Universality in String Theory

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based on

1801.10126 [EH, Petr Vasko]

1611.00787,1604.03514 [EH,PLB]

1609.01385,1707.06609 [EH ,JHEP]

1210.3825,1205.5079 [EH, IY.Park,JHEP,NPB],

outline

- String Theory
- Motivations
- Effective action DBI+CS+WZ
- Exact Math Results
- Brane-Anti Brane
- Open questions & Conclusion

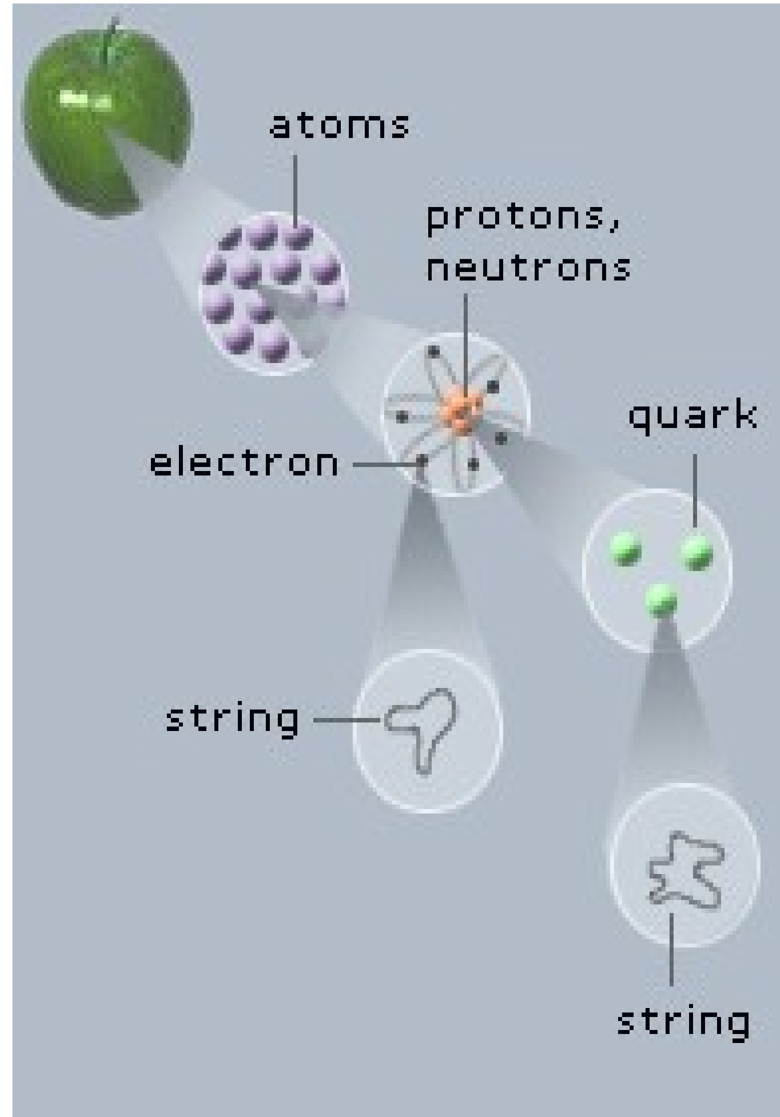
String Theory

A theory to unify Quantum Mechanics and GR

based on the postulate, the elementary constituents of matter are tiny one dimensional **(string-like)** objects instead of point particles

