Wigner's 1939 representation theory of the Poincare group and ongoing foundational changes of QFT

Talk presented at the Wigner 111 Symposium

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Content

1. The Wigner's 1939 positive energy representation theory. The first intrinsic (no reference to classical fields) classification of particle spaces.
2. Adding causal localization. From covariant wave functions to point- and string-local fields.
3. Extensions of renormalized perturbation theory using stringlocal fields. Hilbert space setting for vectormesons
4. Impact on gauge theories.

Ref. (BGL) R. Brunetti, D. Guido and R. Longo, Rev. Math. Phys.**14**, (2002) 759
(MSY) J. Mund, B. Schroer and J. Yngvason, Commun. Math. Phys. **268**, (2006) 621
B. Schroer, *Renormalizability in Hilbert space from string-localization; the case of couplings of vectormesons to scalar particles,* arXiv:1307.3469
B. Schroer, *Dark matter and Wigner's third positive-energy representation class*, arXiv:1306.3876

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- Wigner's proposal: *avoid the parallelism to classical field theory* (Lagrangian quantization,..); access relativistic QT directly.
- Wigner's proposal: classify particles (m, s) through unitary positive energy representations of Poincaré group; early harbinger of modern LQP. His former collaborator P. Jordan dreamed of an intrinsic setting for QFT already in 1929 (Charkov conference proceedings): *"QFT without classical crutches"* (no quantization parallelism, the more foundational QFT should stand on its own feet) but Wigner undertook the first step. The results presented in this talk are fruits of this way of thinking.

- Wigner's work on representation theory was far ahead of his time. Only 15 years later A.S. Wightman and R. Haag directed attention to it. Later S. Weinberg showed in a systematic way how to *convert the unitary representations into covariant fields*. One representation class, namely the class of "infinite" spin representations (ISR) remained outside; their fields were only discovered in 2006.

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- Wigner realized that without understanding "causal localization" within his representation theoretical setting, the physical content of the IS representations cannot be understood. He was perfectly aware that the adaptation of the position operator (the Born localization in QM) to his representation theory (the Newton-Wigner localization) was not what he wanted. What was missing was "modular localization", the recently discovered intrinsic form of causal localization.

- Only after the appearance of modular localization, the ISR problem was finally solved in 2006. It represents a new kind of (zero mass) matter which only exists in the form of noncompact spacetime localization, generated by semi-infinite localized covariant fields $\Psi(x,e)$, loc. on $x+\mathbb{R}_+e$, $e^2=-1$. This noncompact localization of this noninteracting matter is "irreducible"; it cannot be cut into compact localized pieces or approximated by sequences of localized matter.

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- According to standard ideas about particle counters, irreducible noncompact matter cannot activate a counter (violation of causality); for the same reasons it seems to be inert with respect to ordinary matter. The natural arena of action are galaxies. As *positive energy* matter it shares *stability* and *gravitational coupling*; its noncompact extension would cause a galactic change of gravitational balance. Is Wigner's third class stuff the astrophysical dark matter?

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- The use of string-localization in the perturbative renormalization setting of ordinary matter leads to an alternative to the BRST gauge theory: *the use of stringlocal fields in Hilbert space* (instead of Krein spaces).

- 3 classes of positive energy (stability) representations in Hilbert space $H_1 = L^2(H_m^+, d\mu; \mathfrak{h}) = L^2(H_m^+, d\mu) \otimes \mathfrak{h}$, H_m^+ *mass-shell*, $d\mu$ *is L-inv.measure*

$$(U(a,\Lambda)\psi)(p) = e^{ipa} D(R(\Lambda,p))\psi(\Lambda^{-1}p), \; R(\Lambda,p) = B_p^{-1}\Lambda B_{\Lambda^{-1}p}$$

$$D \text{ acts on } \mathfrak{h}, \; B_p\hat{p} = p : \hat{p} = \begin{cases} (1,0,0,0) & m > 0, \\ (1.0.0.1) & m = 0 \end{cases} \; R(\Lambda,p)\hat{p} = \hat{p}$$

