A Wigner-idő.
[The Wigner time.]

VARRÓ Sándor, WFK

2017. November 15,
MTA Székház, Díszterem

$\tau = 2\hbar d\eta / dE$

A Wigner-idő definíciója, késés kvantummechanikai szórásnál. [The definition of the Wigner time, delay during quantum-mechanical scattering.]

Analógia az electromágneses hullámok terjedésénél fellépő késéssel. [Analogy with the delay appearing in the propagation of electromagnetic waves (in dispersive media).]

A Wigner-idő néhány általánosítása. [Some generalization of the Wigner time.]

Mai kísérleti eredmények az attoszekundumos késésre fotoionizáció esetén. [Recent experimental results on the attosecond delay in photoionization.]

The 'Eisenbud-Wigner time delay'. [1948, 1955].

PHYSICAL REVIEW VOLUME 98, NUMBER 1 APRIL 1, 1955

Lower Limit for the Energy Derivative of the Scattering Phase Shift

EUGENE P. WIGNER
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

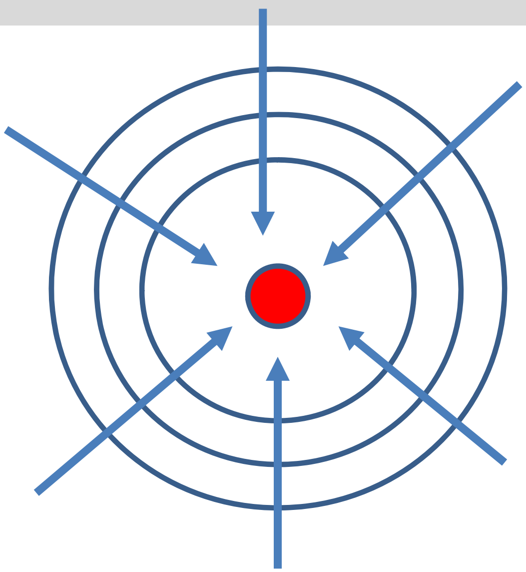
(Received December 10, 1954)

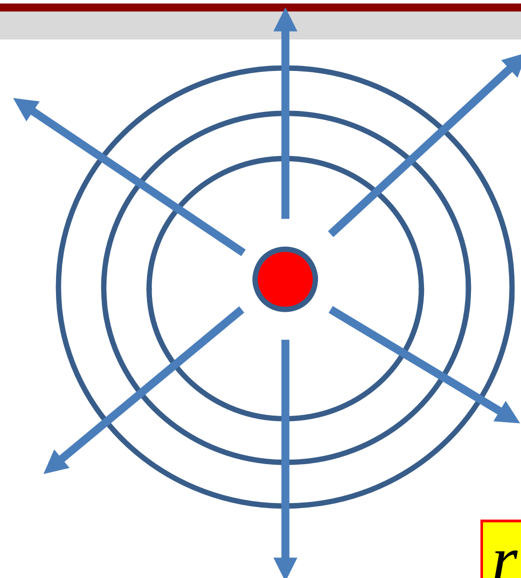
It is shown that the derivative of the scattering phase shift with respect to energy, $d\eta/dE$, must exceed a certain limit if the interaction of scattered particle and scatterer vanishes beyond a certain distance. This limitation of $d\eta/dE$ is, fundamentally, a consequence of the principle of causality; it is derived, however, from a property of the derivative matrix R.

>>The cross section and its angular dependence, as a function of energy, do not seem to determine in general the phase shifts uniquely. It may be useful, therefore, to derive certain general rules about the energy dependence of phase shifts... The relation to be derived here are based, fundamentally, on what has come to be called „the principle of causality". It states that the scattered wave cannot leave the scatterer befor the incident wave has reached it...
Before carrying out the very simple calculation, the general nature of the result will be illustrated by means of Eisenbud's interpretation of the energy derivative of the phase shift as a time delay (3)....<<

Wigner E P 1955, Lower limit for the energy derivative of the scattering phase shift. Physical Review 98(1), 145-147 (1955). References: (2) Wigner E P and Eisenbud L, Phys. Rev. 72, 29 (1947). (3) Eisenbud L, Dissertation, Princeton, June 1948 (unpublished).

The energy derivative of the phase shift (2 h dη / dE) as delay (τ).





>>One sees that the outgoing wave is retarded by a stretch 2dη/dk; it arrives at the point r–2dη/dk at the time it would have arrived at r without the action of the scattering center.<<

$$r = -2(d\eta/dk) + (d\nu/dk)\cdot t$$

$$\psi_{inc} = r^{-1}[e^{-i(k+k')r-i(\nu+\nu')t} + e^{-i(k-k')r-i(\nu-\nu')t}]$$

$$\psi_{out} = r^{-1}[e^{-i(k+k')r-i(\nu+\nu')t+2i(\eta+\eta')} + e^{-i(k-k')r-i(\nu-\nu')t+2(\eta-\eta')}]$$

$$r = -(\nu'/k')t = -(d\nu/dk)\cdot t$$

Wigner-idő: $\tau = 2(d\eta/d\nu) = 2\hbar d\eta/dE$

Wigner E P 1955, Lower limit for the energy derivative of the scattering phase shift. Physical Review 98(1), 145-147 (1955). (References: (2) Wigner E P and Eisenbud L, Phys. Rev. 72, 29 (1947). (3) Eisenbud L, Dissertation, Princeton, June 1948 (unpublished).

An example for the Wigner delay, derived from the Breit-Wigner amplitude.

S: Scattering matrix

$$S = e^{2i\delta}$$

S expressed by the T matrix

$$S = 1 + 2iT$$

Breit-Wigner resonance form.