Every particle boils down to vibrating strings

The difference in vibration makes up each atom and gives them their properties

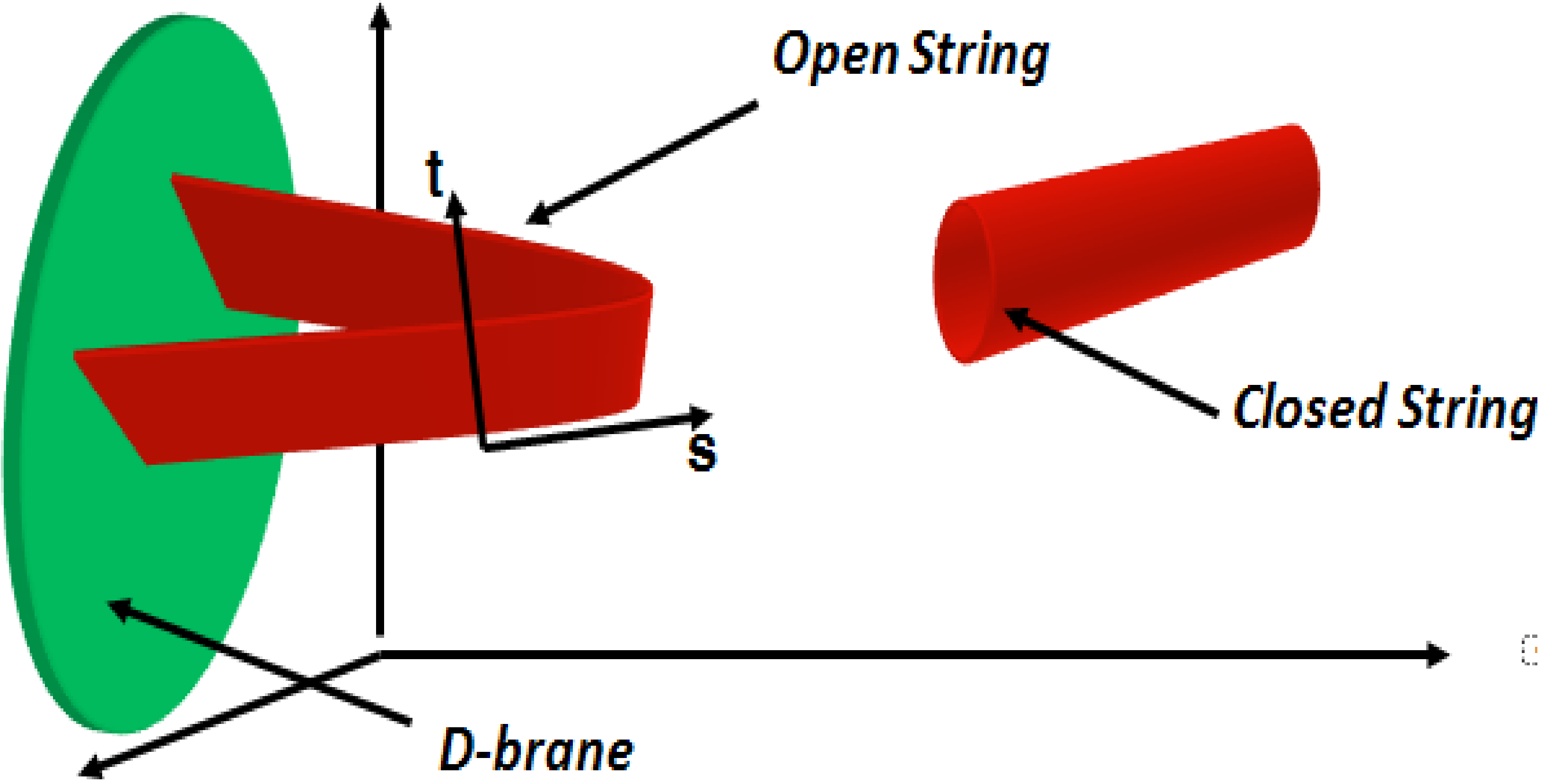


The elementary particles are excitation modes of elementary strings

There are 2 kinds of strings

Open, closed string

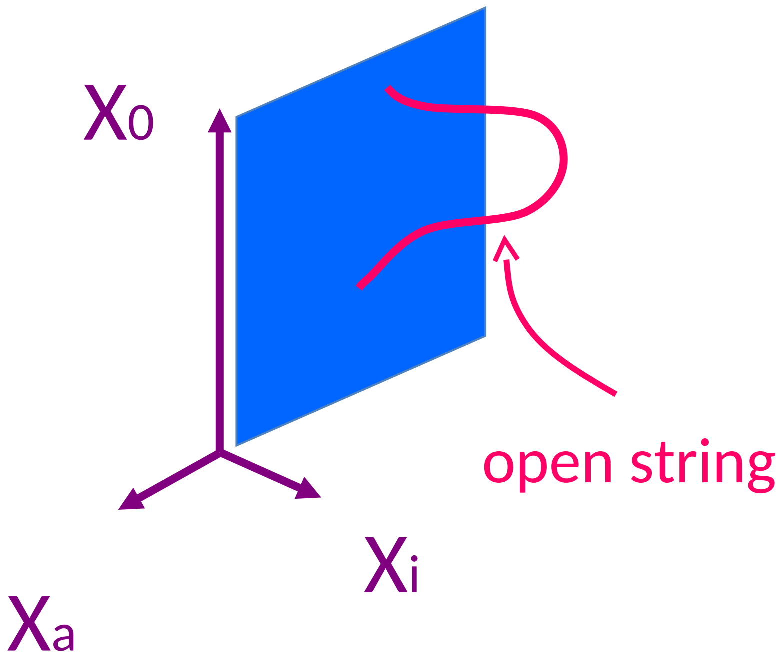
Dp-branes are fundamental non perturbative objects where the end points of open strings attached to them.


Open String
t
s
Closed String
D-brane

Dp-brane (BPS)

A p-dimensional extended object exist in bosonic ,IIA, IIB.

Open string endpoints stick to a D-brane are excitations on it.
To find mass spectrum of open strings we apply Dir. bc on Transverse dirs of D-branes and Neu. bc on WV dirs



$$X^i : a = p + 1, ...9, (D = 10)$$

BPS Dp-branes in II

IIA(IIB),includes BPS branes (stable) with even p (odd p) and the only difference with non BPS branes is the absence of Tachyon.

The charge of a Dp-brane is **Ramond-Ramond (p+1) field**

BPS branes have orientation, while anti branes carry anti charge and their orientation is reverse of BPS branes.

Dp-branes are the source of RR (p+1)-form fields in IIA, IIB. They are of high importance in both theory and phenomenology.

For stable Dp-branes **(p is even in IIA , odd in IIB)** preserve half of supersymmetry.

Stability, Supersymmetry, conserved (RR) charge and having no tachyons are, all properties of these type II branes.

Motivations

Motivated by Witten, based on string amplitudes
we talk about a universal conjecture holds for (BPS , non-BPS) branes

DBI,new WZ,Chern-Simons, Complete form of
D-brane-Anti-D-brane

Contact terms shed new light on understandings
of world-volume dynamics of branes.

Scattering of strings, we point out how to look for new effective actions of all D-branes

New EFT couplings with all their corrections

These new actions/couplings are neither inside Myers' terms nor within pull-back/ Taylor expansions.

Motivations

1) **Universality** for all-order α' corrections to BPS/non-BPS branes

2) It seems that, the description of world volume dynamics of D-brane is still lacking at some fundamental level

3) Holographic QCD Models, Cosmology,...

4) Given a close interplay between open, a closed string is behind AdS/CFT, amplitudes involving mixed of them should be especially worth studying.

5) Dealing with Dualities

6) Mathematical structures behind Scattering amplitudes
(world-sheet integrals)

The world-volume theory of a Dp-brane involves
a massless U(1) vector, $9 - p$ real massless
Scalars, fermions.

At leading order, the low-energy action is DBI.

There are higher $\alpha' = l_s^2$ order corrections.

When derivatives of the Field strength are small on string scale, the action takes BI form

If we consider N coincident Dp-branes, the U(1) symmetry of a Dp-brane gets enhanced to
non-abelian U(N).

The low energy action of Dp-branes consists of 2 parts. Born-Infeld and Chern-Simons

The action for constructing **non-abelian Dp-branes** is

$$S_{\mathrm{BI}} = -T_p \int d^{p+1}\sigma \, \mathrm{STr}\left(e^{-\phi}\sqrt{-\det\left(P\left[E_{ab} + E_{ai}(Q^{-1} - \delta)^{ij}E_{jb}\right] + \lambda F_{ab}\right)\det(Q^i{}_j)}\right),$$

with

$$E_{ab} = G_{ab} + B_{ab} \qquad , \qquad Q^i{}_j \equiv \delta^i{}_j + i\lambda\left[\Phi^i, \Phi^k\right]E_{kj},$$

where $\lambda = 2\pi\ell_s^2$, T_p is the brane tension, P[...] indicates pull-back of background metric and NSNS two-form (a,b = 0, .., 9), F_{ab} is the field strength of gauge field and STr(...) is symmetric trace prescription.

In order to find interactions expected from DBI , we expand the action and set all background field to zero, that is, working on flat empty space background

The second part is the Wess-Zumino action, contains the coupling of the U(N) massless
world volume vectors to RR field

Effective Field Theory on the World-Volume

The states in S-matrix are gauge ,scalars and tachyons from DBI action and RR field from
the WZ action

$$S_{CS} = \mu_p \int P\left[\sum C^{(n)}\, e^{B}\right] e^{2\pi\ell_s^2 F}$$

$$S_{WZ} = \mu_p \int \mathrm{STr}\,\left(P\left[e^{i\lambda\, i_\Phi i_\Phi}(\sum C^{(n)})\right] e^{\lambda F}\right) \; .$$

The scalar are transverse coordinate of the D-brane, These scalars appear in 3 different ways

1st: explicit appearance in the exponential

$$i_\Phi i_\Phi C^{(n)} = \frac{1}{2(n-2)!}[\Phi^i, \Phi^j]\, C^{(n)}_{ji\mu_3\cdots\mu_n} dx^{\mu_3} \cdots dx^{\mu_n},$$

2nd,covariant derivatives pull-back.

