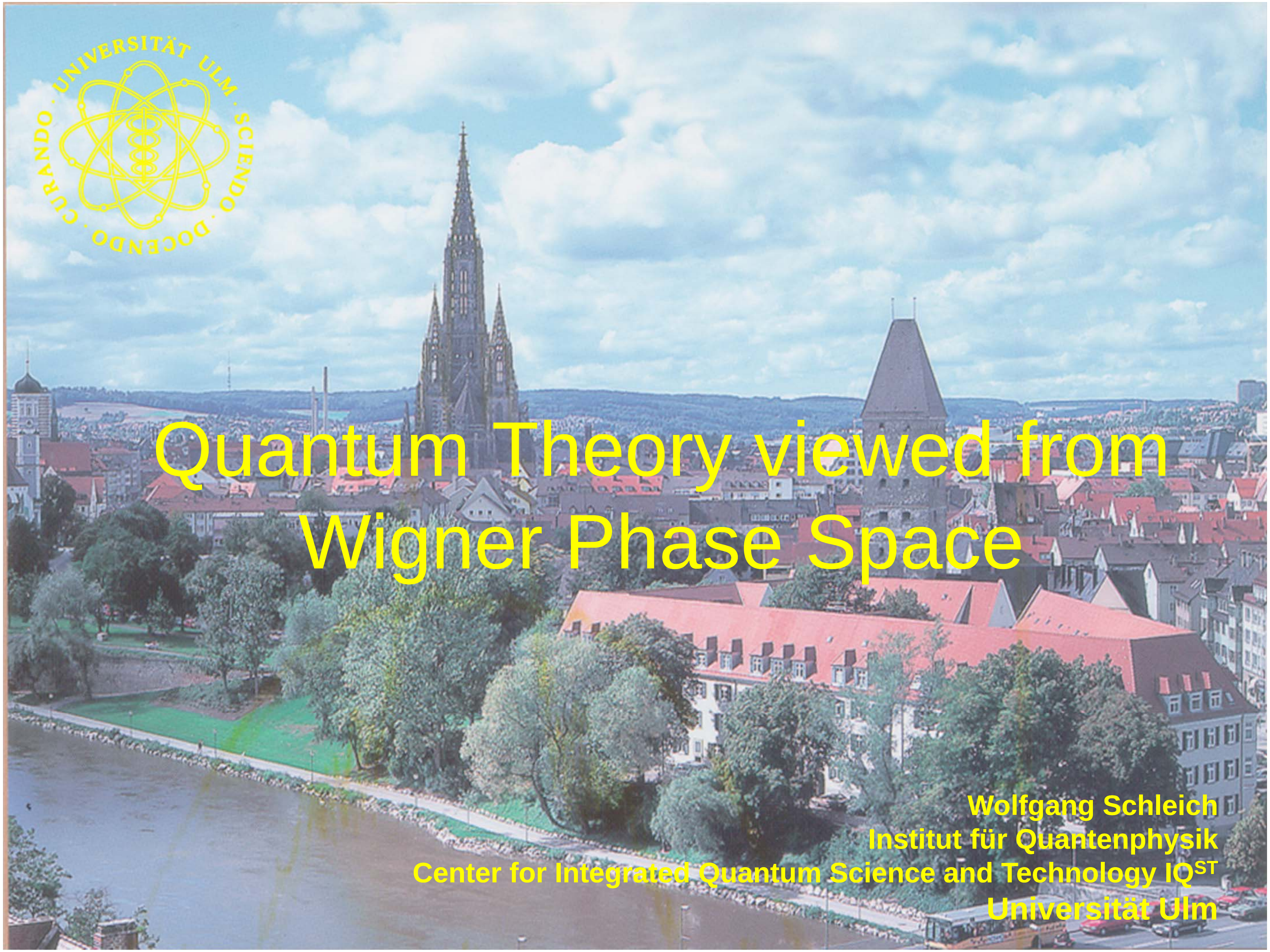


Quantum Theory viewed from Wigner Phase Space

Wolfgang Schleich
Institut für Quantenphysik
Center for Integrated Quantum Science and Technology IQST
Universität Ulm