$$T = \frac{\Gamma/2}{E_R - E - i\Gamma/2}$$

$$\Delta t(E) = \frac{\hbar\Gamma}{(E - E_R)^2 + \Gamma^2/4}$$

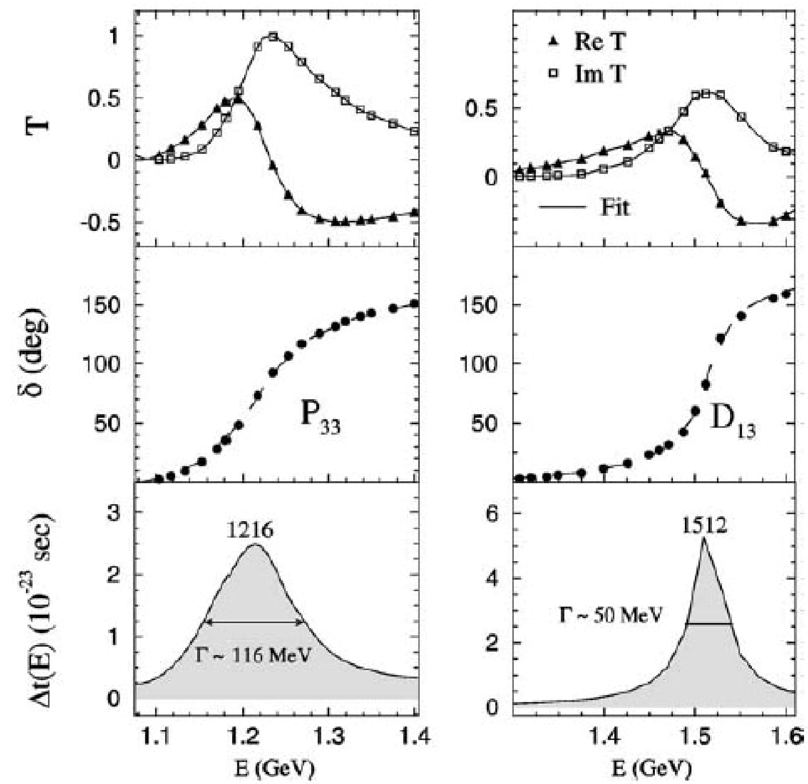


Fig. 1. Single energy values of the real part of the *T -matrix (filled triangles), imaginary part of T (open squares),* phase shifts (filled circles) and the time delay *t evaluated in the P33 and D13 partial waves of πN elastic* scattering. The time delay is evaluated using the *T -matrix given by the solid lines which fit the single energy* values very well.

N.G. Kelkar, *M. Nowakowski , K.P. Khemchandani and* S.R. Jain , Time delay plots of unflavoured baryons. Nuclear Physics A 730 (2004) 121–140.

Planck (1900) and Laue (1905): Wave propagation. Group velocity. Delay.

4. *Ueber irreversible Strahlungsvorgänge;*
von Max Planck.

(Nach den Sitzungsber. d. k. Akad. d. Wissensch. zu Berlin vom 4. Februar 1897, 8. Juli 1897, 16. December 1897, 7. Juli 1898, 18. Mai 1899 und nach einem auf der 71. Naturf.-Vers. in München gehaltenen Vortrage für die Annalen bearbeitet vom Verfasser.)

5. *Die Fortpflanzung der Strahlung in dispergierenden und absorbierenden Medien;*
von M. Laue.[1])

$$Z(t) = \int d\nu \; C_\nu \; e^{i(2\pi\nu t - \vartheta_\nu)}$$

$$\frac{1}{2\pi} \sum_{\text{interfaces}} \left(\frac{d\varphi_\nu}{d\nu} \right)_0$$

$$I_0'(t) = e^{-\kappa_0 x} I_0 \left\{ t - \frac{x}{c} \left[\frac{d(\nu \cdot n_\nu)}{d\nu} \right]_0 \right\}$$

Planck M 1900 Irreversible Strahlungsvorgänge *Annalen der Physik (4)* 1, 69-123 (1900)

Laue M von 1905 Fortpflanzung der Strahlung in dispergierenden und absorbierenden Medien *Annalen der Physik* (4) 18, 523-566 (1905)

Sommerfeld and Brillouin [1907-1914]: on the concept of group velocity.

1914. № 10.

ANNALEN DER PHYSIK.

VIERTE FOLGE. BAND 44.

1. *Über die Fortpflanzung des Lichtes in dispergierenden Medien;* *von A. Sommerfeld.*

§ 1. Einleitung und Ergebnisse.

Die vorliegende Untersuchung, über deren Resultate ich schon auf der Dresdener Naturforscherversammlung[1]) berichtet habe, ist ein gekürzter Abdruck einer Abhandlung, die in der Festschrift zum 70. Geburtstage von Heinrich Weber veröffentlicht ist.[2]) Den Anlaß zu ihrer Neubearbeitung bildete die

1) Unter dem Titel: Ein Einwand gegen die Relativtheorie der Elektrodynamik und seine Beseitigung (Phys. Zeitschr. 8. p. 841. (1907) und Beiblätter 33. p. 413).

„Well, on the basis of the above results, there is no difficulty at all. The front of the signal under any circumstances propagates with the velocity of light in vacuum, c; the main part of the energy propagates necessarily with a lower velocity. This, according to Mr. Brillouin, equals in general with the group velocity, except for the vicinity of the absorption band, in the region of anomalous dispersion. Here, the group velocity as a signal velocity loses its meaning."

Sommerfeld A 1914 Über die Fortpflanzung des Lichtes in dispergierenden Medien *Annalen der Physik* 44 177-202 (1914)

Sommerfeld [1914] : 'Superluminar propagation' and 'Precursors' ['Vorläufer'].

182 *A. Sommerfeld.*

gungen in 2π Zeiteinheiten), k die „Wellenzahl" (Anzahl der Wellenlängen auf 2π Längeneinheiten), V die Phasengeschwindigkeit, U die Gruppengeschwindigkeit bei der Frequenz n und sieht man von der Absorption ab, setzt also k als reell voraus, so ist bekanntlich

$$V = \frac{n}{k}, \qquad U = \frac{dn}{dk}$$

wofür man auch schreiben kann

$$U = \frac{d(Vk)}{dk} = V + k\frac{dV}{dk} = V - \lambda\frac{dV}{d\lambda}.$$

Bei anomaler Dispersion $dV/d\lambda < 0$ wird hiernach $U > V$. Ist also V bereits größer als c so würde ein Fortschreiten des Signals mit der Gruppengeschwindigkeit U erst recht zu einer Überlichtgeschwindigkeitswirkung führen, die relativtheoretisch unmöglich ist.

„Here the group velocity as a signal velocity looses its meaning; the constructed relativistic difficulties are based on the overestimation of the notion of group velocity, in comparison with the wave velocity, which is usually called „the velocity of light."

$$\xi = \omega_p^2 x / 8\pi \cdot c$$

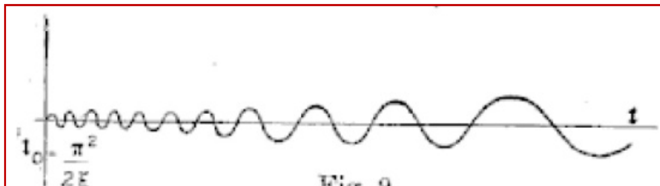


Fig. 9.