$$P[E]_{ab} = E_{ab} + \lambda\, E_{ai}\, D_b\Phi^i + \lambda\, E_{ib}\, D_a\Phi^i + \lambda^2\, E_{ij} D_a\Phi^i D_b\Phi^j\,,$$

3rd, the action includes derivatives of closed string fields through the Taylor expansion of these fields

$$\begin{aligned} G_{\mu\nu} &= \exp\left[\lambda\Phi^i\,\partial_{x^i}\right] G^0_{\mu\nu}(\sigma^a, x^i)|_{x^i=0} \\ &= \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} \Phi^{i_1}\cdots\Phi^{i_n}\,(\partial_{x^{i_1}}\cdots\partial_{x^{i_n}})G^0_{\mu\nu}(\sigma^a, x^i)|_{x^i=0}\,. \end{aligned}$$

non trivial interactions, we need 3O1C

Thus we expect from the field theory, the processes where two open string scalar states scatter to emit a massless virtual particle (gauge field)

which is going to be absorbed by a lower order world-volume interaction involving the RR field and gauge/scalar or even tachyons.

To study these effective actions, we use the
S-matrix method (Scattering amplitudes)

Using CFT, we evaluate the amplitudes to find
all closed string couplings to open strings
on Dp-branes

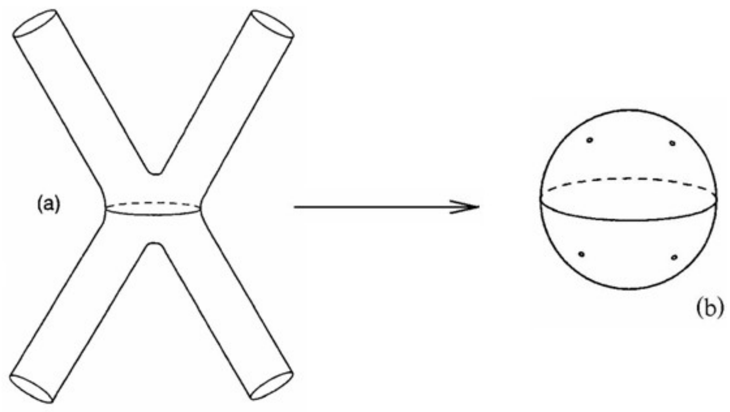
One narrates scattering of RR from a brane

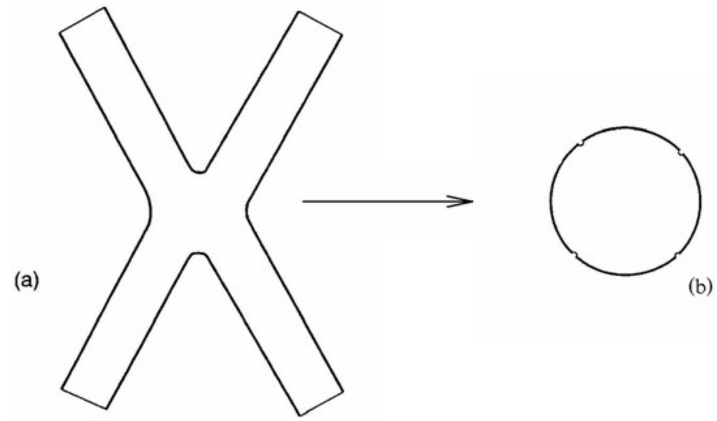
The string background is taken to be flat space, interactions of RR with a Dp-brane are described by world-sheets with boundary. Its boundary must be fixed to the surface at position of brane.

We consider Dir. bcs on the fields transverse to brane and Neumann bcs on fields along
world volume of brane

Four Point amplitude

We calculate scattering amplitudes of strings by CFT methods





From CFT, one evaluates the correlation functions of all fields

TTTT we consider a world-sheet with the topology of a disk with vertex operator insertions on its boundary

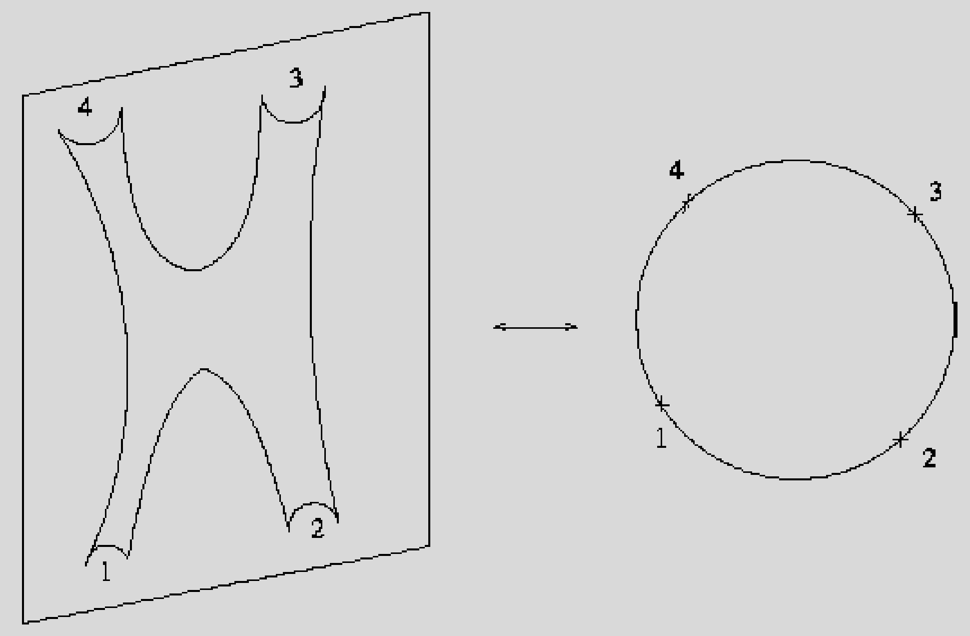


Fig. 1: World-sheet corresponding to the scattering of four open strings.

