_{ik}, but extra 5th *D* was introduced (10 GR + 4 EM vector potential + 1 for g_{55}). Setting $g_{55}=1$ violates the 'free choice of coordinate systems'. But, Maxwell eqs. can be carry out in a weak-field approx.

C Brans & RH Dicke (1961) introduced a time dependent GR constant G(t), thus EM and GR can be unified within one geometrical picture.

A Chodos & S Detweiler (1980) proper separation of 5th D first time, discovered 15 geometrical parameter can be separate as 10+4+1 with a scalar field in 5th D.

B Lukács & T Pacher (1985) pointed out, B&D theory is limited by the *e/m* ratio of the p. This rule out EM being of the charge, but opened a new direction to identify the 5th direction as hypercharge ($Y=B+S$)or strangeness (S).

N Kan & K Shiraishi (2002) Model for compact star with extra *D*s inside.

Symmetries, we have...

… in case of a (compact) star:

Spherical: invariant under *O(3)* rotations

Static: the element of metric & T^{ik} tensor are *t* independent.

Ideal relativistic fluid: only diagonal elements, $T^{ik}{}_{,i} = 0$

Isotropic: no explicit φ & θ dependence of the components of g^{ik} and T^{ik}.

Extra dimension(s): need for extra assumptions/statements.

GGB, B Lukács, P Lévai: astro-ph/0312330 astro-ph/0312332
Asrton.Nachr. 328 (2007) 809
J.Phys.Conf.Ser. 218 (2010) 012010

Introducing extra dimensions...

Assumptions on Kaluza–Klein-like extra dimension(s):

(i) $(3+d_C)+1$ dimensional space-time, dimensions are space-like except the last one, which is time-like.

(ii) The GR is the same as we learned in *3+1 D*, especially the form of 'Equivalence Principle' is unchanged.

(iii) All causality postulates, including the lightcone structure are as they were in *3+1* case.

(iv) The extra, space like d_C dimensions are microscopical.

(v) There is complete Killing symmetry in the d_C-dimensional microscopical subspace.

Let's focus on the simplest $d_C=1_C$ case.

GGB, B Lukács, P Lévai: J.Phys.Conf.Ser. 218 (2010) 012010

Metric for $3+1+1_C$ D space-time

The symmetries of the metrical tensor in $1+4$ dimension:

(i) spherical symmetry $\Longrightarrow$ $\mathcal{O}(3)$ symmetry
(ii) Static picture $\Longrightarrow$ $g_{i0}=0$ and $g_{ik,0}=0$
(iii) the 4 D g_{ik} is x^5 independent
(iv) Killing transformations $\Longrightarrow$ $g_{01}=0,\ g_{51}=0$

$$g_{ik}=\begin{pmatrix} g_{00} & g_{01} & 0 & 0 & g_{05} \\ g_{01} & g_{11} & 0 & 0 & g_{15} \\ 0 & 0 & g_{22} & 0 & 0 \\ 0 & 0 & 0 & g_{22}\sin^2\vartheta & 0 \\ g_{05} & g_{15} & 0 & 0 & g_{55} \end{pmatrix}, \qquad ($$

The $t=x^0$, $r=x^1$, $\vartheta=x^2$, $\varphi=x^3$, $\chi=x^5$ coordinates,
and with $\nu(r)$,$\lambda(r)$ and $\Phi(r)$ radial functions

$$g_{ik}=\ \mathrm{diag}\ \left(\mathrm{e}^{2\nu},\ -\mathrm{e}^{2\lambda},\ -r^2,\ -r^2\sin^2\vartheta,\ -\mathrm{e}^{2\Phi}\right)$$

T_{ik} for the $3+1+1_C$ D matter

(a) **Energy-momentum tensor for anisotrop liquid:**

$$T_{ik} := \epsilon\, u_i u_k - p\,(g_{ik} - u_i u_k + v_i v_k) - p_5\, v_i v_k$$

the liquid is isotrop for 3 dimension $T_1^1 = T_2^2 = T_3^3 = p$,
but **pressure,** $T_5^5 = p_5$ **in the** 5^{th} **direction is anisotrop** ,
with ϵ **energy-density** :

$$T_{ik} = \ \mathrm{diag}\ \left(\epsilon\, \mathrm{e}^{2\nu}\,,\, p\, \mathrm{e}^{2\lambda}\,,\, p\, r^2\,,\, p\, r^2 \sin^2\vartheta\,,\, p_5\, \mathrm{e}^{2\Phi}\right)$$

(b) **Let's make the** R_{ik} **Ricci tensor, and the** R **Ricci scalar:**

$$R := R_i^i = R_0^0 + R_1^1 + R_2^2 + R_3^3 + R_5^5$$

Einstein Equations for $3+1+1_C\, D$

$$R_{ik} - \frac{1}{2} R\, g_{ik} = -\gamma\, T_{ik}, \quad \text{ahol} \quad \gamma = \frac{8\pi G}{c^4}$$

Components of the Einstein equation: :

$$-\gamma\, \epsilon = \mathrm{e}^{-2\lambda} \left[\Phi'' + \Phi'^2 - \lambda'\Phi' + \frac{2\Phi'}{r} - \frac{2\lambda'}{r} + \frac{1}{r^2} \right] - \frac{1}{r^2},$$

$$-\gamma\, p = \mathrm{e}^{-2\lambda} \left[-\nu'\Phi' - \frac{2\Phi'}{r} - \frac{2\nu'}{r} - \frac{1}{r^2} \right] + \frac{1}{r^2},$$

$$-\gamma\, p = \mathrm{e}^{-2\lambda} \left[-\nu'' - \nu'^2 + \nu'\lambda' - \Phi'' - \Phi'^2 - \nu'\Phi' + \lambda'\Phi' - \frac{\nu'}{r} + \frac{\lambda'}{r} - \frac{\Phi'}{r} \right],$$

$$-\gamma\, p_5 = \mathrm{e}^{-2\lambda} \left[-\nu'' - \nu'^2 + \nu'\lambda' - \frac{2\nu'}{r} + \frac{2\lambda'}{r} - \frac{1}{r^2} \right] + \frac{1}{r^2}.$$