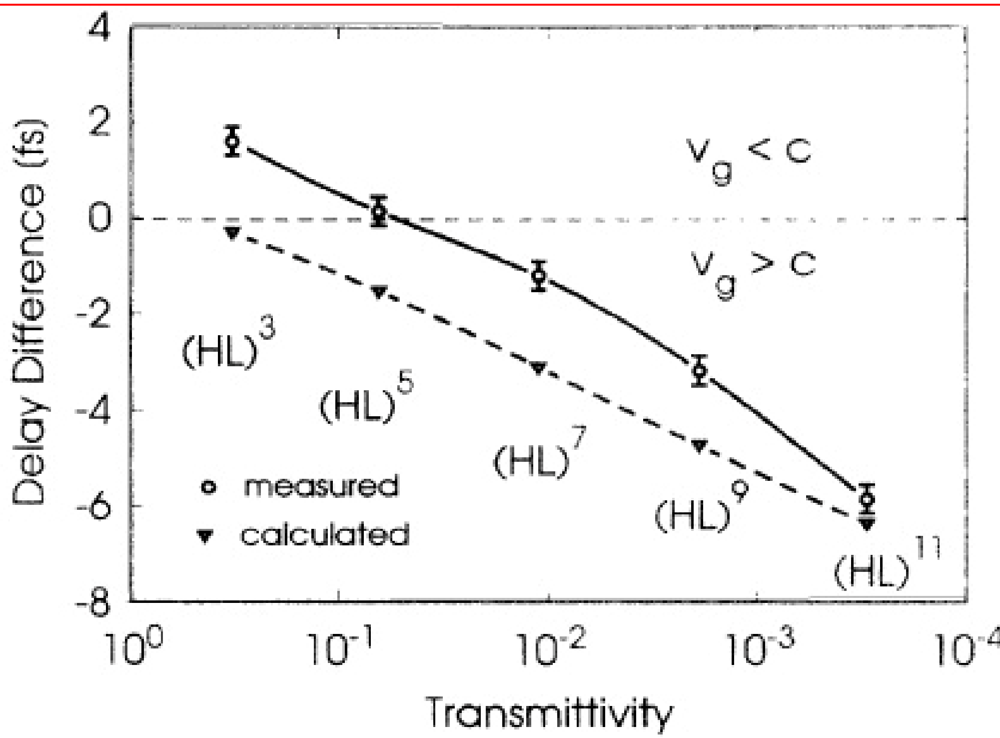
J_1 allmählich an, wie in der schematischen Fig. 9 angedeutet, wobei indessen im Auge zu behalten ist, daß

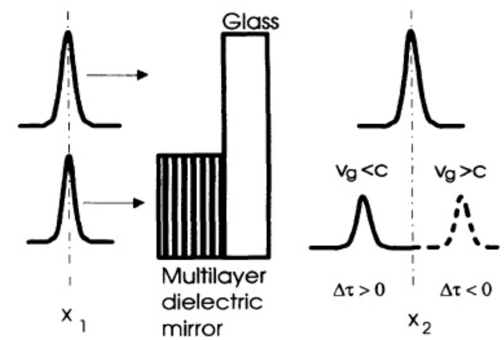
$$f(x,t) = \frac{2\pi}{\tau}\sqrt{\frac{\mathrm{t}}{\xi}}\, J_1(2\sqrt{\mathrm{t}\xi})$$

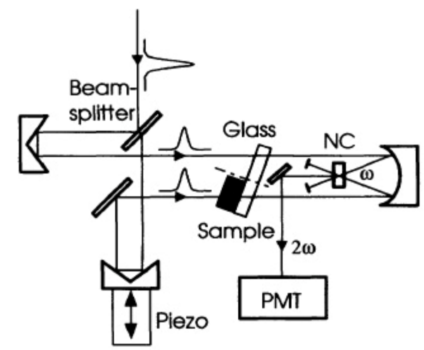
$$\mathrm{t} = t - x/c$$

Sommerfeld A 1914 Über die Fortpflanzung des Lichtes in dispergierenden Medien *Annalen der Physik* 44 177-202 (1914)

Delay and „superluminar propagation” by photonic band-gap crystals. [1994].







(b)

FIG. 3. Measured and calculated (for an angle of incidence of 20°) difference $\Delta\tau$ between the tunneling time and the corresponding vacuum time for multilayer dielectric coatings of various thicknesses.

Spielmann Ch, Szipőcs R, Stingl A and Krausz F 1994 Tunneling of optical pulses through photonic band gaps. *Phys. Rev. Lett.* 73 2308-2311 (1994).

"Lifetime matrix" : F. T. Smith [1960]

PHYSICAL REVIEW VOLUME 118, NUMBER 1 APRIL 1, 1960

Lifetime Matrix in Collision Theory*

FELIX T. SMITH
Stanford Research Institute, Menlo Park, California

(Received October 16, 1959)

The duration of a collision is usually a rather ill-defined concept, depending on a more or less arbitrary choice of a collision distance. If the *collision lifetime* is defined as the limit, as $R\to\infty$, of the difference between the time the particles spend within a distance R of each other and the time they would have spent there in the absence of the interaction, a well-defined quantity emerges which is finite as long as the interaction vanishes rapidly enough at large R.

In quantum mechanics, using steady-state wave functions, the average time of residence in a region is the integrated density divided by the total flux in (or out), and the lifetime is defined as the difference between these residence times with and without interaction. Transformation properties require construction of the *lifetime matrix*, $\mathbf{Q}$. If the wave functions ψ_i are normalized to unit total flux in and out through a sphere at $R\to\infty$, the matrix elements are

$$Q_{ij}=\lim_{R\to\infty}\left[\int^{r<R}\psi_i\psi_j^*d\tau-R(v_i^{-1}\delta_{ij}+\Sigma_k S_{ik}v_k^{-1}S_{jk}^*)\right]_{\mathrm{Av}},$$

where the average value is taken to eliminate oscillating terms at large R, S_{ik} is an element of the unitary scattering matrix, $\mathbf{S}$, and v_i is the velocity in the ith channel. $\mathbf{Q}$ is Hermitian; a diagonal element Q_{ii} is the average lifetime of a collision beginning in the ith channel. As a function of the energy $\mathbf{Q}$ is related to $\mathbf{S}$: $\mathbf{Q}=-i\hbar\mathbf{S}\,d\mathbf{S}^\dagger/dE$; $\mathbf{Q}$ and $\mathbf{S}$ contain the same information, from complementary points of view. When $\mathbf{Q}$ is diagonalized, its proper values, q_{ii}, are the lifetimes of metastable states if they are large compared to $\hbar/E$; for a sharp resonance, the measured lifetime is the average of $q_{ii}(E)$ over a distribution in energy. The corresponding eigenfunctions, Ψ_i, are the proper functions to describe these metastable states. The causality principle appears directly from an inequality involving the integral expression for Q_{ii} or q_{ii}, and it is shown how some of its consequences for inelastic collisions can be deduced.

$$Q=-i\hbar S\cdot dS^{+}/dE$$

Smith F T 1960 Lifetime matrix in collision theory *Phys. Rev.* 118 349-356 (1960).