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- The 3 classes differ in the "little group" (l.g.). For m>0, l.g. is SO(3) D=D^s, s=spin; for m=0, l.g. E(2), repr. D splits into

$$deg.\ repr.\ D^h(R(\theta)) = e^{ih\theta}, \;\; h \ helicity$$
$$faithfull\ repr.\ (D^\kappa(c,R(\theta))\varphi)(k) = e^{ic\cdot k}\varphi(R^{-1}(\theta)k)$$
$$on\ \mathfrak{h} = \;\; L^2(R^2, \delta(k^2-\kappa^2)d^2k), \;\; \kappa \ Pauli - Lub.\ inv.$$

Terminology: m>0 first class repr., $m = 0$, h 2nd class and ISR 3rd class

Standard method to construct pointlike covariant fields from Wigner repr. (see Weinberg's book on QFT)

- Use Wigner representation to compute *intertwiners* $u(p)$, matrix-valued function with ($m > 0, s$ integer). Weinberg uses group theory to compute the $u^{A,\dot{B}}(p)$ $|A - B| \leq s \leq A + B$

$$D^s(R(\Lambda, p))u^{A,\dot{B};s}(\Lambda^{-1}p) = D^{A\dot{B}}(\Lambda^{-1})u^{A,\dot{B};s}(p),$$

$$A^{A,\dot{B}}(x) = \int \left(e^{ipx}u^{A,\dot{B}}(p) \cdot a^*(p) + e^{-ipx}u_c^{A,\dot{B}}(p) \cdot a(p)\right) \frac{d^3p}{2p_0} \; covar.$$

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- For $(m = 0, \; h)$ the Wigner rotation D is diagonal 2×2 matrix, multiplication with helicity phase factors $exp \pm ih\theta(\Lambda, p)$. In this case the relation between A, B are more restrictive $h = |A - B|$; e.g. no $A_\mu(x)$ but stringlocal potentials $A_\mu(x, e)$.

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- Group theoretic method breaks down for ISR

modular localization

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- MSY: construction of generating stringlike free fields for m>0 and ISR

$$\Psi(x,e) = \int \left(e^{ipx} u(p,e) \cdot a^*(p) + e^{-ipx} u_c(p,e) \cdot a(p)\right) \frac{d^3p}{2p_0}$$

$$D(R(\Lambda,p))u)(\Lambda^{-1}p,e) = u(p,\Lambda e)$$

$$U(a,\Lambda)\Psi(x,e)\,U^{-1}(a,\lambda) = \Psi(\Lambda x + a, \Lambda e)$$

the same construction of *scalar* stringlocal fields works for (m>0, s), but for $m = 0$, $h \geq 1$ tensor indices are indispensable.

It is not evident from the intertwining properties of $u(p,e)$ that the field obeys string locality

$$\left[\Psi(x.e),\Psi(x'.e')\right]=0 \;\; x+\mathbb{R}_+e ><x'+\mathbb{R}_+e'$$

One has to use analytic properties of $u(p,e)$.
It is somewhat surprising that massive fields for arbitrary s can be described in terms of *scalar stringlocal fields*; this is not possible for massless $(m=0,h)$ fields. Whereas for ordinary matter the intertwiners are rational $\mathfrak{h}$-valued functions in p and e, the ISR intertwiners are more complicated

$$u(p,e)(k)=e^{-i\pi\alpha/2}\int d^2ze^{ikz}(B_p\xi(z)\cdot e)^{\alpha},\;\; Re\alpha<0$$
$$\xi(z)=(\frac{1}{2}(z^2+1),z_1,z_2,\frac{1}{2}(z^2-1))$$

The resulting 2-point functions cannot be computed in closed form, whereas for ordinary matter the stringlocal 2-point functions are rather simple (see later).

modular

The missing intrinsic form of causal localization (which Wigner looked for in vain) is is *modular localization* . The terminology originates from its close relation to the *Tomita-Takesaki modular theory of operator algebras.*

- Its characteristic aspect is that it encodes spacetime localization properties into domain properties of certain unbounded operators (Tomita S-operators). The subspace of the Wigner repr. space of wave functions $sub(\mathcal{O}) \subset H_1$ localized in a spacetime region $\mathcal{O}$ is equal to domain of $S_{\mathcal{O}}$

$$S_{\mathcal{O}}\psi = \psi_c, \; sub(\mathcal{O}) = domS_{\mathcal{O}}, \; dense \; in \; H_1$$