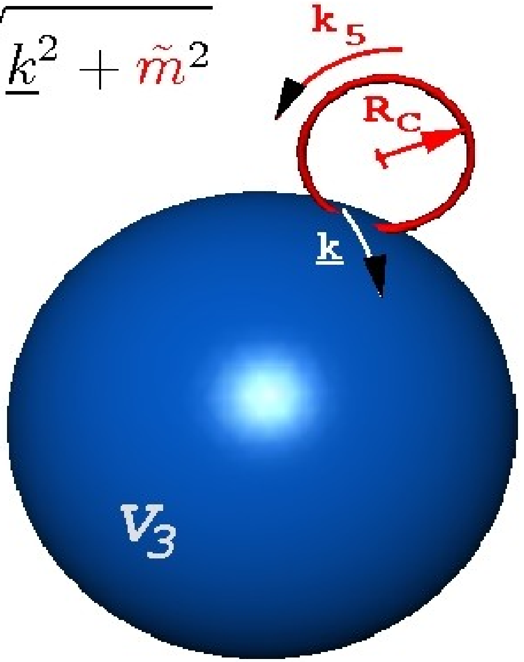
'Observation' of the 5th Dimension

(a) **Strangeness as a new degrees of freedom:**
change for $1+3+1_C$ dimensional space-time, where 1_C non-observable macroscopically $\Longrightarrow$ compactified

(b) **Connection between $\tilde{m} = m_s$ and R_c radius**

$$E_5 = \sqrt{\underline{k}^2 + \left(\frac{n}{R_c}\right)^2 + m^2} = \sqrt{\underline{k}^2 + \tilde{m}^2}$$

$$\tilde{m}^2 = \left(\frac{n}{R_c}\right)^2 + m^2$$



m: light (u,d) quark mass;
n: excitation number $n=1$;
R_c: compactified radius;
$\tilde{m}$: heavy (s) quark mass.

(c) **If $m - m_s \approx 150$MeV, then $R_c \approx 10^{-13}$cm**

Thermodynamics for $3+1+1_C$ D

(a)**Extra 5^{th} D is compactified in a S^1 circle with radius R_c**

$\Longrightarrow$ periodical boundary condition:

$$\Psi(\xi) \approx \mathrm{e}^{ik_5 \cdot \xi} \quad \textbf{and} \quad \Psi(\xi + 2\pi R_c) \sim \Psi(\xi) \implies k_5 = \frac{n}{R_c}, \quad \textbf{, where}$$

k_5 is the momenta in the 5^{th} direction, ξ is the coordinate and $n \in \mathbb{N}$

(b)**Thermodynamical potential for $4+1$D Fermion gas**

$$\Omega_5 = -2\frac{V_4}{\beta} \int \frac{\mathrm{d}^4 \underline{k}}{(4\pi)^4} \left[\ln \left(1 + \mathrm{e}^{-\beta\left(\sqrt{\underline{k}^2 + \tilde{m}^2} - \mu\right)} \right) + (\mu \leftrightarrow -\mu) \right]$$

$\tilde{m}^2 = (n/R_c)^2 + m^2$ excited mass

$\int \mathrm{d}k_5 \rightarrow \frac{1}{R_c} \sum_n$: discretization

$V_4 = 2\pi R_c V_5$: 4-volume

TOV equations for a $3+1+1_C D$

(1) **Thermodynamical potential and its quantities:**

$$\Omega_5 = \sum_n \Omega_4 \left(\sqrt{m^2 + \frac{n^2}{{R_c}^2}} \right) = \Omega_4(\tilde{m})$$

$$p = -\frac{1}{2\pi R_c} \frac{\partial \Omega_5}{\partial V} \qquad p_5 = -\frac{1}{2\pi V} \frac{\partial \Omega_5}{\partial R_c} \qquad \epsilon = \frac{U}{V_4}$$

(2) **The TOV equations at $\Phi' = 0$ special case:**

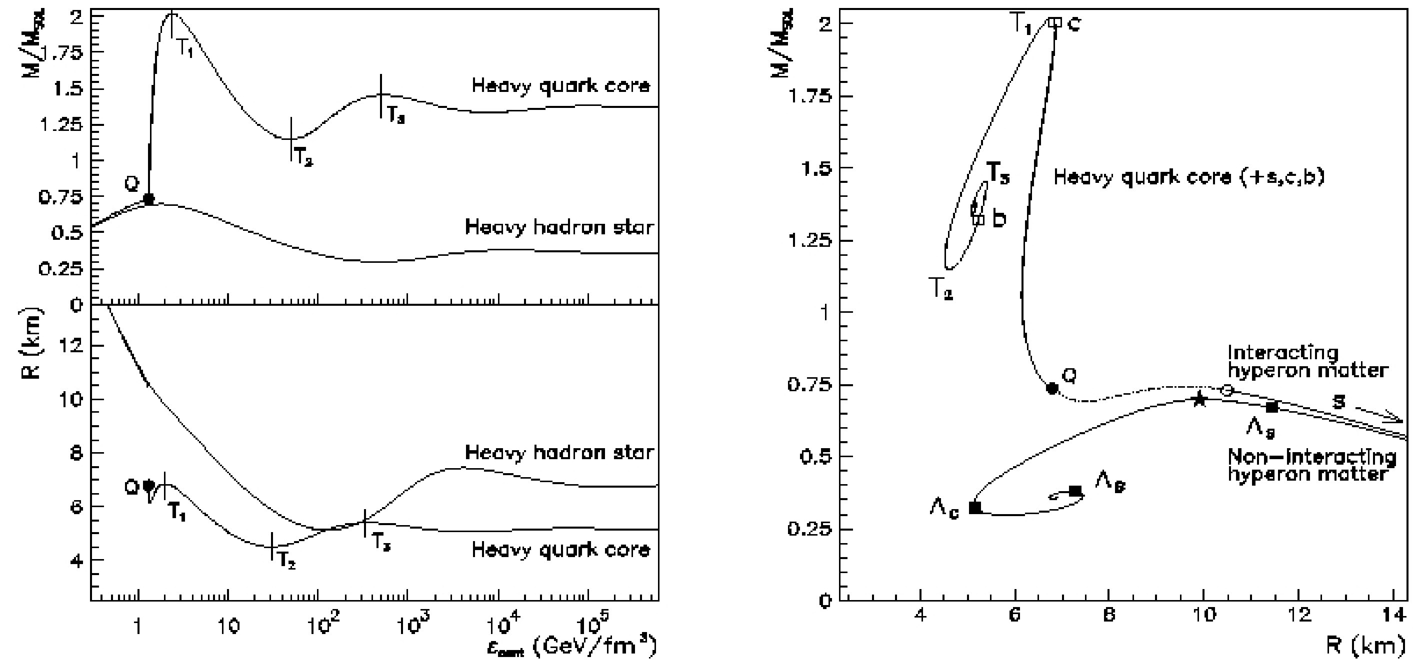
$$\frac{dp(r)}{dr} = -\frac{[p(r) + \varepsilon(r)] \cdot [M(r) + 4\pi r^3 p(r)]}{r \cdot [r - 2M(r)]}$$