Lippmann's derivation of the delay from a 'time operator' [1966].

SECOND SERIES, VOL. 151, NO. 4 — 25 NOVEMBER 1966

Operator for Time Delay Induced by Scattering*

B. A. LIPPMANN
DRC Incorporated, Santa Barbara, California
(Received 6 July 1966)

The operator that gives the time delay induced by a scattering process is exhibited explicitly.

$$\tau = \frac{m}{2p}\left(r_p + \frac{i\hbar}{p}\right) + \left(r_p + \frac{i\hbar}{p}\right)\frac{m}{2p}$$

$$r_p = \frac{1}{p}(\mathbf{p}\cdot\mathbf{r})$$

$$\tau(t) = e^{iH_0 t}\tau(0)e^{-iH_0 t} = t + \tau(0)$$

$$S = \exp[2i\delta(E)]$$

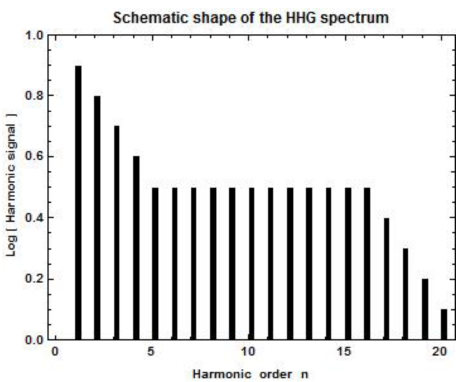
$$\left\langle \Psi_a(0) \middle| e^{iH_0 t}\tau(0)e^{-iH_0 t}\Psi_a(0)\right\rangle = t + \left\langle \phi_a(0) \middle| S^{-1}\tau(0)S\phi_a(0)\right\rangle$$

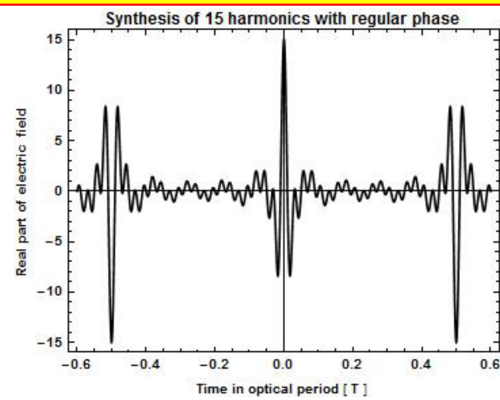
$$\left\langle \Psi_a(t) \middle| \tau(0)\Psi_a(t)\right\rangle = t - 2\frac{\partial\delta(E)}{\partial E} + \left\langle \phi_a(0) \middle| \tau(0)\phi_a(0)\right\rangle$$

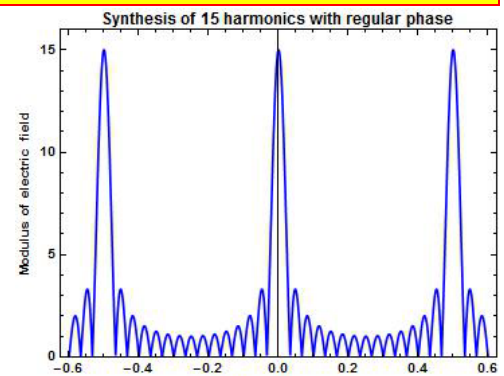
Lippmann B A 1966 Operator for time delay induced by scattering *Phys. Rev*. 82 664-679 (1951) 151 1023-1024.

Phase-locking of the higher-harmonic components [Farkas & Tóth (1992)]

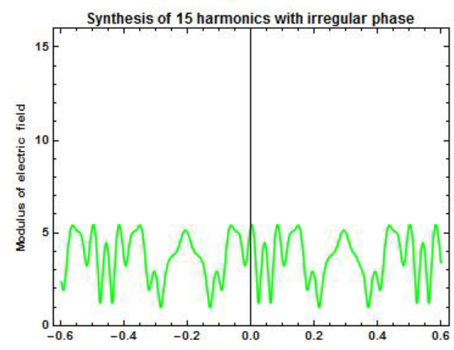
$$E(t) = \sum_{n=n_1=2k_1+1}^{n=n_2=2k_2+1} A_n e^{i\phi_n} e^{-2\pi i n(t/T)} \approx A_{n_1} \sum_{n=n_1}^{n=n_2} e^{-2\pi i n(t/T)+i\phi_n}. \quad (5.12)$$





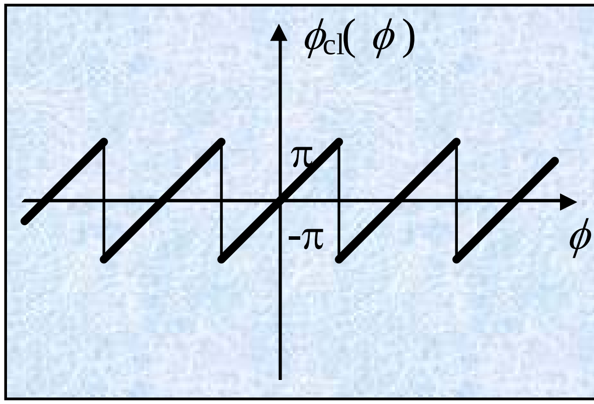


Synthesis of N = 15 higher-harmonics of the same intensity at the plateau region, according to (5.12); Real part, imaginary part and modulus with phase-locking. In the last figure the phase difference of the components are not constant (random).

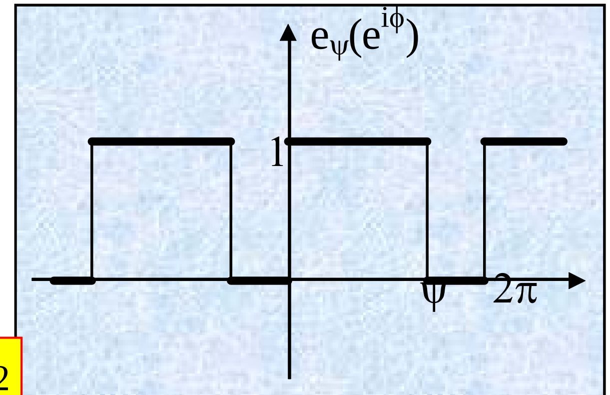


Farkas Gy and Tóth Cs, Proposal for attosecond light pulse generation using laser-induced multiple-harmonic conversion processes in rare gases. *Phys. Lett. A* 168, 447 (1992). [Also: SV and Farkas Gy : Attosecond electron pulses from interference of above-threshold de Broglie waves. *Laser and Particle Beams* 26, 9 (2008).]