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- It was instrumental for *discovering* the QFT associated with the ISR Wigner representation and for replacing the BRST gauge setting by a new gauge-independent Hilbert space formulation, but it is not necessary for *presenting* these results. Its main properties in the context of Wigner representation will be briefly sketched on the

An abstract Tomita S operator is an closed antilinear closed involutive operator with a dense domain of definition

$$Def: \; S\; Tomita: S\; antilin,\; densely\; def.,\; closed,\; involutive\; S^2 \subseteq \mathbf{1}$$
$$polar\; decomp.\; S = J\Delta^{1/2},\; J\; modular\; reflection,\; \Delta^{it}\; mod.\; group$$

Such S occur in the theory of (von Neumann) operator algebras $\mathcal{A}$

$$SA\Omega = A^*\Omega,\; A \in \mathcal{A},\; acts\; cyclic\; i.e. \overline{\mathcal{A}\Omega} = H$$
$$S = J\Delta^{1/2},\; J\; modular\; reflection,\; \Delta^{it} = e^{-itH_{mod}}\; mod.\; group$$

S is uniquely defined by this formula iff the algebra is "standard" i.e. acts cyclic and separating (no annihilator in $\mathcal{A},\; i.e.\; A\Omega = 0 \curvearrowright A = 0$). Thanks to the Reeh-Schlieder theorem (see R. Haag's book "Local Quantum Physics") this is true for any pair $(\mathcal{A}(\mathcal{O}), \Omega_{vac})$ where $\mathcal{A}(\mathcal{O})$ is algebra of observables localized in $\mathcal{O}$.
Modular theory generalizes the (uni)*modularity* of Haar measures in group repr. to von Neumann algebras in standard position. The Tomita-Takesaki theorem states that the adjoint action of Δ^{it} is an automorphism of A and AdJ maps the algebra into its commutant $\mathcal{A}'$

- The modular group describes the strong entanglement of the global vacuum after its restriction to a local subalgebra $\mathcal{A}(\mathcal{O})$ QFT: the ensembles of observables localized in $\mathcal{A}(\mathcal{O})$ becomes an impure KMS (statistical mechanics like) state to the Hamiltonian H_{mod}. The reflection J generalizes the TCP invariance.

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- The application of modular theory to positive energy Wigner representations leads to modular localized subspaces. For the wedge region $W_0 = \{x_3 > |x_0|\}$, S can be defined in terms of Wigner representation data: Let $U(j_W)$ be the antiunitary reflection $repr(\mathcal{P}_+^\uparrow)$ which maps the wedge into its causal complement and $\Lambda_{W_0}(\chi)$ the Lorentz subgroup which leaves the wedge invariant. These two operators commute and the definition

$$J_{W_0} := U(j_{W_0}),\ \Delta_{W_0}^{it} := U(\Lambda_{W_0}(-2\pi t)),\ def.\ S = J_{W_0}\Delta_{W_0}^{1/2}$$

is easily shown to be a Tomita operator. By using the covariance properties of S and intersecting wedges one can construct $S_{\mathcal{O}}$ whose domain consist precisely of the dense subspace of $\mathcal{O}$-localized wave functions in the Wigner one particle space H_1.

The new scope

For constructing pointlike fields there is no gain from modular theory, Weinberg's group theoretical method based on covariance (no direct use of localization) are simpler. The discovery of ISR stringlocal Wigner ("galactic") matter with *modular localization* was a "door-opener" for obtaining new insights for dealing with ordinary matter:

- Resolves the clash between localization and Hilbert space for $m = 0.h \geq 1$; instead of pointlike vectorpotentials in Krein spaces (BRST) use stringlocal potentials in Hilbert space; no special role of $s = 1$, all of QFT united under one foundational principle: causal locality.