$$p_5 = 1 - \frac{2M(r)}{r} + \frac{1}{r} \ln \left[1 - \frac{2M(r)}{r} \right] - 2p$$

Description of a hybrid star in *3+1 D*

(b)**Description of hybrid stars: n^0,Λ^0 $+q$ core $\Longrightarrow$ STABLE**

Hybrid star: neutron/hyperon star (+ heavy quark core)



Bottom star: $\mu > m_b \approx 175$ GeV; Top star: $\mu > m_t \approx 4.5$ GeV

Compact Star in Kaluza–Klein World

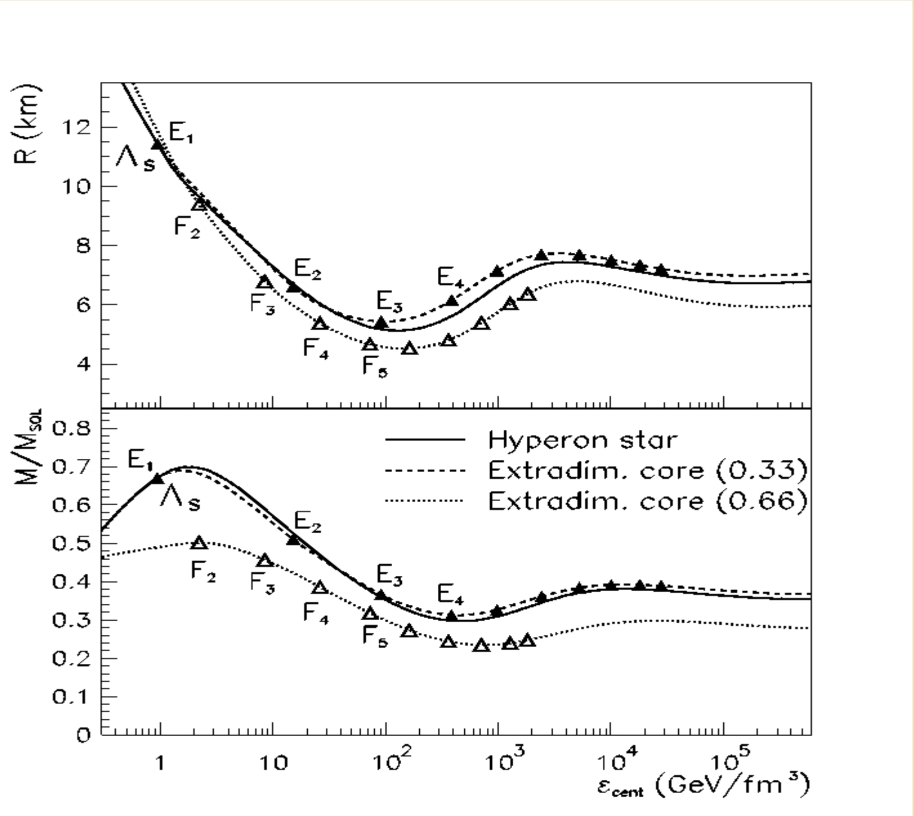
Compact star with 1_C extra dimension vs. hyperon star

Hyperon star (n^0, Λ^0, ...)

M(R) with n^th excited state

$R_C = 0.33$ fm ($E_1, E_2, E_3 \ldots$)

$R_C = 0.66$ fm ($F_1, F_2, F_3, \ldots$)



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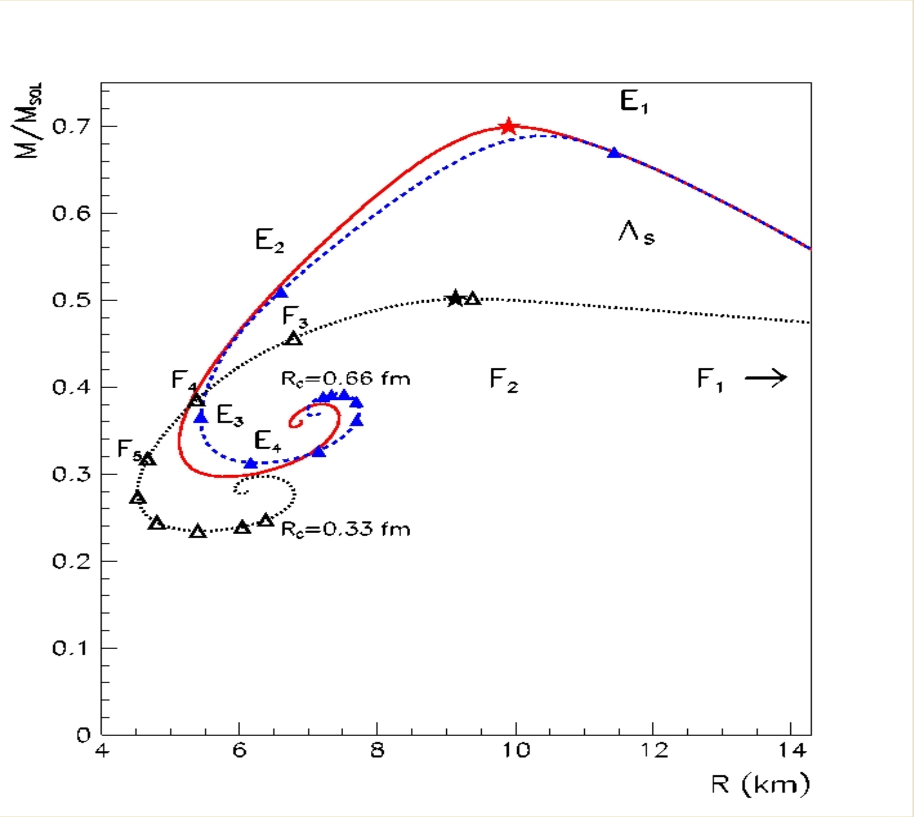
- R_C=0.66 fm (F_1, F_2, F_3,...)