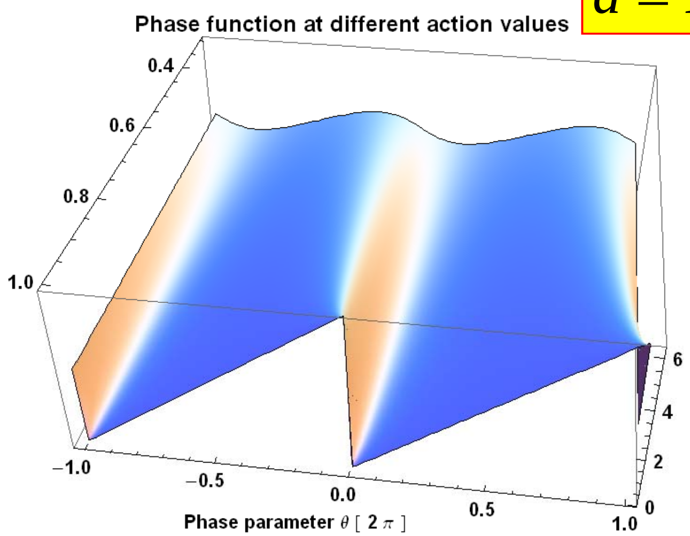
Detour. The question of quantum-mechanical phase uncertainties, 'absolut phase' and the 'phase projectors'. A recently proposed construction: SU(1,1) sampling states, generated by a parametric process. For larger intensities the 'sampling is sharper'.

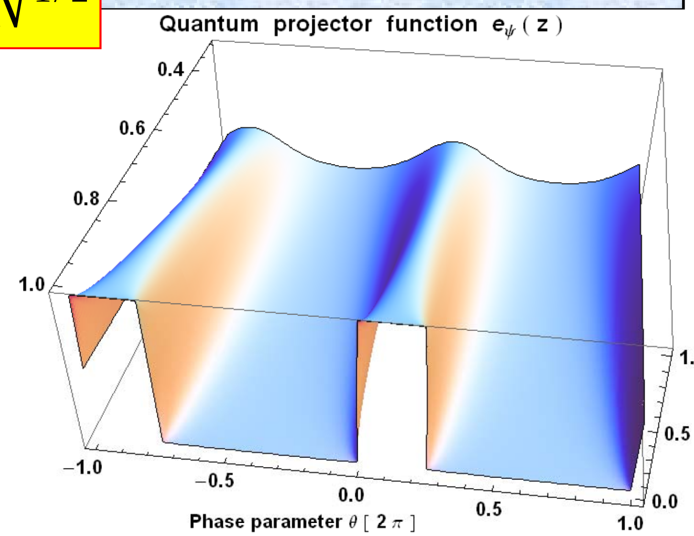


$$z = e^{i\varphi}\sqrt{z^* z} = |z|\, e^{i\varphi}$$



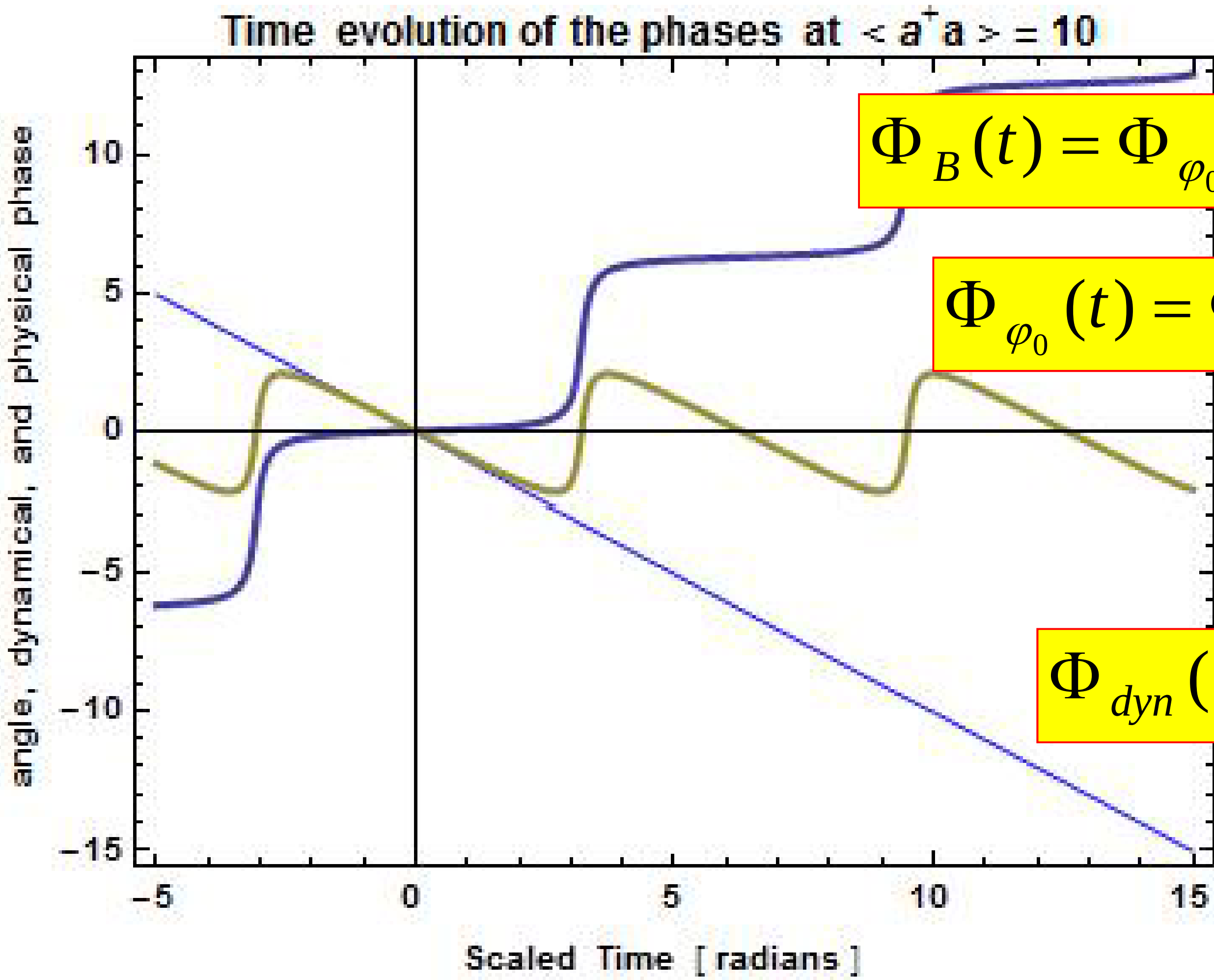
$$\hat{a} = \hat{E}\sqrt{\hat{a}^+ \hat{a}} \neq e^{i\hat{\Phi}}\, \hat{N}^{1/2}$$





Figures taken from: S. V., Regular phase operator and SU(1,1) coherent states of the harmonic oscillator. *Physica Scripta* 90 (7), 074053 (2015). Figs. 2-3.