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- Improves short distance behavior even for massive fields. Pointlocal fields have $d_{sd} = s + 1$; exist always $d_{sd} = 1$ stringlocal fields in same localization class. Illustration for $s = 1$

$$F_{\mu\nu}(x) = \partial_\mu A_\nu^P(x) - \partial_\nu A_\mu^P(x), \quad A_\mu(x, e) \equiv \int_0^\infty d\lambda F_{\mu\nu}(x + \lambda e) e^\nu$$

$$U(\Lambda, a) A_\mu(x, e) U(\Lambda, a)^* = \left(\Lambda^{-1}\right)_\mu^{\ \nu} A_\nu(\Lambda x + a, \Lambda e)$$

- By construction $A_\mu(x,e)$ is localized on $x+\mathbb{R}_+e$, both fields $A_\mu^P(x)$ and $A_\mu(x,e)$ are members of the same locality class (Borchers class). They are linearly related

$$A_\mu(x,e) = A_\mu^P(x) + \partial_\mu\phi(x,e), \quad \phi(x,e) = \int A_\mu^P(x+\lambda e)e^\mu d\lambda$$

$$\left\langle A_\mu(x,e)A_\nu(x',e')\right\rangle, \ \left\langle\phi(x,e)\phi(x',e')\right\rangle, \ \left\langle A_\mu(x,e)\phi(x',e')\right\rangle, \ ...$$

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- Adiabatic equivalence property (AE): linear relation within local equivalence (Borchers) class; maintained in presence of interaction; extension to matter φ (g coupling)

$$AE: A_\mu(x,e) = A_\mu^P(x) + \partial_\mu\phi, \ \varphi(x,e) = e^{ig\phi}\varphi^P(x),$$

  intuitive: $\partial_\mu\phi$ "peels off" leading short distance behavior $d_{sd}^P = 2 \rightarrow d_{sd}^S = 1$ and AE breaks down for $m \rightarrow 0$ (no scalar strings for $h \geq 1$)

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- AE generalizes the conceptual content of gauge theory (the BRST Q-gauge charge formalism), but now in a Hilbert space and by using nothing more than the foundational causal localization principle of QFT. It cannot undo the unlimited increase of short distance dimension d_{sd}^{p} with perturbative order, but at least it describes the same finite coupling parameter situation as its stringlike sibling (singular pointlike field "coordinatization", Jaffe fields?).

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- The extension of the relation between point- and string-like fields to massive Y-M couplings requires nonlinear coupling-dependent equivalence class relations

$$A_\mu(x,e) = U^{-1}(x,e) A_\mu^P(x) U(x,e) - \frac{i}{g} U^{-1} \partial_\mu U(x,e)$$

$$A_\mu := A_\mu^a T_a, \ \phi = \phi^a T_a, \ [T_a, T_b] = f_{abc} T_c, \ U(x,e) = e^{ig\phi(x,e)}$$

massive QED

- Use operator setting of Stückelberg-Bogoliubov-Epstein-Glaser renormalization setting. AE property for scattering matrix for massive vectormesons

$$S_{scat}^{P}=S_{scat}^{S},\ \ \int T\mathcal{L}^{P}..\mathcal{L}^{P}=\int T\mathcal{L}^{S}..\mathcal{L}^{S}$$

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- In first and second order AE (notation $\mathcal{L}=\mathcal{L}^{S}$)

$$\mathcal{L}^{P}(x)=\mathcal{L}(x,e)-\partial_{\mu}V^{\mu}\ \ or\ \ d_{e}\{..\}=0\ \ \ from\ A^{P}-A\ relation$$

$$show\ \ (d_{e}+d_{e'})\left(T\mathcal{L}\mathcal{L}'-\partial_{\mu}TV^{\mu}\mathcal{L}'-\partial_{\nu}'\mathcal{L}V'^{\nu}+\partial_{\mu}\partial_{\nu}'TV^{\mu}V'^{\nu}\right)=0$$

d_{e} differential acting on zero forms, (...) *defines* second order pointlike $\mathcal{T}\mathcal{L}^{P}\mathcal{L}^{P}\,'$. Equivalent (use symmetry in e,e') to

$$d_{e}(T\mathcal{L}\mathcal{L}'-\partial_{\mu}TV^{\mu}\mathcal{L}')=0,\ \ d_{e}(\partial_{\nu}'\mathcal{L}V'^{\nu}-\partial_{\mu}\partial_{\nu}'TV^{\mu}V'^{\nu})=0$$

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- peeling off" singularities in form of *derivatives* from $T\mathcal{L}^p\mathcal{L}'^p$, vanishing surface terms in adiabatic limit.