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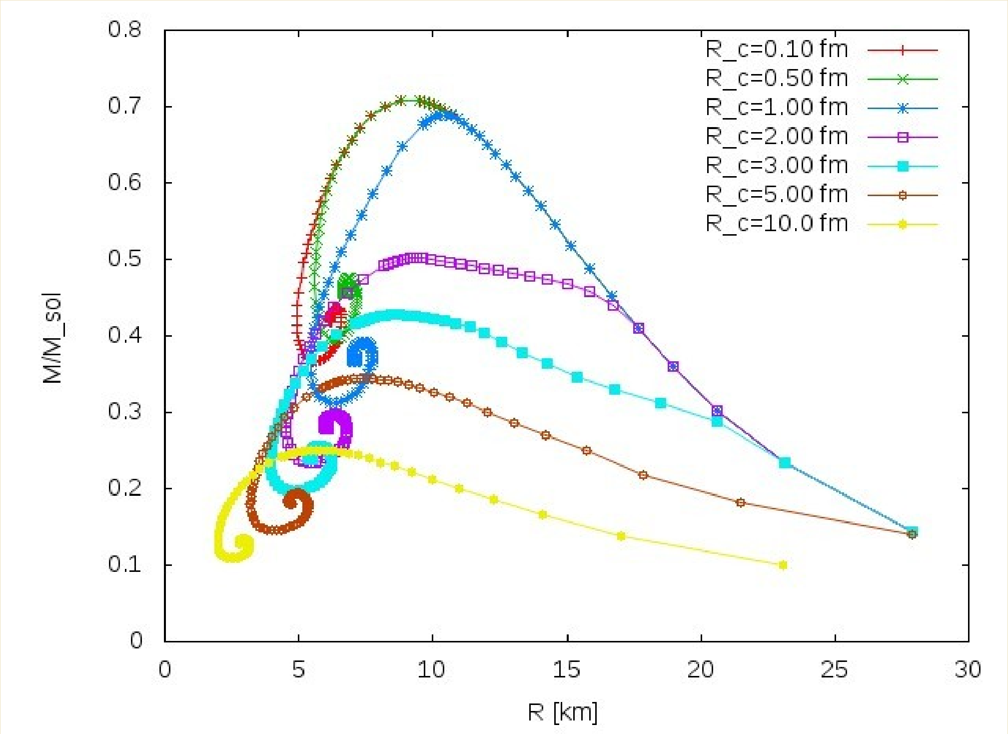
Smooth variation of the compactified *D*

Change of the R_C :

- Excitations with $\Delta m_n \sim n/R_C$
- Larger the R_C , smaller excitations to m_n .
- Similar solutions (shape, stability, etc.)
- More extra *D*, more complex structure $\Delta m_{n12} \sim O\,(n/\,R)$



Limited by the $\mu_0 > m$, but strong statement on extra D.

Possible experiments to test...

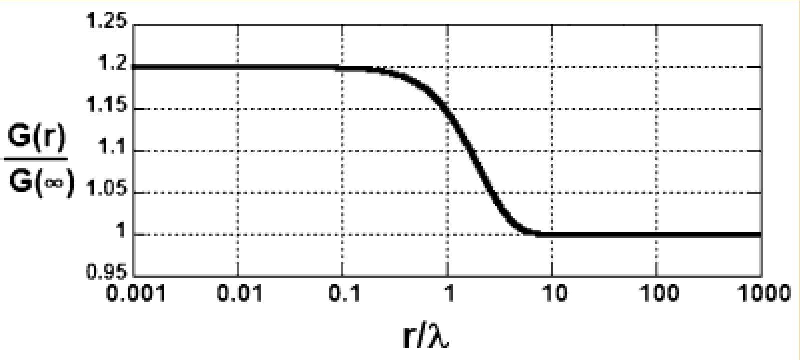
- Without violating the theories: deviant Newtonian motion