Detour on the time evolution of the physical phase of a ‘quantum clock’. p.1



Unbounded

$$\Phi_B(t) = \Phi_{\varphi_0+\omega t}(0) - \Phi_{\varphi_0}(0)$$

$$\Phi_{\varphi_0}(t) = \Phi_{dyn}(t) + \Phi_B(t)$$

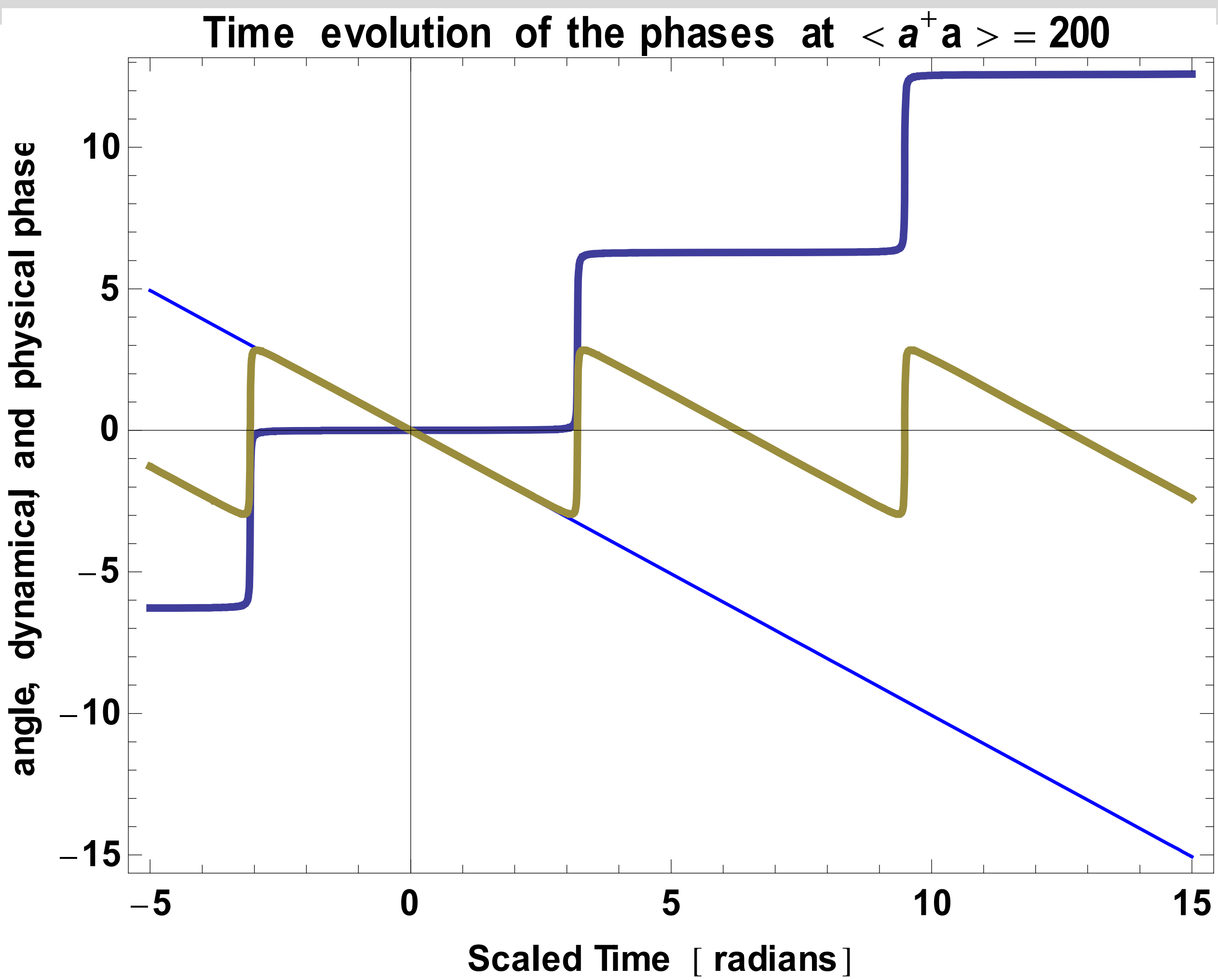
Sum: Periodic

$$\Phi_{dyn}(t) = \Phi_{\varphi_0}(0) - \omega t$$

Unbounded

S. V., *Physica Scripta* 90 (7), 074053 (2015). The quantum phase of the photon a Haar integral on the Blaschke group.
[Seminar 7 on Quantum Information of LPHYS-16, 11-15 July 2016]

Detour on the time evolution of the physical phase of a ‘quantum clock’. p.2



Unbounded

Sum: Periodic

Unbounded

S. V., *Physica Scripta* 90 (7), 074053 (2015). The quantum phase of the photon a Haar integral on the Blaschke group.
[Seminar 7 on Quantum Information of LPHYS-16, 11-15 July 2016]

~ 21 attosecond delay of photoelectrons from different initial states. (2010)