We can find effective action on the D-branes by their scattering

Vertex Operators

Vertex includes information about the properties of strings.
Form of every vertex is calculated by using **the conformal invariance of S-matrix**.

$$V_{RR}^{(-1)}(z,\bar{z}) \;=\; (P_-\not{H}_{(n)}M_p)^{\alpha\beta}e^{-\phi(z)/2}S_\alpha(z)e^{ip\cdot X(z)}e^{-\phi(\bar{z})/2}S_\beta(\bar{z})e^{ip\cdot D\cdot X(\bar{z})}\otimes\sigma_3\sigma_1$$

$$\not{H}_{(n)} = \frac{a_n}{n!}H_{\mu_1\ldots\mu_n}\gamma^{\mu_1}\ldots\gamma^{\mu_n}$$

$$V_A^{(0)}(x) \;=\; \xi_i\Big(\partial X^i(x) + 2ik\cdot\psi\psi^i(x)\Big)e^{2ik.X(x)}\lambda\otimes I \qquad k^2=0, \qquad k.\xi=0$$

λ is the external CP matrix in the U(N) group.

$$I=\int_{\mathcal{H}^+} d^2z|1-z|^a|z|^b(z-\bar{z})^c(z+\bar{z})^d,$$

where $d=0,1,2$ and a,b,c should be computed in terms of the Mandelstam variables. Since we are talking about disk level amplitude the integrations must be done on upper half plane. The necessary conditions for these integrals must be taken into account as

$$a+b+c\leq -2$$
$$a+b+d\leq -2$$

To remove integrals on x,y we may use the following definitions

$$|z|^b=\frac{1}{\Gamma(-\frac{b}{2})}\int_0^\infty du\, u^{-\frac{b}{2}-1}e^{-u|z|^2},$$
$$|1-z|^a=\frac{1}{\Gamma(-\frac{a}{2})}\int_0^\infty ds\, s^{-\frac{a}{2}-1}e^{-s|1-z|^2}.$$

where $z = x + iy$

$$I_y = \int_0^\infty dy\, y^c e^{-(s+u)y^2} = \frac{\Gamma(\frac{1+c}{2})}{2(s+u)^{\frac{1+c}{2}}},$$

$$F(\lambda) = \int_{-\infty}^\infty dx e^{-(s+u)x^2+2\lambda x} = \sqrt{\frac{\pi}{s+u}} e^{\frac{\lambda^2}{s+u}}.$$

Here we rewrite down the integration on x

$$I_x = 2^d \int_{-\infty}^\infty dx\, x^d e^{-s} e^{-(s+u)x^2+2sx} = 2^d e^{-s} \int_{-\infty}^\infty dx\, x^d e^{-(s+u)(x-\frac{s}{s+u})^2+\frac{s^2}{s+u}}.$$

so after all the integration on x will be appeared as

$$I_x = 2^d e^{-\frac{us}{s+u}} \int_{-\infty}^{\infty} dx\, x^d e^{-(s+u)(x-\frac{s}{s+u})^2} = e^{-s} \frac{d}{d^d\lambda} F(\lambda)|_{\lambda=s}.$$

$d = n, \;\; n \in Z,$

$$I_x = 2^d e^{-\frac{us}{s+u}} \frac{\sqrt{\pi}}{(s+u)^{\frac{1}{2}}} \begin{cases} 1 & , d = 0 \\ \frac{s}{(s+u)} & , d = 1 \end{cases}$$

For simplicity we just do the integration for $d = 0$ and finally we show our results for $d = 1, 2$. So for $d = 0$ after replacing those steps mentioned above and doing the integrals over x, y, collecting them and replacing in the general integration on I, we will have

$$I = \frac{\sqrt{\pi}(2i)^{c+1} 2^0 \Gamma(\frac{1+c}{2})}{2\Gamma(\frac{-a}{2})\Gamma(\frac{-b}{2})} \int_0^{\infty}\int_0^{\infty} \frac{dsdu}{(s+u)^{1+c/2}} u^{-b/2-1} s^{-a/2-1} e^{-\frac{us}{s+u}},$$

We might use the following change of variables

$$s = \frac{x}{t}, \quad u = \frac{x}{1-t}, \quad dsdu = Jdxdt = \frac{xdxdt}{(t(1-t))^2}$$

Replacing the change of variables in the Jacobian, we find

$$I = \frac{\pi^{1/2}(2i)^{c+1}\Gamma(\frac{1+c}{2})}{2\Gamma(\frac{-a}{2})\Gamma(\frac{-b}{2})}\int_0^\infty dx e^{-x} x^{\frac{-4-(a+b+c)}{2}} \int_0^1 dt t^{(c+a)/2}(1-t)^{(c+b)/2},$$

$$I = (2i)^c\,\pi\frac{\Gamma(1+\frac{b+c}{2})\Gamma(1+\frac{a+c}{2})\Gamma(-1-\frac{a+b+c}{2})\Gamma(\frac{1+c}{2})}{\Gamma(-\frac{a}{2})\Gamma(-\frac{b}{2})\Gamma(2+c+\frac{a+b}{2})}.$$

The following relations have been used

$$\int_0^1 dx x^{\beta-1}(1-x)^{\alpha-1} = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}, \quad \Gamma(z) = (z-1)!,$$

Eventually we obtain the result for $d=1$ as

Eventually we obtain the result for $d=1$ as

$$I = (2i)^c 2\pi \frac{\Gamma(2+\frac{b+c}{2})\Gamma(1+\frac{a+c}{2})\Gamma(-1-\frac{a+b+c}{2})\Gamma(\frac{1+c}{2})}{\Gamma(-\frac{a}{2})\Gamma(-\frac{b}{2})\Gamma(3+c+\frac{a+b}{2})},$$

Therefore one can write them down in a closed form as

$$\int d^2z|1-z|^a|z|^b(z-\bar{z})^c(z+\bar{z})^d = (2i)^c 2^d \pi \frac{\Gamma(1+d+\frac{b+c}{2})\Gamma(1+\frac{a+c}{2})\Gamma(-1-\frac{a+b+c}{2})\Gamma(\frac{1+c}{2})}{\Gamma(-\frac{a}{2})\Gamma(-\frac{b}{2})\Gamma(2+c+d+\frac{a+b}{2})}.$$

$$\int d^2z|1-z|^a|z|^b(z-\bar{z})^c(z+\bar{z})^d = (2i)^c 2^d \pi \frac{J_1+J_2}{\Gamma(-\frac{a}{2})\Gamma(-\frac{b}{2})\Gamma(d+2+c+\frac{a+b}{2})}.$$

where

$$\begin{aligned} J_1 &= \frac{1}{2}\Gamma(d+\frac{b+c}{2})\Gamma(d+\frac{a+c}{2})\Gamma(-d-\frac{a+b+c}{2})\Gamma(\frac{1+c}{2}) \\ J_2 &= \Gamma(d+1+\frac{b+c}{2})\Gamma(1+\frac{a+c}{2})\Gamma(-1-\frac{a+b+c}{2})\Gamma(\frac{1+c}{2}). \end{aligned}$$

Universality in all-order α' higher derivative corrections of non-BPS and BPS branes

There exists a regularity in the higher derivative expansions.
"prescription"

1st we find S-matrix of desired amplitudes which are either non-BPS or BPS.

Then, using Mandelstam variables, we rewrite the amplitudes such that all poles can be seen in a clear way.

3rd step is finding leading couplings from DBI action.

The last step is to express the symmetric trace in term of ordinary trace and applying the higher derivative corrections on them.

where

$$\begin{aligned}
\mathcal{D}_{nm}(EFGH) &\equiv D_{b_1}\cdots D_{b_m}D_{a_1}\cdots D_{a_n}EFD^{a_1}\cdots D^{a_n}GD^{b_1}\cdots D^{b_m}H,\\
\mathcal{D}'_{nm}(EFGH) &\equiv D_{b_1}\cdots D_{b_m}D_{a_1}\cdots D_{a_n}ED^{a_1}\cdots D^{a_n}FGD^{b_1}\cdots D^{b_m}H.
\end{aligned}$$

The crucial step seems to extract the symmetric trace in term of ordinary trace and applying the higher derivative corrections $\mathcal{D}_{nm}, \mathcal{D}'_{nm}$ on them.