- Search for extra dimensions at the LHC

  Maximal LHC energy just reach the LXD regime

- Modified GR potential

$$F(r) = G\frac{m_1\ m_2}{r^{2+\epsilon}}$$



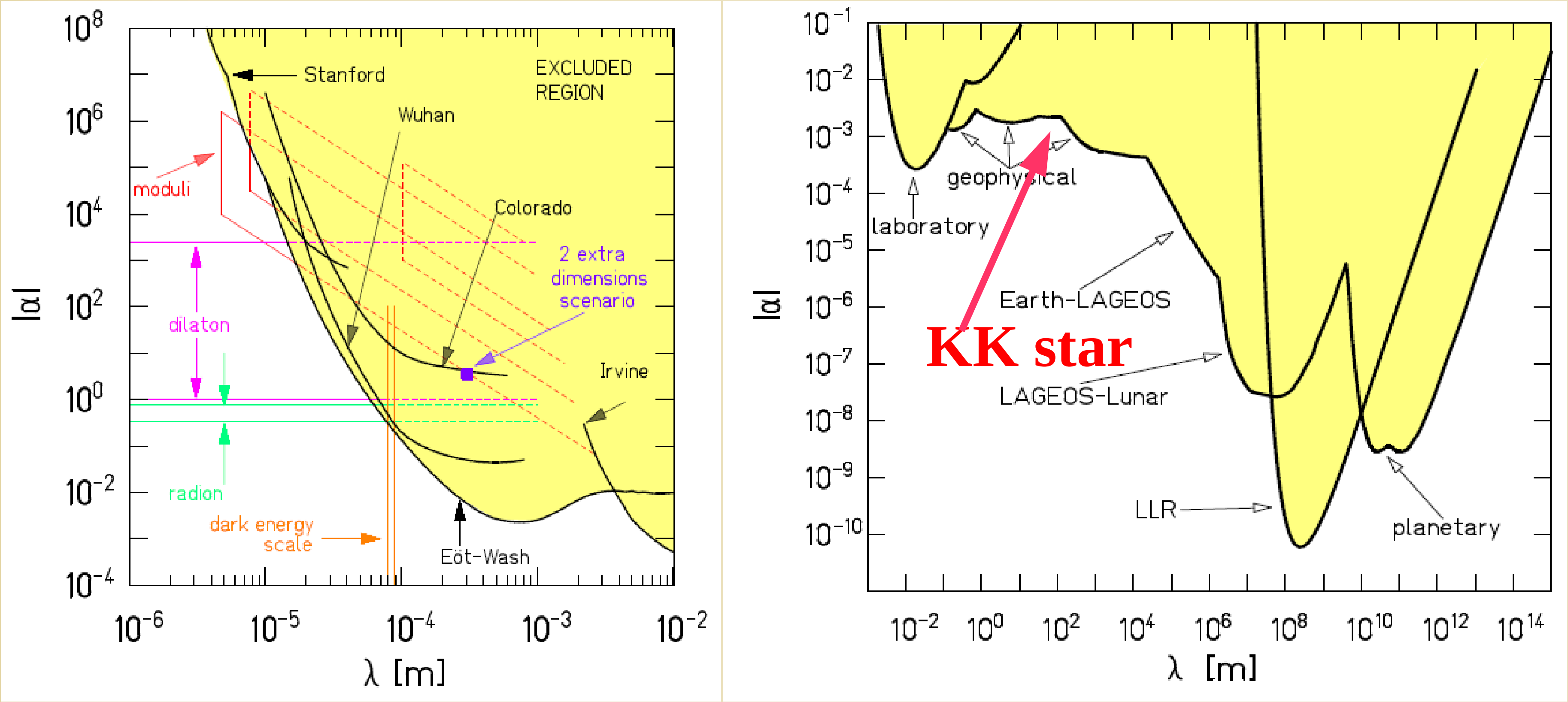
$$V(r) = -G\frac{m_1\ m_2}{r}\left[1 + \alpha_N\left(\frac{r_0}{r}\right)^{N-1}\right]$$

$$V(r) = -G\frac{m_1\ m_2}{r}\left[1 + \alpha\ e^{-r/\lambda}\right]$$

$$F(r) = \frac{Gm_1m_2}{r^2}\left(1+\alpha\left(1+\frac{r}{\lambda}\right)e^{-r/\lambda}\right) = \frac{G(r)m_1m_2}{r^2}$$

Possible experiments to test...

Already excluded regions by gravitational experiments:



P. Raffai et al. arXiv:1109.4258 [gr-gc]

E.G. Adelberger: Ann. Rev. Nucl. Part. Sci. 53. 77 (2003)

Summary

- Compact stars in $1+3+1_C\ D$ were analyzed:

  Static, spherical Schwarzschild-like space-time

  TOV-like eqs. with specific, but exact (stable) solution

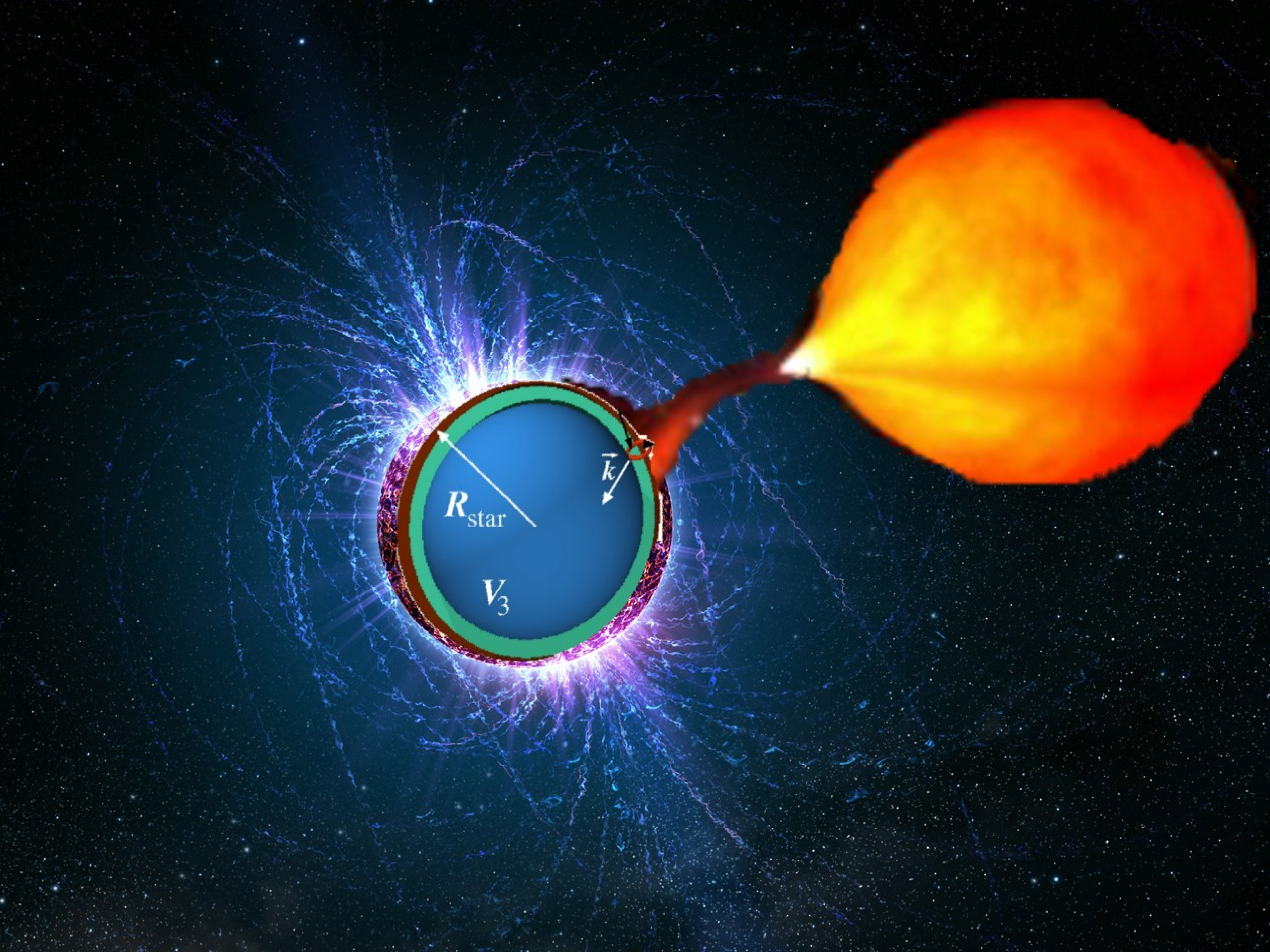
  Solutions overlap with strange star models if R_C is set to the mass of strangeness: 10^{-13} cm $< R_C < 10^{-9}$ cm