Overview

- Wigner function essentials
- focusing wave packets
- quantum dynamics and kinematics in phase space
- quantum tunneling
- Wigner function in higher dimensions

JUNE 1, 1932 *PHYSICAL REVIEW* *VOLUME 40*

On the Quantum Correction For Thermodynamic Equilibrium

By E. Wigner
Department of Physics, Princeton University
(Received March 14, 1932)

If a wave function $\psi(x_1 \cdots x_n)$ is given one may build the following expression[2]

$$P(x_1, \cdots, x_n; p_1, \cdots, p_n) = \left(\frac{1}{h\pi}\right)^n \int_{-\infty}^{\infty} \cdots \int dy_1 \cdots dy_n \psi(x_1 + y_1 \cdots x_n + y_n)^* \psi(x_1 - y_1 \cdots x_n - y_n) e^{2i(p_1 y_1 + \cdots + p_n y_n)/h} \quad (5)$$

and call it the probability-function of the simultaneous values of $x_1 \cdots x_n$ for the coordinates and $p_1 \cdots p_n$ for the momenta.

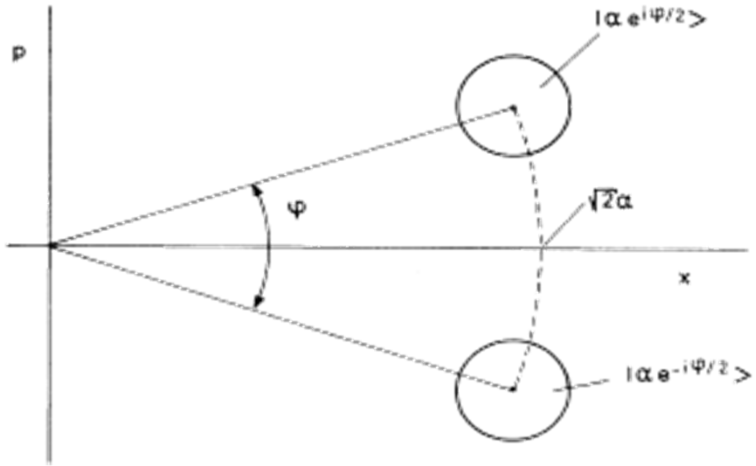
[2] This expression was found by L. Szilard and the present author some years ago for another purpose.

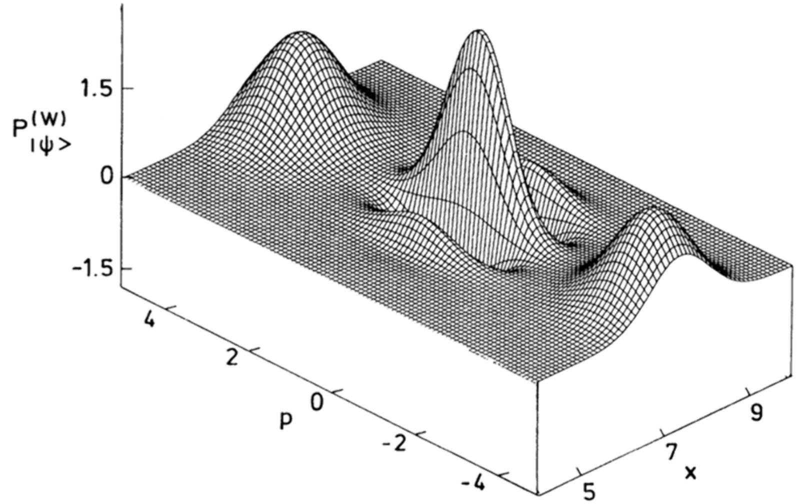
PHYSICAL REVIEW A VOLUME 44, NUMBER 3 1 AUGUST 1991

Nonclassical state from two pseudoclassical states

W. Schleich, M. Pernigo, and Fam Le Kien
Max-Planck-Institut für Quantenoptik, D-8046 Garching bei München, Federal Republic of Germany
and Center for Advanced Studies, Department of Physics and Astronomy, University of New Mexico,
Albuquerque, New Mexico 87131
(Received 3 December 1990)

The quantum-mechanical superposition of two coherent states of identical mean photon number but different phases yields a state that can exhibit sub-Poissonian and oscillatory photon statistics, as well as squeezing.