For $j_\mu = \bar{\psi}\gamma_\mu\psi$ see forthcoming paper by J. Mund. New phenomenon for scalar matter $j_\mu = i\varphi^* \overleftrightarrow{\partial}_\mu \varphi$; expect gauge covariant completion $\partial_\mu \rightarrow D_\mu = \partial_\mu - ieA_\mu(x,e)$ as a consequence of AE

- Relation $d_e T\mathcal{L}\mathcal{L}' \overset{!}{=} d_e \partial_\mu TV^\mu \mathcal{L}'$ holds for all $x \neq x'$ and uncontracted Wick-ordered contribution, potentially violated by δ-terms on the diagonal $x = x'$. Undetermined C-counterterm in tree approximation on left hand side from

$$
\begin{aligned}
lhs: &\ \left\langle T\partial_\mu \partial'_\nu \varphi^*(x)\varphi(x')\right\rangle = \partial_\mu \partial'_\nu \left\langle T\varphi^*(x)\varphi(x')\right\rangle + cg_{\mu\nu}\delta(x-x') \\
rhs: &\ \partial^\mu \left\langle T\partial_\mu \varphi^*(x)\varphi(x')\right\rangle = \partial^\mu \partial_\mu \left\langle T\varphi^*(x)\varphi(x')\right\rangle = \delta(x-x') + ..
\end{aligned}
$$

  is matched with δ-contribution (K-G acting on time ordered function), this fixes c

For $j_\mu = \bar{\psi}\gamma_\mu\psi$ see forthcoming paper by J. Mund. New phenomenon for scalar matter $j_\mu = i\varphi^* \overleftrightarrow{\partial}_\mu \varphi$; expect gauge covariant completion $\partial_\mu \rightarrow D_\mu = \partial_\mu - ieA_\mu(x, e)$ as a consequence of AE

- Relation $d_e T\mathcal{L}\mathcal{L}' \stackrel{!}{=} d_e \partial_\mu T V^\mu \mathcal{L}'$ holds for all $x \neq x'$ and uncontracted Wick-ordered contribution, potentially violated by δ-terms on the diagonal $x = x'$. Undetermined C-counterterm in tree approximation on left hand side from

$$lhs: \ \left\langle T\partial_\mu \partial'_\nu \varphi^*(x)\varphi(x')\right\rangle = \partial_\mu \partial'_\nu \left\langle T\varphi^*(x)\varphi(x')\right\rangle + cg_{\mu\nu}\delta(x - x')$$
$$rhs: \ \partial^\mu \left\langle T\partial_\mu \varphi^*(x)\varphi(x')\right\rangle = \partial^\mu \partial_\mu \left\langle T\varphi^*(x)\varphi(x')\right\rangle = \delta(x - x') + ..$$

  is matched with δ-contribution (K-G acting on time ordered function), this fixes c

- The 2.order contact term on left hand side combined with the first order $\mathcal{L}^P$ on right hand side gives the desired $\hat{\mathcal{L}}^P := i\varphi^* \overleftrightarrow{D}_\mu \varphi A^\mu$. The loop contributions lead to the expected counterterms for mass and wave function renormalization.

- Can physical stringlocal field also be described inside the pointlike BRST formalism? Formal answer: yes; it corresponds to the expression (Jordan, Dirac,..)

$$\textit{fundamental } \varphi(x,e)|_{m\to 0} \simeq \textit{composite } \varphi^K(x)e^{ig\int_0^\infty d\lambda A_\mu^K(x+\lambda e)e^\mu}$$

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- Old point of view: QED is simpler than massive QED, but pointlike Krein space description of charged φ pointlike is void of physical meaning. Basic physical fields are stringlocal.
- New message: massive QED is conceptually simpler since it is a renormalizable theory with stringlocal physical matter fields without infrared problems; it has the standard particle interpretation (LSZ scattering). Its conceptual simplicity is useful for the description of QED in the limit $m \to 0$; use m as a natural covariant infrared regularization parameter (instead of ad hoc noncovariant cutoffs).