1st example

In [1003.0314], it was shown that the string theory result of (CAAA) is reproduced by

four-gauge field couplings

$$-T_p(2\pi\alpha')^4 S\mathrm{Tr}\left(-\frac{1}{8}F_{bd}F^{df}F_{fh}F^{hb}+\frac{1}{32}(F_{ab}F^{ba})^2\right).$$

The closed form of the higher derivative corrections of four gauge fields to all orders of α' (which must be added to DBI) was shown to be

Extension of these interaction vertices to higher derivative couplings reproduc all **infinite massless poles**

$$(2\pi\alpha')^4 \frac{1}{8\pi^2} T_p \left(\alpha'\right)^{n+m} \sum_{m,n=0}^{\infty} (\mathcal{L}_5^{nm} + \mathcal{L}_6^{nm} + \mathcal{L}_7^{nm}),$$

with

$$\mathcal{L}_5^{nm} = -\mathrm{Tr}\left(a_{n,m}\mathcal{D}_{nm}[F_{bd}F^{df}F_{fh}F^{hb}] + b_{n,m}\mathcal{D}'_{nm}[F_{bd}F_{fh}F^{df}F^{hb}] + h.c. \right),$$
$$\mathcal{L}_6^{nm} = -\mathrm{Tr}\left(a_{n,m}\mathcal{D}_{nm}[F_{bd}F^{df}F^{hb}F_{fh}] + b_{n,m}\mathcal{D}'_{nm}[F_{bd}F^{hb}F^{df}F_{fh}] + h.c. \right),$$
$$\mathcal{L}_7^{nm} = \frac{1}{2}\mathrm{Tr}\left(a_{n,m}\mathcal{D}_{nm}[F_{ab}F^{ab}F_{cd}F^{cd}] + b_{n,m}\mathcal{D}'_{nm}[F_{ab}F^{cd}F^{ab}F_{cd}] + h.c \right),$$

where the higher derivative operators D_{nm} and D'_{nm} are defined

These corrections are checked by explicit computations of the amplitude of one RR and **3 gauge fields** [E.H,1003.0314,JHEP].

Another demonstration that shows the pure SYM vertices
(two scalar two gauge field couplings)
produce the same massless poles as the corresponding
correlator of one RR field vertex and 3 SYM vertices

The same is true for two scalar fields and
two gauge fields with

$$-\frac{T_p(2\pi\alpha')^4}{2}\mathrm{STr}\left(D_a\phi^i D^b\phi_i F^{ac}F_{bc}-\frac{1}{4}(D_a\phi^i D^a\phi_i F^{bc}F_{bc})\right)$$

The 2nd example is **two scalar and two gauge field** couplings [E.H,IY Park 1203.5553,PRD] (CAA\phi)

$$-\frac{T_p(2\pi\alpha')^4}{2}\mathrm{STr}\left(D_a\phi^i D^b\phi_i F^{ac}F_{bc} - \frac{1}{4}(D_a\phi^i D^a\phi_i F^{bc}F_{bc})\right).$$

Implementing the crucial step mentioned above, the closed form of the higher derivative corrections of two scalars and two gauge fields (which must be added to DBI) turned out to be

$$(2\pi\alpha')^4\frac{1}{2\pi^2}T_p\,(\alpha')^{n+m}\sum_{m,n=0}^{\infty}(\mathcal{L}_8^{nm}+\mathcal{L}_9^{nm}+\mathcal{L}_{10}^{nm}),$$

$$\mathcal{L}_8^{nm} = -\mathrm{Tr}\left(a_{n,m}\mathcal{D}_{nm}[D_a\phi^i D^b\phi_i F^{ac}F_{bc}] + b_{n,m}\mathcal{D}'_{nm}[D_a\phi^i F^{ac}D^b\phi_i F_{bc}] + h.c.\right),$$
$$\mathcal{L}_9^{nm} = -\mathrm{Tr}\left(a_{n,m}\mathcal{D}_{nm}[D_a\phi^i D^b\phi_i F_{bc}F^{ac}] + b_{n,m}\mathcal{D}'_{nm}[D_a\phi^i F_{bc}D^b\phi_i F^{ac}] + h.c.\right),$$
$$\mathcal{L}_{10}^{nm} = \frac{1}{2}\mathrm{Tr}\left(a_{n,m}\mathcal{D}_{nm}[D_a\phi^i D^a\phi_i F^{bc}F_{bc}] + b_{n,m}\mathcal{D}'_{nm}[D_a\phi^i F_{bc}D^a\phi_i F^{bc}] + h.c\right),$$

As usual, the above couplings are valid up to total derivative terms and terms such as $\partial_a\partial^a FFD\phi D\phi$ that vanish on-shell.

$$-T_p(2\pi\alpha')^4\mathrm{STr}\left(-\frac{1}{4}D_a\phi^iD_b\phi_iD^b\phi^jD^a\phi_j+\frac{1}{8}(D_a\phi^iD^a\phi_i)^2\right)$$

Applying our prescription, one can easily determine their higher derivative forms by noting universality property that was present in the previous works as follows

$$(2\pi\alpha')^4\frac{1}{4\pi^2}T_p\,(\alpha')^{n+m}\sum_{m,n=0}^{\infty}(\mathcal{L}_{11}^{nm}+\mathcal{L}_{12}^{nm}+\mathcal{L}_{13}^{nm})$$

$$\mathcal{L}_{11}^{nm}=-\mathrm{Tr}\left(a_{n,m}\mathcal{D}_{nm}[D_a\phi^iD_b\phi_iD^b\phi^jD^a\phi_j]+\ b_{n,m}\mathcal{D}'_{nm}[D_a\phi^iD^b\phi^jD_b\phi_iD^a\phi_j]+h.c.\right)$$

$$\mathcal{L}_{12}^{nm}=-\mathrm{Tr}\left(a_{n,m}\mathcal{D}_{nm}[D_a\phi^iD_b\phi_iD^a\phi^jD^b\phi_j]+\ b_{n,m}\mathcal{D}'_{nm}[D_b\phi^iD^b\phi^jD_a\phi_iD^a\phi_j]+h.c.\right)$$

$$\mathcal{L}_{13}^{nm}=\mathrm{Tr}\left(a_{n,m}\mathcal{D}_{nm}[D_a\phi^iD^a\phi_iD_b\phi^jD^b\phi_j]+\ b_{n,m}\mathcal{D}'_{nm}[D_a\phi^iD_b\phi^jD^a\phi_iD^b\phi_j]+h.c\right)$$