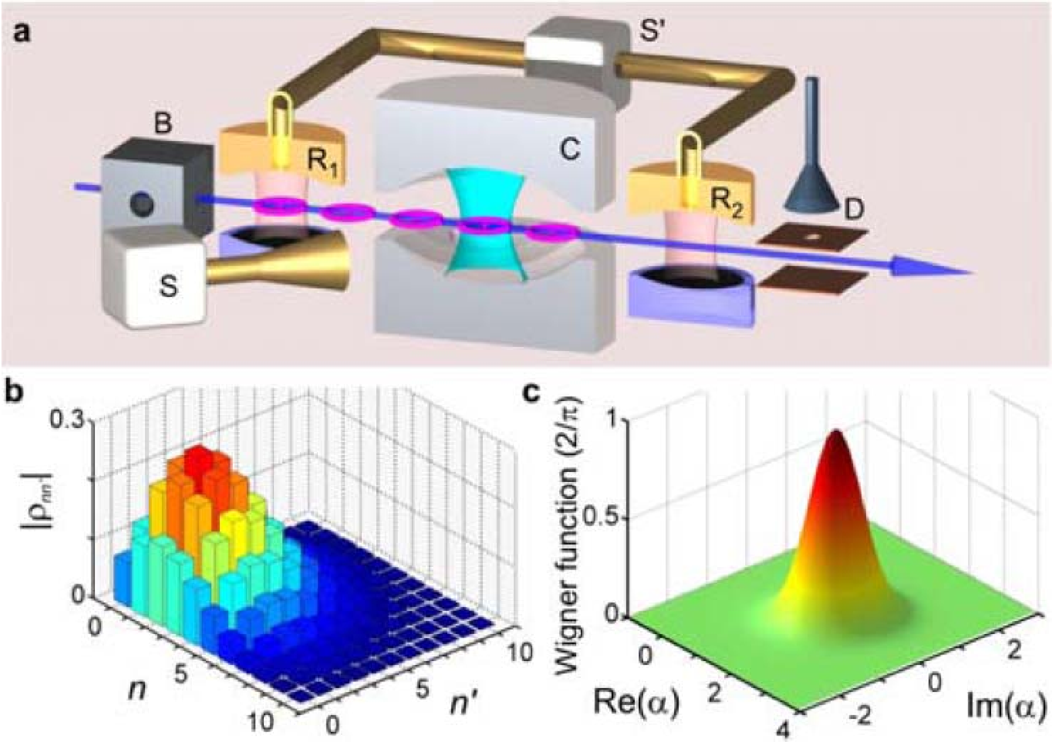
$$|\psi\rangle \sim \sum_n w_n e^{in\varphi}|n\rangle + \sum_n w_n e^{-in\varphi}|n\rangle$$