4. chargeless matter

Massive vectormesons coupled to neutral scalar field

- Since $A_\mu^P H \partial^\mu H = \frac{1}{2}\partial^\mu (A_\mu^p H^2)$ vanishes in AE setting, the only $\mathcal{L}^P$ coupling which is AE equivalent to a renormalizable $\mathcal{L}$ interaction is

$$\mathcal{L}^p = m(\left(A_\mu A^\mu\right)^P H + aH^3), \ \ with \ A_\mu^P = A_\mu - \partial_\mu \phi \ \ obtain:$$

$$\mathcal{L}^p = \mathcal{L} - \partial^\mu V_\mu, \ \ with \ \mathcal{L}_1 {=} m(A_\mu A^\mu H + A^\mu \phi \overleftrightarrow{\partial_\mu} H - \frac{m_H^2}{2m}\phi^2 H + aH^3)$$

$$and \ \ V_\mu = m(A_\mu \phi H + \frac{1}{2}\phi^2 \overleftrightarrow{\partial}_\mu H)$$

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new: explicit appearance of ϕ in stringlocal coupling $\mathcal{L}$

- Second and third order induce local $\delta(x-y)Op.$ terms, net result is

$$\mathcal{L}_2 = A_\mu A^\mu(H^2 + m^2\phi^2) - \frac{m_H^2}{2}\phi^2(m^2\phi^2 - H^2) - \frac{m_H^2}{2m^2}H^4$$

$$g\mathcal{L}_1 + \frac{1}{2}g^2\mathcal{L}_2 = A_\mu\ terms + V,\ \ V = -g^2\frac{m_H^2}{8m^2}(H^2 + \frac{2m}{g}H + m^2\phi^2)^2 -$$

V a Mexican hat potential, but not input rather induced by locality.

Further comments:
Our result confirms result obtained by the Zürich group (Aste, Dütsch, Scharf) within the BRST setting. This de-mystifies the Higgs mechanism, the Higgs field is nothing else than a *scalar neutral field coupled to a massive vectormeson*. This model is renormalizable (in BRST and stringlocal setting) as it stands. In particular the *H does not create masses of other particles*; there is no conceptual difference to charged coupling, except that neutrality leads to more induced terms. The Higgs mechanism is a metaphoric way to obtain such a theory but it creates more questions than it is capable to answer.

What about LHC, was'nt the validity of the Higgs mechanism verified?

- What was seen was a neutral scalar particle. On the side of theory there are second order calculations within the BRST (see Scharf's book) gauge setting which claim that the BRST setting by itself is inconsistent, but that this can be patched up by coupling other fields (outside massive Y-M); this has no relation to the metaphoric spontaneous symmetry breaking of the "Higgs mechanism", rather it shows that in order to preserve gauge invariance in the presence of massive gluons one need the additional coupling of a neutral scalar particle with the same coupling strength. It is important to check its validity in the conceptual clearer locality-based Hilbert space description (in progress).

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- In case that the new stringlocal Hilbert space formulation does not confirm this result there are other foundational QFT ideas which could account for the LHC result. One such idea is that the intrinsic physical (Hilbert space) charge neutral scalar Stückelberg field has scalar bound states. Such a solution would be attractive since it uses only Y-M degrees of freedom.

- Schwinger's *screening* idea and Swieca's "screening theorem" (the "Schwinger-Higgs screening") of the 70s holds *for all interactions involving massive vectormesons*: screening of the Maxwell charge ($\neq$ from charge counting charge) which is different from the "global counting charge". The Higgs ideas missed this important intrinsic aspect of massive vectormesons.

$$Q^{Max} = \int j_0^{Max}(x)d^3x \equiv 0, \; j_\mu^{Max} = \partial^\nu F_{\mu\nu}, \; Q^{glob} = \int j_0(x)d^3x \neq 0$$

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- The new setting which uses stringlocal fields (of $d_{sd} = 1$ independent of s) in Hilbert space does not only *recover the democracy* between particles ("nuclear democracy" of particles from the times of the S-matrix bootstrap approach), but also re-unites non-gauge and gauge QFT under the same conceptual roof: the foundational causal localization principle in a Hilbert space setting. The gauge setting was the result of insisting in a pointlike description which is not possible in a Hilbert space.