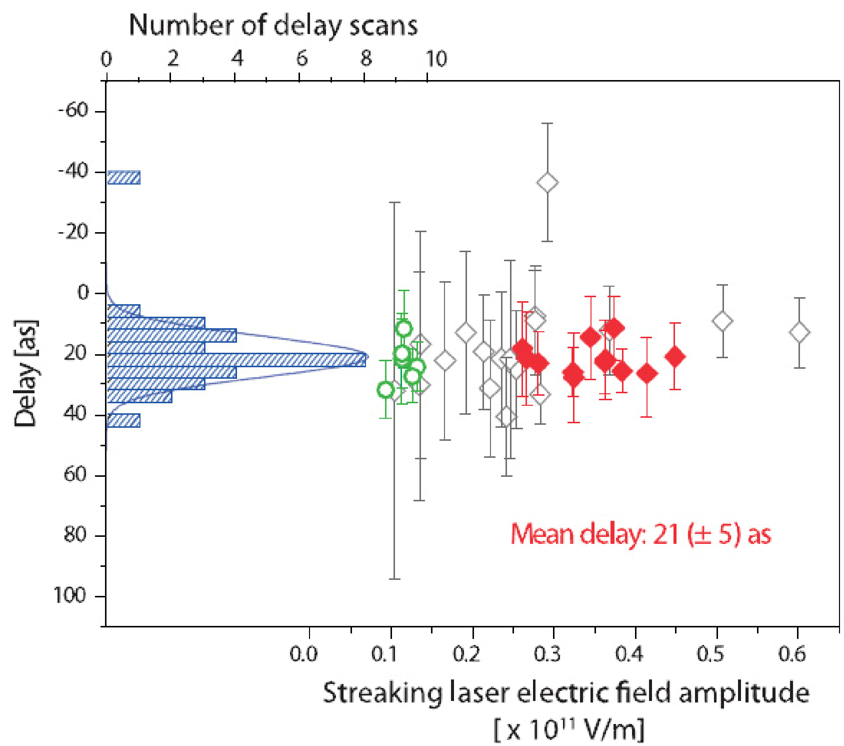
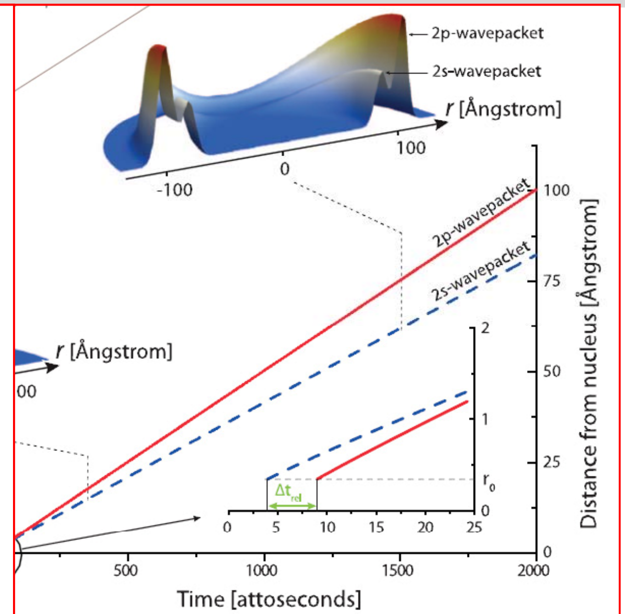


Fig. 3. The relative delay between photoemission from the 2*p* and 2*s* subshells of Ne atoms, induced by sub–200-as, near–100-eV XUV pulses. The depicted delays are extracted from measured attosecond streaking spectrograms by fitting a spectrogram, within the strong-field approximation, with parameterized NIR and XUV fields. Our optimization procedure matches the first derivatives along the time delay



2p gyorsabb, de később emittálódik.
2s lassabb, de előbb emittálódik.

$$\Phi = Z\log(2kr)/k + \delta_l(k)$$

$$\tau = \hbar \partial \delta_l(E)/\partial E \ \ ?$$

$$\delta_l(k) = \arg\{\Gamma(l+1-iZ/k)\}$$

Schultze M, Fieß M, Karpowicz N *et al* 2010 Delay in photoemission. *Science* 328, 1658–1662 (2010).

Measurement of the delay caused by the continuum-continuum transitions. (2011).

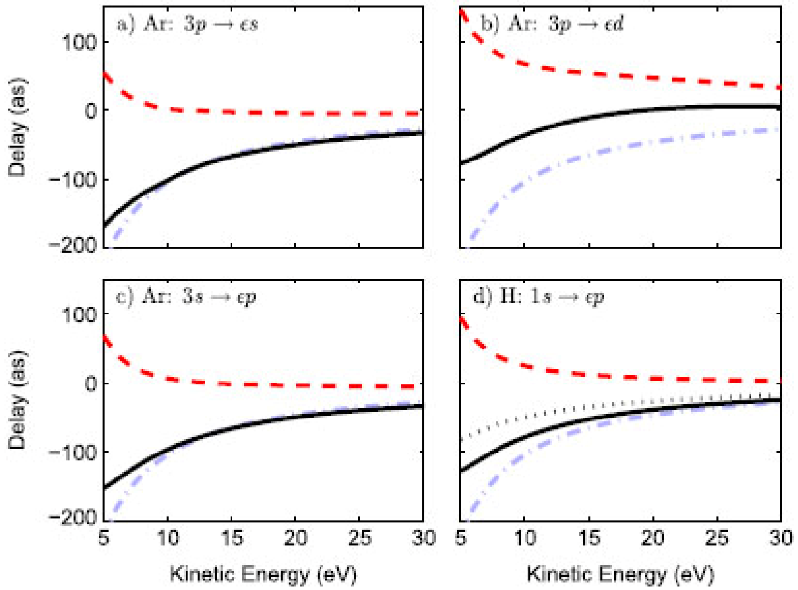


FIG. 3 (color online). Computed delays associated with the following ionization channels: (a) $3p \to \epsilon s$, (b) $3p \to \epsilon d$, (c) $3s \to \epsilon p$ in Ar, and (d) $1s \to \epsilon p$ in H. The dashed lines (red) are the one-photon Wigner time delays. The dash-dotted lines (blue) represent the estimated delays induced by the measurement τ_{cc}. The sum of the two delays is shown as a solid line (black). The dotted line (black) in (d) is the result of an exact calculation in H.

PRL 106, 143002 (2011) PHYSICAL REV

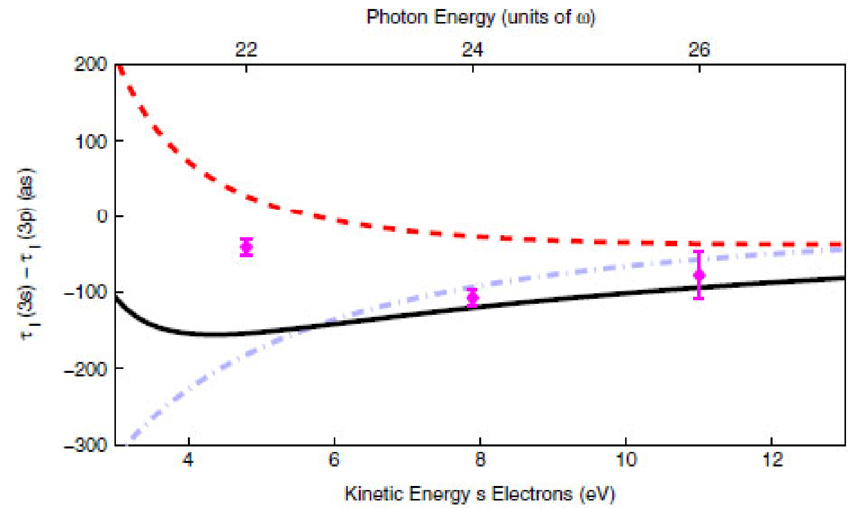


FIG. 4 (color online). Comparison between the measured delay differences for ionization of Ar from the $3s$ and $3p$ shells (diamonds) with calculations performed according to the approximate theory developed in this work (solid black line). Also shown is the delay expected for one-photon ionization (dashed red line) and the laser-driven continuum-continuum transition (dash-dotted blue line).

$$\tau_I = \tau_W + \tau_{cc}$$

Klünder K, Dahlström J M, Gisselbrecht M *et al* 2011 Probing single-photon ionization on attosecond time scale *Phys. Rev. Lett.* 106 143002 (2011). Continuum – continuum transitions are also significant.