  Mass limits: Determinate by R_C , D and the μ & larger the R_C result more compact compact star

- Possible ongoing tests:

  Might see a signal for X*D*s at the LHC: $hc/E_{beam} \sim 10^{-18}$ cm

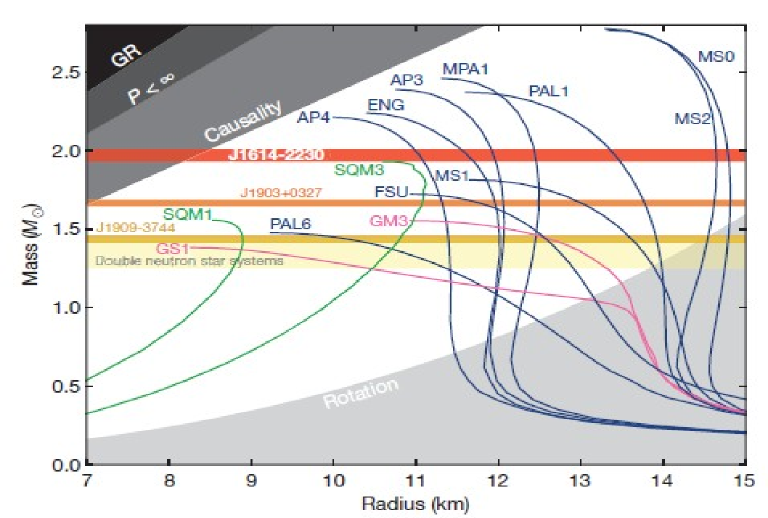
  Compact Stars or GW detectors: deviation from GR potential