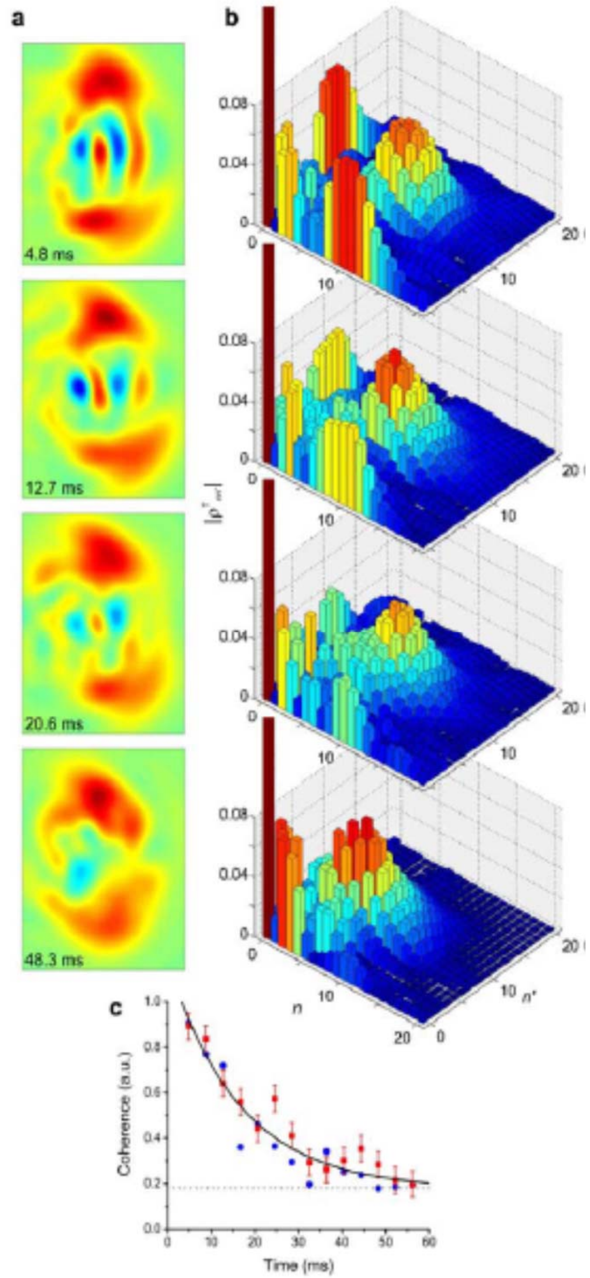
Vol 455|25 September 2008|doi:10.1038/nature07288

LETTERS

Reconstruction of non-classical cavity field states with snapshots of their decoherence

Samuel Deléglise[1], Igor Dotsenko[1,2], Clément Sayrin[1], Julien Bernu[1], Michel Brune[1], Jean-Michel Raimond[1] & Serge Haroche[1,2]