Measurement of an ‘effective’ Wigner delay in photoionization of noble gases. [2014].

J. Phys. B: At. Mol. Opt. Phys. **47** (2014) 245003 C Palatchi *et al*

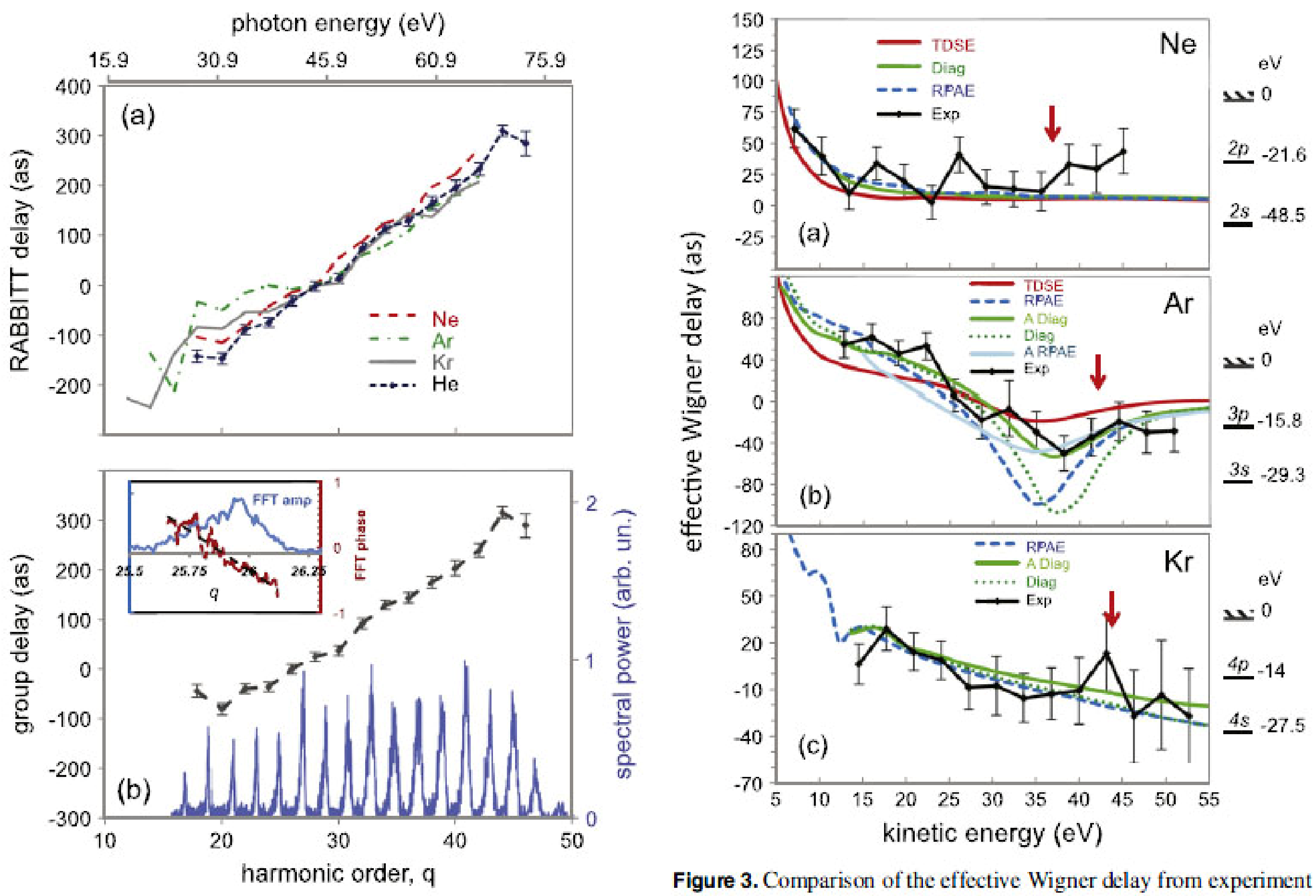


Figure 3. Comparison of the effective Wigner delay from experiment

>>...The extracted delays are compared with several theoretical predictions and the results are consistent within 30 as over the energy range from 10 to 50 eV. An ‘effective’ Wigner delay over all emission angles is found to be more consistent with our angle-integrated measurements near Cooper minimum..<<

Palatchi C, Dahlström J M, Kheifets A S, Ivanov I A, Canaday D M, Agostini P and DiMauro L F, Atomic delay in helium neon argon and krypton. J. Phys. B: At. Mol. Opt. Phys. 47 (2014) 245003 (7pp).

A historical remark on the quantum-mechanical theory of photoelectric effect [1926-30]

Zur Theorie des photoelektrischen Effekts.

Von G. Wentzel in Leipzig.

Mit 1 Abbildung. (Eingegangen am 19. November 1926.)

Über den Photoeffekt in der K-Schale der Atome, insbesondere über die Voreilung der Photoelektronen

Von A. Sommerfeld und G. Schur

Über die nichtstationäre Behandlung des Photoeffekts

Von H. Bethe

$$(14)\quad \begin{cases} u = -i\dfrac{Ahe}{4mc}\cdot\dfrac{1}{r}\cdot N(W_0)\,A^{+}_{W_0}\,e^{-\frac{2\pi i}{h}(W_0+m_0c^2)t+i(k_0r+\gamma_0\lg k_0r)} & \text{für } r < v_0 t \\ u = 0 & \text{für } r > v_0 t. \end{cases}$$

Wentzel G 1926 Zur Theorie des photoelektrischen Effekts *Zeitschrift für Physik* 40 574-589 (1926).
Wentzel G 1927 Über die Richtungsverteilung der Photoelektronen *Zeitschrift für Physik* 41, 828- (1927).
Sommerfeld A, 1929 *Atombau und Spektrallinien. Wellenmechanische Ergänzungsband* (Druck und Verlag von Friedr. Vieweg & Sohn Akt.-Ges., Braunschweig, 1929). Kapitel II. §4. Photo-effekt .
Sommerfeld A und Schur G, 1930 Über den Photoeffekt in der K-Schale..., *Ann. der Physik* (5) 4, 409-
Bethe H 1930 Über die nichtstationare Behandlung des Photoeffekts *Annalen der Physik* (5) 4 443-449

Accumulation time? Decay time?
Retention time?

$$\tau = 2\hbar d\eta / dE$$