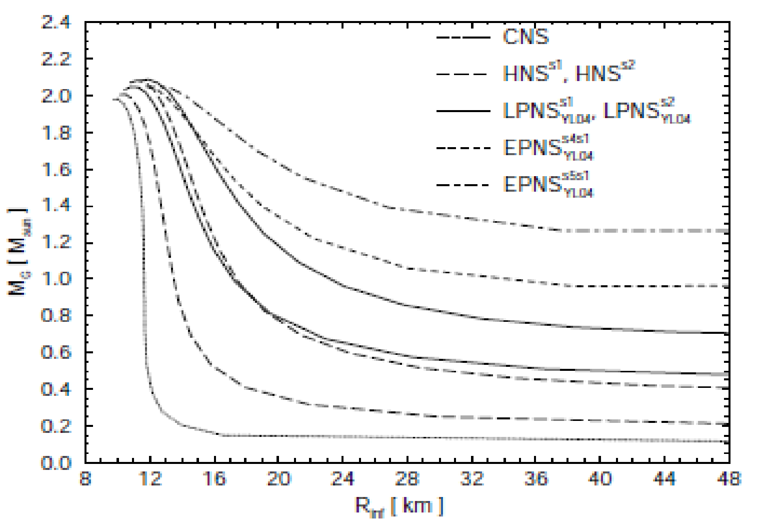


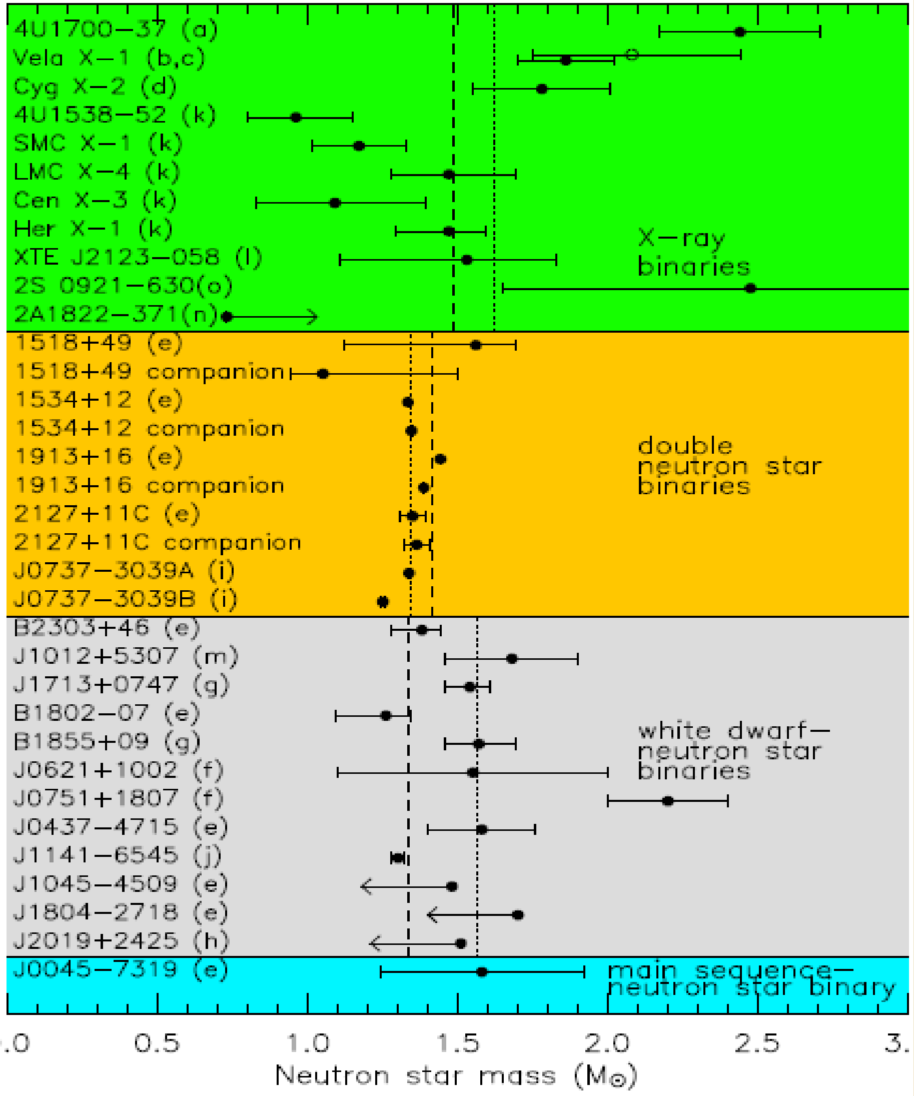
$R_{\rm star}$
$\vec{k}$
V_3

BACKUP SLIDES

Notes on Limiting Mass







Description of a hybrid star in *3+1 D*

(a)**Typical description:** n^0,Λ^0, ... q ... $\Longrightarrow$ **TOV equations**

Einstein equations (symmetries + metric + T_{ik} tensor)

Equation-of-State (EoS – matter + particle interactions)

Tollman – Oppenheimer – Volkov-equations:

$$\frac{\mathrm{d}p(r)}{\mathrm{d}r} = -\frac{[p(r)+\varepsilon(r)]\cdot[M(r)+4\pi r^3 p(r)]}{r\cdot[r-2M(r)]}$$

Solution exists: n^0,Λ^0 $\Longrightarrow$ σ, ω mean-field theories

u,d,s,c,e,μ $\Longrightarrow$ MIT Bag model

For heavy quarks $\Longrightarrow$ NOT stable for radial perturbations

Full Historical Background for KK

Unifications of gravity *et al.* rooted back to ~100 yrs:

A Einstein (1916) developed GR intudocue gravity into a geometrical approach with a symmetric g^{ik}. Later he tried to unify GR & EM.

E Cartan (1923) after Einstein, his generalization, gives up the symmetry of the g^{ik} , means 16 free parameter (10 for gravity, 6 for Maxwell tensor). This generalization is trivial, but leads to non-physical consequences.

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N Kan & K Shiraishi (2002) Provide a model for neutron stars with extra Ds structure inside.

GGB, B Lukács, P Lévai: J.Phys.Conf.Ser. 218 (2010) 012010

Space-time & metric tensor

- Using a standard form of the 'ansatz': $\lambda(r),\ \nu(r),\ \Phi(r)$
- Assuming the Killing symmetries the metric is:

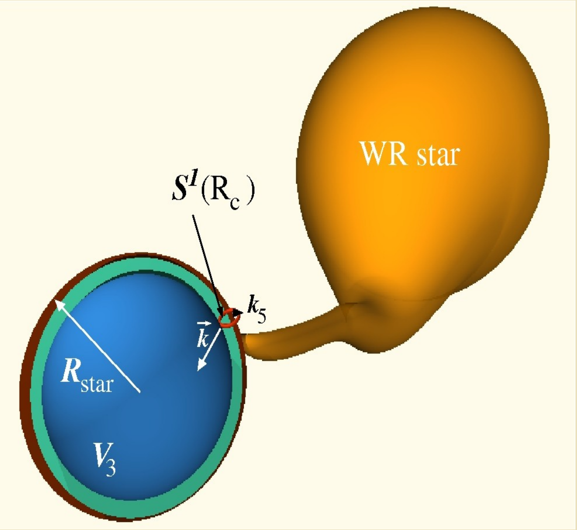
$$g_{ik} = \begin{pmatrix} g_{00} & g_{01} & 0 & 0 & g_{05} \\ g_{01} & g_{11} & 0 & 0 & g_{15} \\ 0 & 0 & g_{22} & 0 & 0 \\ 0 & 0 & 0 & g_{22}\sin^2\vartheta & 0 \\ g_{05} & g_{15} & 0 & 0 & g_{55} \end{pmatrix}$$

- More symmetries will make diagonal:

$$g_{ik} = \operatorname{diag}\left(e^{-2\nu}, -e^{2\lambda}, -r^2, -r^2\sin^2\theta, -e^{2\Phi}\right)$$

- Line element:

$$ds^2 = e^{-2\nu}dt^2 - e^{2\lambda}dr^2 - r^2 d\Omega^2 - e^{2\Phi}d\chi^2$$



Space-time & curvature for $3+1+1_C$

- After assuming the symmetries, the metric is:

$$g_{ik} = \mathrm{diag}\left(e^{-2\nu}, -e^{2\lambda}, -r^2, -r^2\sin^2\theta, -e^{2\Phi}\right)$$