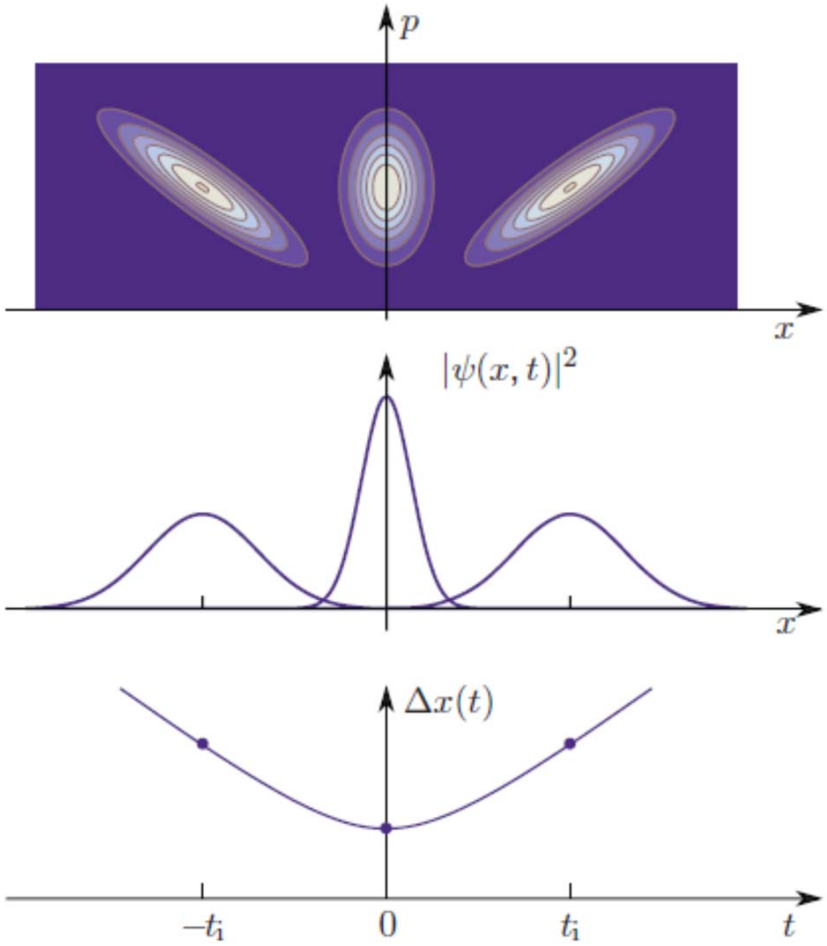
Wigner function: essentials

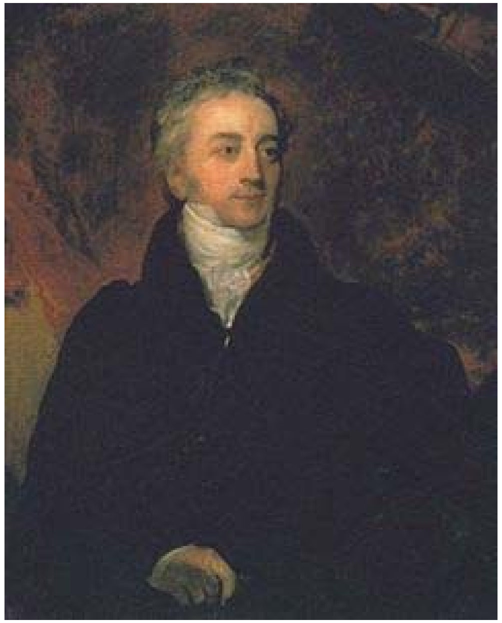
$$W(x,p) \equiv \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} dy\, e^{-ipy/\hbar} \psi^*(x - y/2)\psi(x + y/2)$$

$$\int_{-\infty}^{\infty} dp\, W(x,p) = |\psi(x)|^2$$

$$W(x,p;t) = W_0(x - \frac{p}{m}t, p)$$

Contracting states: classical correlations





Thomas Young (1773-1829)

PHILOSOPHICAL

TRANSACTIONS.

I. *The* Bakerian *Lecture.. Experiments and Calculations relative to physical Optics. By* Thomas Young, *M. D. F. R. S.*

Read November 24, 1803.

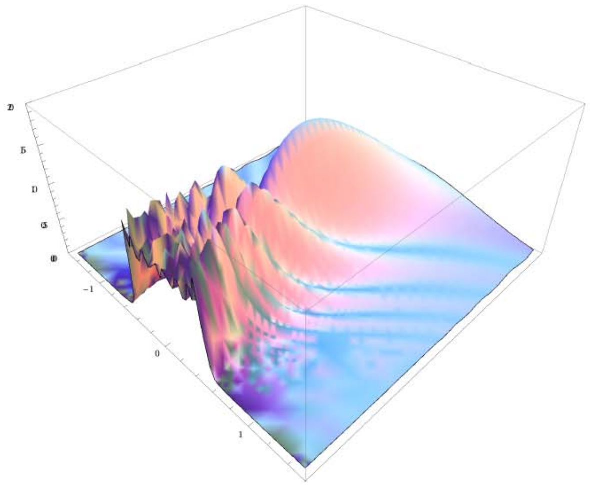
I. EXPERIMENTAL DEMONSTRATION OF THE GENERAL LAW OF THE INTERFERENCE OF LIGHT.

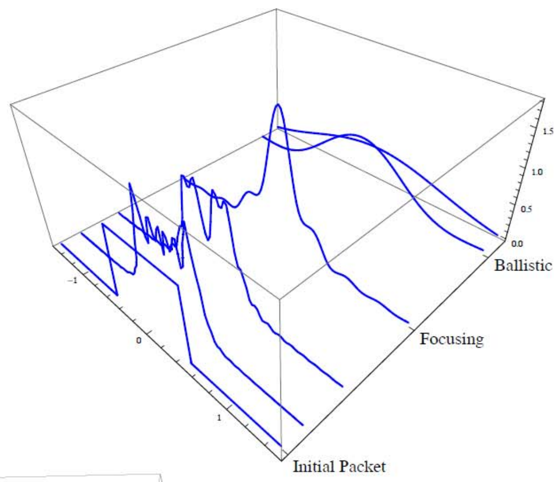
In making some experiments on the fringes of colours accompanying shadows, I have found so simple and so demonstrative a proof of the general law of the interference of two portions of light, which I have already endeavoured to establish, that I think it right to lay before the Royal Society, a short statement of the facts which appear to me so decisive. The proposition on which I mean to insist at present, is simply this, that fringes of colours are produced by the interference of two portions of light; and I think it will not be denied by the most prejudiced, that the assertion is proved by the experiments I am about to relate, which may be repeated with great ease, whenever the sun shines, and without any other apparatus than is at hand to every one.