4th example : **two tachyons and two scalar fields on N non-BPS D-branes** [E.H,1203.1329,PRD].

$$2T_p(\pi\alpha')^3\mathrm{STr}\left(m^2T^2(D_a\phi^iD^a\phi_i)+\ D^\alpha TD_\alpha TD_a\phi^iD^a\phi_i-2D_a\phi^iD_b\phi_iD^bTD^aT\right)$$

the all-order vertices turned out to be

$$\mathcal{L} \;=\; -2T_p(\pi\alpha')(\alpha')^{2+n+m}\sum_{n,m=0}^{\infty}(\mathcal{L}_1^{nm}+\mathcal{L}_2^{nm}+\mathcal{L}_3^{nm}+\mathcal{L}_4^{nm})$$

where

$$\begin{aligned}
\mathcal{L}_1^{nm} &= m^2\mathrm{Tr}\Big(a_{n,m}[\mathcal{D}_{nm}(T^2D_a\phi^iD^a\phi_i)+\mathcal{D}_{nm}(D_a\phi^iD^a\phi_iT^2)]\\
&\quad+b_{n,m}[\mathcal{D}'_{nm}(TD_a\phi^iTD^a\phi_i)+\mathcal{D}'_{nm}(D_a\phi^iTD^a\phi_iT)]+h.c.\Big)\\
\mathcal{L}_2^{nm} &= \mathrm{Tr}\Big(a_{n,m}[\mathcal{D}_{nm}(D^\alpha TD_\alpha TD_a\phi^iD^a\phi_i)+\mathcal{D}_{nm}(D_a\phi^iD^a\phi_iD^\alpha TD_\alpha T)]\\
&\quad+b_{n,m}[\mathcal{D}'_{nm}(D^\alpha TD_a\phi^iD_\alpha TD^a\phi_i)+\mathcal{D}'_{nm}(D_a\phi^iD_\alpha TD^a\phi_iD^\alpha T)]+h.c.\Big)\\
\mathcal{L}_3^{nm} &= -\mathrm{Tr}\Big(a_{n,m}[\mathcal{D}_{nm}(D^\beta TD_\mu TD^\mu\phi^iD_\beta\phi_i)+\mathcal{D}_{nm}(D^\mu\phi^iD_\beta\phi_iD^\beta TD_\mu T)]\\
&\quad+b_{n,m}[\mathcal{D}'_{nm}(D^\beta TD^\mu\phi^iD_\mu TD_\beta\phi_i)+\mathcal{D}'_{nm}(D^\mu\phi^iD_\mu TD_\beta\phi_iD^\beta T)]+h.c.\Big)\\
\mathcal{L}_4^{nm} &= -\mathrm{Tr}\Big(a_{n,m}[\mathcal{D}_{nm}(D^\beta TD^\mu TD_\beta\phi^iD_\mu\phi_i)+\mathcal{D}_{nm}(D^\beta\phi^iD^\mu\phi_iD_\beta TD_\mu T)]\\
&\quad+b_{n,m}[\mathcal{D}'_{nm}(D^\beta TD_\beta\phi^iD^\mu TD_\mu\phi_i)+\mathcal{D}'_{nm}(D_\beta\phi^iD_\mu TD^\mu\phi_iD^\beta T)]+h.c.\Big)
\end{aligned}$$

The prescription works even for brane / anti brane.

This phenomenon seems quite universal and must have deep origins going back to the relation of open string and a closed string.

It should originate from the composite nature of a closed string state in terms of open string states.

Two Important comments on Myers paper
[arxiv:hep-th/9910053, JHEP, 0010122, JHEP]

In order to find all **new couplings/contact interactions** to all orders of α' one has to have **the complete form** of the amplitudes.

Note that the result of the amplitudes at leading orders is not very useful [see 1211.2413,EH,JHEP],as lots of poles have been overlooked, new couplings are derived within new techniques

$< V_C V_\phi V_\phi V_\phi >$ has some extra terms that are absent in

$$< V_C V_A V_A V_A >$$

As a comment we were not able to reproduce all contact terms of four point amplitude with usual pull-back. Thus it is a hint shows that **pull-back must be Modified.**

From the poles we see that the scattering amplitude has infinite massless poles . The low energy expansion is carried out by sending all Mandelstam variables to zero.

The above results cannot entirely be derived by applyingT-duality to the previous result that was obtained for (CAAA).

All the terms that contain the transverse components of the momentum of RR are not present in the corresponding result of [E.H,1003.0314,JHEP].

Several remarks on T-duality are in order.

T-duality can be straightforwardly employed to deduce a pure open string tree amplitude of scalar vertex operators from gauge amplitudes.

Once one considers an amplitude of a mixture of open and closed strings, direct computation is necessary because of the subtleties associated with T-duality.

Two subtleties exist in the very construction of RR C vertex operator

First, the construction of the C vertex operator was such that one set of oscillators was used instead of two.

The second issue is that the C vertex operator does not contain winding modes, this must be related to the fact that we have pointed out in [E.H,I.Park,1203.5553,PRD],

The terms that contain pi are absent in (**CAAA**) amplitude.

Once we are dealing with open-closed amplitudes T-duality transformation is not very effective and in fact direct computations of those amplitudes are inevitable.

For example we have shown that it is not possible to derive C3S from C3A.

In particular we have seen that the terms including momentum of closed string RR in transverse directions p^i, p^j are not appeared in **C3A.**

We also found new couplings which are neither inside Myers' terms nor within pull back/Taylor expansions.

$$S_3 = \frac{\lambda^3\mu_p\pi}{12}\int d^{p+1}\sigma\frac{1}{(p-3)!}(\varepsilon^v)^{a_0\cdots a_p}\left(\frac{\alpha'}{2}\right) \times C^{(p-3)}_{a_0\cdots a_{p-4}}\mathrm{Tr}\left(F_{a_{p-3}a_{p-2}}(D^aD_a)\left[D_{a_{p-1}}\phi^iD_{a_p}\phi_i\right]\right)$$

and

$$S_4 = \frac{\lambda^3\mu_p\pi}{6}\int d^{p+1}\sigma\,(\alpha')\,\mathrm{Tr}\left(C_{p-3}\wedge D^{b_1}F\wedge D_{b_1}\left[D\phi^i\wedge D\phi_i\right]\right)$$

Can be found just **by S-matrix and their coefficients should be set by applying Scattering approach not any other tools [1211.2413, JHEP,1302.5024,EH,NPB]**

NON-BPS Branes

This action was proposed based on S-Matrix method which is a generalization of DBI action for gauge fields and MSF's on WV of branes.

$$L = -V(T)\sqrt{-det(\eta_{\mu\nu} + 2\pi\alpha' F_{\mu\nu} + 2\pi\alpha'\partial_\mu T\partial_\nu T)}, \quad V(T) = 1 - \frac{T^2}{4} + O(T^4)$$

where for non –BPS branes in II ST this potential will produce tachyon's mass on branes very precisely.