- Riemann tensor components:

$$\begin{aligned}
\Gamma^2{}_{12} = \Gamma^2{}_{21} = \Gamma^3{}_{13} = \Gamma^3{}_{31} &= r^{-1}, \\
\Gamma^3{}_{23} = \Gamma^3{}_{32} &= \mathrm{ctg}\,\vartheta, \\
\Gamma^5{}_{15} = \Gamma^5{}_{51} &= \Phi', \\
\Gamma^0{}_{10} = \Gamma^0{}_{01} &= \nu', \\
\Gamma^2{}_{33} &= -\sin\vartheta\cos\vartheta, \\
\Gamma^1{}_{00} &= \mathrm{e}^{-2\lambda+2\nu}\,\nu', \\
\Gamma^1{}_{11} &= \lambda', \\
\Gamma^1{}_{22} &= -\mathrm{e}^{-2\lambda}\,r, \\
\Gamma^1{}_{33} &= -\mathrm{e}^{-2\lambda}\,r\sin^2\vartheta, \\
\Gamma^1{}_{55} &= -\mathrm{e}^{-2\lambda+2\Phi}\,\Phi',
\end{aligned}$$

$$R_{ik} = R^l{}_{ilk} = \Gamma^l{}_{ik,l} - \Gamma^l{}_{il,k} + \Gamma^n{}_{ik}\Gamma^l{}_{nl} - \Gamma^n{}_{il}\Gamma^l{}_{nk},$$

$$\begin{aligned}
R_{00} &= e^{-2\lambda+2\nu}\left[-\nu'' - \nu'^2 + \nu'\lambda' - \nu'\Phi' - \frac{2\nu'}{r}\right] \\
R_{11} &= \nu'' - \nu'^2 - \nu'\lambda' - \lambda'\Phi' + \Phi'^2 + \Phi'' - \frac{2\lambda'}{r} \\
R_{22} &= e^{-2\lambda}(1 - r\lambda' + r\nu' + r\Phi') - 1 \\
R_{33} &= R_{22}\cdot\sin^2\theta \\
R_{55} &= e^{-2\lambda+2\nu}\left[\nu'\Phi' - \lambda'\Phi' + \Phi'' + \Phi'^2 + \frac{2\Phi'}{r}\right]
\end{aligned}$$

- Ricci scalar:

$$R = 2\,\mathrm{e}^{-2\lambda}\left[-\nu'' - \nu'^2 - \Phi'' - \Phi'^2 + \nu'\lambda' + \Phi'\lambda' - \nu'\Phi' + 2\frac{\lambda' - \nu' - \Phi'}{r} - \frac{1}{r^2}\right] + \frac{2}{r^2}.$$

EoS for the 1_C extra dimension

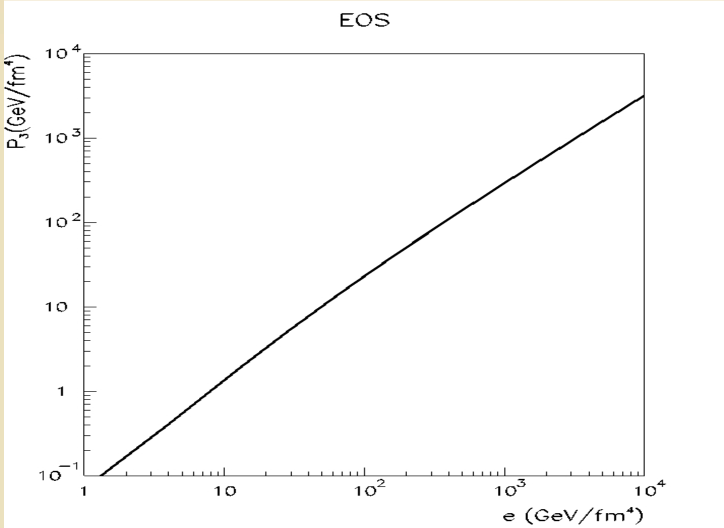
- EoS for the normal matter

$$\widetilde{\epsilon} = \frac{\widetilde{g}}{(2\pi)^4}\int_0^{\widetilde{k_F}} \widetilde{\varepsilon}\, \mathrm{d}^4\widetilde{\mathbf{k}}\bigg|_{T=0} =$$

$$= \frac{\widetilde{g}}{16\pi^3 b}\sum_{\kappa=0}^{1}\left[\widetilde{\mu}\,\sqrt{\widetilde{\mu}^2-\widetilde{m}^2}\left(\widetilde{\mu}^2-\frac{1}{2}\widetilde{m}^2\right)+\frac{\widetilde{m}^4}{2}\ln\left|\frac{\widetilde{m}}{\widetilde{\mu}+\sqrt{\widetilde{\mu}^2-\widetilde{m}^2}}\right|\right]$$

$$\widetilde{p} = -\frac{1}{2\pi b}\frac{\partial\widetilde{\Omega}}{\partial V}\bigg|_{T=0} =$$

$$= \frac{\widetilde{g}}{48\pi^3 b}\sum_{\kappa=0}^{1}\left[\widetilde{\mu}\,\sqrt{\widetilde{\mu}^2-\widetilde{m}^2}\left(\widetilde{\mu}^2-\frac{5}{2}\widetilde{m}^2\right)-\frac{3}{2}\widetilde{m}^4\ln\left|\frac{\widetilde{m}}{\widetilde{\mu}+\sqrt{\widetilde{\mu}^2-\widetilde{m}^2}}\right|\right].$$



- EoS for the extra D

  Excited states of n → n*

  EoS getting softer

$$\widetilde{p}_5 = -\frac{1}{2\pi V}\frac{\partial\widetilde{\Omega}}{\partial b}\bigg|_{T=0} = \frac{\widetilde{g}}{2\pi}\sum_{\kappa=0}^{1}\frac{\partial}{\partial b}\widetilde{f}(\widetilde{m},\widetilde{\mu}) =$$

$$= \frac{\widetilde{g}}{48\pi^3 b^3}\left[\widetilde{\mu}\sqrt{\widetilde{\mu}^2-\widetilde{m}^2}\left(5\frac{\widetilde{\mu}^2}{\widetilde{m}^2}-11\right)+6\widetilde{m}^2\ln\left|\frac{\widetilde{m}}{\widetilde{\mu}+\sqrt{\widetilde{\mu}^2-\widetilde{m}^2}}\right|\right]$$