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- An important issue not presented is the enormous conceptual simplification of the confinement problem and the resulting contrasting relation with the third class dark matter: whereas

The Hilbert space setting leads to new ideas about the meaning of confinement. The relevant question is: what distinguishes QED strings from Y-M strings?

- QED strings are reducible, they can be approximated by local operators since abelian potentials $A_\mu(x, e)$ are line integrals over observable field strength. As a consequence the infrared divergences do not appear in (off-shell) correlations but only appear as $\log m$ terms in the perturbative (on-shell) scattering amplitudes. By summing up the leading logs and afterwards $m \to 0$, these vanish. On has to combat this vanishing by forming photon-inclusive cross sections before $\to 0$.

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- Y-M strings are irreducible since they have no linear relation to observables. In that case the infrared manifestations are stronger: $\log m$ terms in off-shell gluon correlations. Confinement picture: sum up leading terms and show that correlation vanish for $m \to 0$. The only way to avoid such vanishing for quark correlation is to take $\psi(x, e) - \psi^*(x', e)$ pairs with e in the $x - x'$ spacelike direction. Note that the string bridge is already contained in the $q - \bar{q}$ pair, it does not have to be added.

Conclusions

- The perturbative use of stringlocal fields in a Hilbert space goes beyond gauge theory since it also contains stringlocal physical matter fields. It restores the unity of QFT by removing the separation between gauge and non-gauge models.
- It replaces the metaphoric Higgs mechanism by the clear picture of massive vectormesons interacting with chargeless (hermitian) scalar fields (no symmetry has been broken, but the neutral coupling has more induced couplings which can be written in the Mexican hat form). It rediscovers important old results (the screening of massive Maxwell currents) which have been lost in the maelstrom of time.
- As a consequence of the existence of stringlocal matter fields in Hilbert space, it presents a good starting point for understanding spacetime aspect of perturbative infrared divergencies

Since the core area of this Wigner symposium were foundations of quantum theory with special emphasis on problems of entanglement, it may be interesting to add some remarks about the significant modifications these concepts undergo in QFT. The starting point is the fact that the vacuum state in the second quantization (Fock space setting) of quantum mechanics is inert (cannot be entangled) under subdivisions whereas in quantum field theory the restriction of the global vacuum to a sub-ensemble of O-localized observables leads to a KMS state (from modular theory in previous section) which is entangled in such a strong way that a density matrix description is not possible; i.e. localization-entanglement in QFT is outside the quantum information setting. In order to recover a density matrix one has to "split" $\mathcal{A}(\mathcal{O})$ from its causal complement $\mathcal{A}(\mathcal{O}')$ by a spacelike split distance ε in order to obtain a density matrix and a corresponding split-localization entropy

$$loc.entropy \simeq \frac{area}{\varepsilon^2}$$

where in the case of the causal shadow $\mathcal{O}$ of a spatial sphere the area is the surface area of the sphere.

The most surprising aspect of modular localization is that the concept of probability is a consequence of causal localization. The vacuum restricted to the ensemble of local observables in $\mathcal{A}(\mathcal{O})$ has a natural ensemble probability which is that of a statistical mechanics state. With such ensemble probabilities Einstein had never any problem, his "God does not throw dice" philosophical rejection of Born's quantum mechanic probability was based on the idea that (quantum) mechanics describes an individual irreducible system and there is no actual ensemble to which the Born probability refers; it is rather a "Gedanken-ensemble" which the observer constructs from the statistics of repeated measurements.

If these aspects of QFT known at the time of Einstein, perhaps the probability dispute would have taken a different turn. In fact the 1925 subvolume fluctuation dispute between Einstein and Jordan which led Jordan to his discovery of QFT, which afterwards was referred to as the Einstein-Jordan conundrum (an early harbinger of the Unruh effect), is only fully understood in terms of modular localization (B. Schroer, Eur. Phys. J. H. **38**, (2013) 137; arXiv:1101.0569).