In order to have consistency with S-Matrix method we have to generalize tachyonic action so that it would namely reproduce all desired couplings in non-BPS branes and brane –anti brane systems.

FOR CTTA, using the momentum conservation along the world volume of brane one finds

$$s+t+u=-p^ip_i-\frac{1}{2}$$

In general, the momentum expansion of a S-matrix element should be around

$$(k_i+k_j)^2 \to 0 \text{ and/or } k_i \cdot k_j \to 0$$

$$(k_i+k_j)^2 \to 0$$

$(k_i+k_j)^2$-channel

$$s \to -\frac{1}{4}, \quad t \to -\frac{1}{4}, u \to 0$$

Using the on-shell relations

Field Theory of Brane-Anti brane

The effective action of a *brane-anti brane in* **IIA(IIB)** *theory is given by* extension of the DBI action and the WZ terms which include the tachyon fields.

$$S_{DBI} = -\int d^{p+1}\sigma \mathrm{Tr}\left(V(T)\sqrt{-\det(\eta_{ab} + 2\pi\alpha' F_{ab} + 2\pi\alpha' D_a T D_b T)}\right)$$

The trace in the above action should be completely symmetric between all matrices of $F_{ab}, D_a T$, and individual T of the tachyon potential.

$$F_{ab} = \begin{pmatrix} F_{ab}^{(1)} & 0 \\ 0 & F_{ab}^{(2)} \end{pmatrix}, \quad D_a T = \begin{pmatrix} 0 & D_a T \\ (D_a T)^* & 0 \end{pmatrix}, \quad T = \begin{pmatrix} 0 & T \\ T^* & 0 \end{pmatrix}$$

$$F_{ab}^{(i)} = \partial_a A_b^{(i)} - \partial_b A_a^{(i)} \qquad D_a T = \partial_a T - i(A_a^{(1)} - A_a^{(2)})T$$

Consistency with S-Matrix imposed

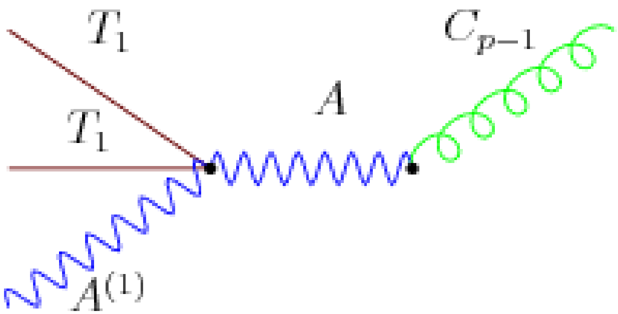
If one uses ordinary trace ,instead, the above action reduces to the action proposed by A.Sen after making the kinetic term symmetric and performing the trace. This latter action is not consistent with S-matrix calculation. The tachyon potential which is consistent with S-matrix element calculations has

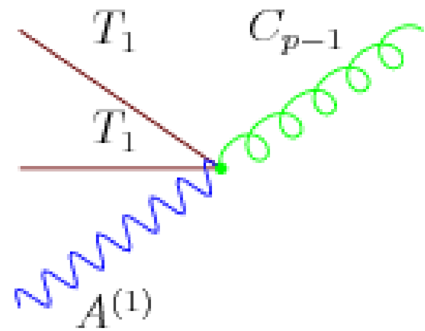
$$V(|T|) \;=\; 1 + \pi\alpha' m^2 |T|^2 + \frac{1}{2}(\pi\alpha' m^2 |T|^2)^2 + \cdots$$

consistent with the tachyon potential of BSFT

The terms of the above action which have contribution to the S-matrix CTTA

$$
\begin{aligned}
\mathcal{L}_{DBI} = & -T_p(2\pi\alpha')\left(m^2|T|^2 + DT\cdot(DT)^* - \frac{\pi\alpha'}{2}\left(F^{(1)}\cdot F^{(1)} + F^{(2)}\cdot F^{(2)}\right)\right) + T_p(\pi\alpha')^3 \\
& \times\Big(\frac{2}{3}DT\cdot(DT)^*\left(F^{(1)}\cdot F^{(1)} + F^{(1)}\cdot F^{(2)} + F^{(2)}\cdot F^{(2)}\right) \\
& +\frac{2m^2}{3}|\tau|^2\left(F^{(1)}\cdot F^{(1)} + F^{(1)}\cdot F^{(2)} + F^{(2)}\cdot F^{(2)}\right) \\
& -\frac{4}{3}\left((D^\mu T)^* D_\beta T + D^\mu T(D_\beta T)^*\right)\left(F^{(1)\mu\alpha}F^{(1)}_{\alpha\beta} + F^{(1)\mu\alpha}F^{(2)}_{\alpha\beta} + F^{(2)\mu\alpha}F^{(2)}_{\alpha\beta}\right)\Big)
\end{aligned}
$$





$$
\begin{aligned}
\mathcal{L}_{DBI} = & -T_p(2\pi\alpha')\left(m^2|T|^2 + DT\cdot(DT)^* - \frac{\pi\alpha'}{2}\left(F^{(1)}\cdot F^{(1)} + F^{(2)}\cdot F^{(2)}\right)\right) + T_p(\pi\alpha')^3 \\
& \times\Big(\frac{2}{3}DT\cdot(DT)^*\left(F^{(1)}\cdot F^{(1)} + F^{(1)}\cdot F^{(2)} + F^{(2)}\cdot F^{(2)}\right) \\
& +\frac{2m^2}{3}|\tau|^2\left(F^{(1)}\cdot F^{(1)} + F^{(1)}\cdot F^{(2)} + F^{(2)}\cdot F^{(2)}\right) \\
& -\frac{4}{3}\left((D^\mu T)^* D_\beta T + D^\mu T(D_\beta T)^*\right)\left(F^{(1)\mu\alpha}F^{(1)}_{\alpha\beta} + F^{(1)\mu\alpha}F^{(2)}_{\alpha\beta} + F^{(2)\mu\alpha}F^{(2)}_{\alpha\beta}\right)\Big)
\end{aligned}
$$

$$
\begin{aligned}
V_b(A^{(1)}, A^{(1)}, T_1, T_1) = & \frac{4i}{3}T_p(\pi\alpha')^3 k_b\left[(s+1/4)(2k_2\cdot\xi) + (t+1/4)(2k_3\cdot\xi)\right] + 2iT_p(\pi\alpha')^3 \times \\
& \times\left[k_{2b}(t+1/4)(2\xi.k_3) + k_{3b}(s+1/4)(2\xi.k_2) - \xi_b(s+1/4)(t+1/4)\right] \\
V_b(A^{(2)}, A^{(1)}, T_1, T_2) = & \frac{2i}{3}T_p(\pi\alpha')^3 k_b\left[(s+1/4)(2k_2\cdot\xi) + (t+1/4)(2k_3\cdot\xi)\right] + 2iT_p(\pi\alpha')^3 \times \\
& \times\left[k_{2b}(t+1/4)(2\xi.k_3) + k_{3b}(s+1/4)(2\xi.k_2) - \xi_b(s+1/4)(t+1/4)\right]
\end{aligned}
$$

So just this Lagrangian could consistently produce the CTTA amplitude.

Note that the term **F^1.F^2** in the tachyon DBI action is necessary for the above consistency ,

it cannot be derived by field redefinition of fields nor by Sen's action.

The above consistency indicates that the coupling $C_{p-1} \wedge F$ has no higher derivative correction.

Non-BPS branes

Field strength and **Covariant derivative of the Tachyons** are defined respectively, as

$$F=\frac{1}{2}F_{ab}dx^a\wedge dx^b,\quad DT=(\partial_a T-i[A_a,T])dx^a$$

Using **S-matrix method** the kinetic term of tachyon appears in the DBI action as

$$S_{DBI}\ \sim\ \int d^{p+1}\sigma \mathrm{STr}\left(V(T^iT^i)\sqrt{1+\frac{1}{2}[T^i,T^j][T^j,T^i])}\right.$$
$$\left.\times\sqrt{-\det(\eta_{ab}+2\pi\alpha' F_{ab}+2\pi\alpha' D_aT^i(Q^{-1})^{ij}D_bT^j)}\right)$$

$$V(T^iT^i)\ =\ e^{-\pi T^iT^i/2}=1-\pi T^iT^i/2+\frac{1}{2}(-\pi T^iT^i/2)^2$$

The above expansion is consistent with tachyonic potential coming from BSFT

$$m^2 = -1/(2\alpha')$$

$$Q^{ij} \;=\; I\delta^{ij} - i[T^i, T^j] \qquad\qquad i,j = 1,2, \quad T^1 = T\sigma_1, \quad T^2 = T\sigma_2$$

The above action is consistent with the momentum expansion of the S-matrix element of TTTT, the S-matrix element of CTTT and with the momentum expansion
of the S-matrix element of TTTTA.

While here due to presence of Tachyon pole the desired expansion is not low energy expansion , this certain expansion gives us effective actions for tachyons,

where we observe that it has a correct expansion in the sense that it has consistent form for amplitudes including tachyons.

A proposal by A. Sen for brane anti brane effective action when branes are coincident is

$$S = -\int d^{p+1}\sigma V(|\tau|)\left(\sqrt{-\det \mathbf{A}^{(1)}} + \sqrt{-\det \mathbf{A}^{(2)}}\right),$$

where

$$\mathbf{A}^{(n)}_{\mu\nu} = \eta_{\mu\nu} + 2\pi\alpha' F^{(n)}_{\mu\nu} + \pi\alpha'\left(D_\mu\tau(D_\nu\tau)^* + D_\nu\tau(D_\mu\tau)^*\right)$$

On D-brane anti D-brane effective actions [1211.5538,EH]

Discovering all higher derivative corrections to produce all the infinite scalar poles of <V_C V_{\phi} V_T V_T>.

we explore the presence of new term involving

$$D\phi^{i(1)}.D\phi_i^{(2)}$$

in DBI action of brane anti brane systems where this new coupling and its all order α' higher derivative corrections can be discovered just by applying S-matrix method of this paper. Having set the tachyon to zero, both ordinary and symmetric trace effective actions become equivalent.

The symmetric trace effective action has a non-zero coupling (like above) while this coupling does not exist in ordinary trace effective action.

The only consistent effective action for D-brane anti D-brane systems, **based on direct S-matrix computations** of (CATT)

was appeared in **[1211.5538 EH, JCAP]**

In that paper, we have shown that there was a non-zero coupling between F ^(1) .F^(2)
we found all their infinite higher derivative corrections.

Note that for ordinary trace prescription, this coupling does not exist. It is important to emphasize that SEN's effective action is not consistent with S-matrix computations and in fact symmetrized trace works out for superstring computations.

The reason :

By using it not only are we unable to produce all the tachyon poles of the (CATT) but also the structure and forms of some of the new couplings like $F^{(1)}.F^{(2)}$
(confirmed by S-matrix) have been overlooked.

New couplings in brane anti brane systems without ambiguity

$$\mu_p \lambda (2\pi\alpha') \int_{\sum_{(p+1)}} \mathrm{Tr}\left(\partial_i C_{p-1} \wedge F(\phi_1 + \phi_2)^i\right)$$

$$\boxed{\frac{i}{2}\mu_p \beta^2 (2\pi\alpha')^3 \mathrm{Tr}\left(\partial_i C_{p-1} \wedge DT \wedge DT^*(\phi^1 + \phi^2)^i\right)}$$

$$\partial_i C_{p-1} \wedge DT \wedge DT^*(\phi^1 + \phi^2)^i$$

$$\frac{i}{2}\mu_p (2\pi\alpha')(\alpha')^2 \left(\frac{\pi^2}{6} - 8ln2^2\right) \mathrm{Tr}\left(\partial_i C_{p-1} \wedge D^a D_a (DT \wedge DT^*)(\phi^1 + \phi^2)^i\right)$$

$$\epsilon^{a_0 \cdots a_p} H_{i a_0 \cdots a_p} (\phi^{(1)} - \phi^{(2)})^i T T^*$$

Open Questions

1) **We might be able** to predict something with (CAAAA or C4\phi)

2) **Some progress** in understanding the form of this action has been made [E.H,JHEP,1307.3520].

3) **Pull-Back should be modified** (with in 4 point function it is **clear**).

4) At the moment we cannot say any thing about the explicit form of Myers terms in higher point functions, their appearance in higher point function should be modified.

5) Unsolved problems : CT3\phi

6) Ambiguity on Tachyonic action whether (CTTTT) needs Symmetrized trace or ordinary trace.

7) Fermionic amplitudes and universality in Type IIA?

Note that a supersymmetric generalization of DBI action is still unknown.

Conclusion

We have analyzed all the 3,4 and 5 point functions of
BPS and non-BPS branes

We found the field theory vertices that reproduce all infinite contact terms of 2 and 3- point functions.

We could produce all poles of the four-point function, but we could produce the contact terms only for $p + 4 = n$ case.

It is entirely not clear how to produce all infinite contact terms of 4-point function for $p + 2 = n$, $p = n$ cases

Possibly pull back may need modification.

We found universality in all order α' higher derivative corrections of non-BPS and BPS branes and the universality played an important role in the determination of field theory vertices.

remarks on T-duality :

Once one considers an amplitude of a mixture of open and closed strings, **direct computation is necessary** and

C vertex operator does **not contain winding modes**, and this must be related to the fact that we have pointed out , the terms that contain p^i are absent in C3A.

In fact those new terms could have been obtained provided C vertex contains **winding modes** as well.

Thank you for your attention