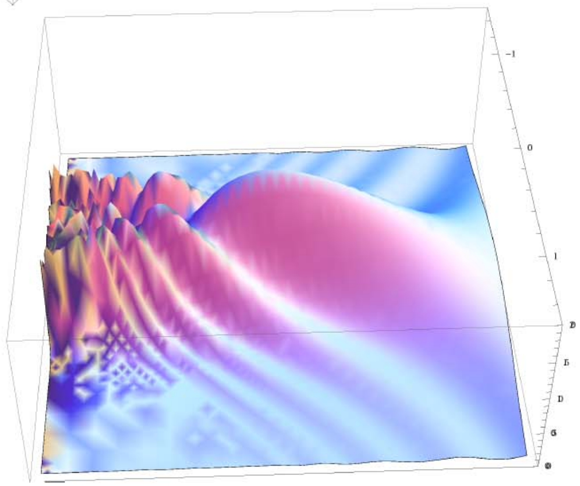
Isaak Newton (1643-1727)

Those who are attached to the NEWTONIAN theory of light, or to the hypotheses of modern opticians, founded on views still less en-larged, would do well to endeavour to imagine any thing like an explanation of these experiments, derived from their own doctrines; and, if they fail in the attempt, to refrain at least from idle declamation against a system which is founded on the accuracy of its application to all these facts, and to a thousand others of a similar nature.

Diffraction from a single slit







A diffractive mechanism of focusing

W. B. Case,[1] E. Sadurni,[2,4,*] and W. P. Schleich[2,3]

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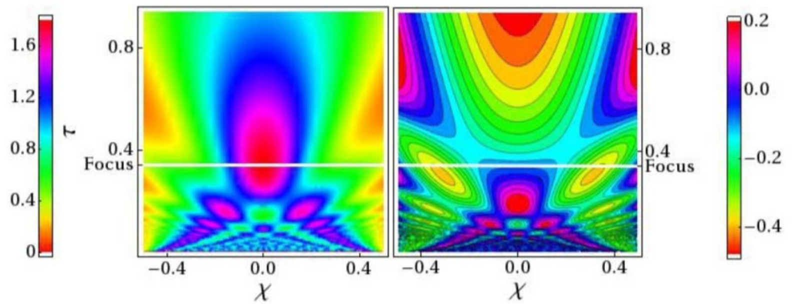
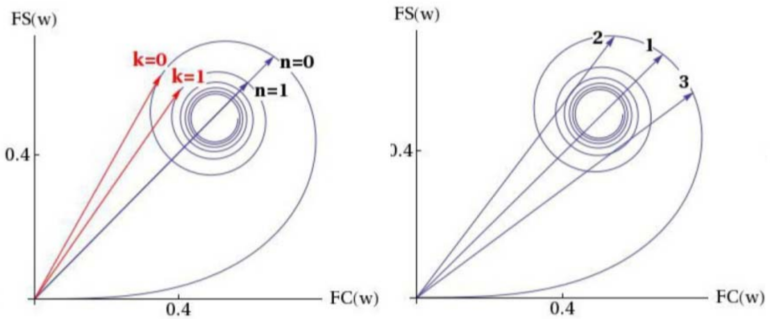
[2] *Institut für Quantenphysik and Center for Integrated Quantum Science and Technology (IQ^{ST}), Universität Ulm, Albert-Einstein-Allee 11, D-89081 Ulm, Germany*
[3] *wolfgang.schleich@uni-ulm.de*
[4] *Instituto de Física, Benemérita Universidad Autónoma de Puebla, Apartado Postal J-48, 72570 Puebla, Mexico*



(a) (b)



(c) (d)

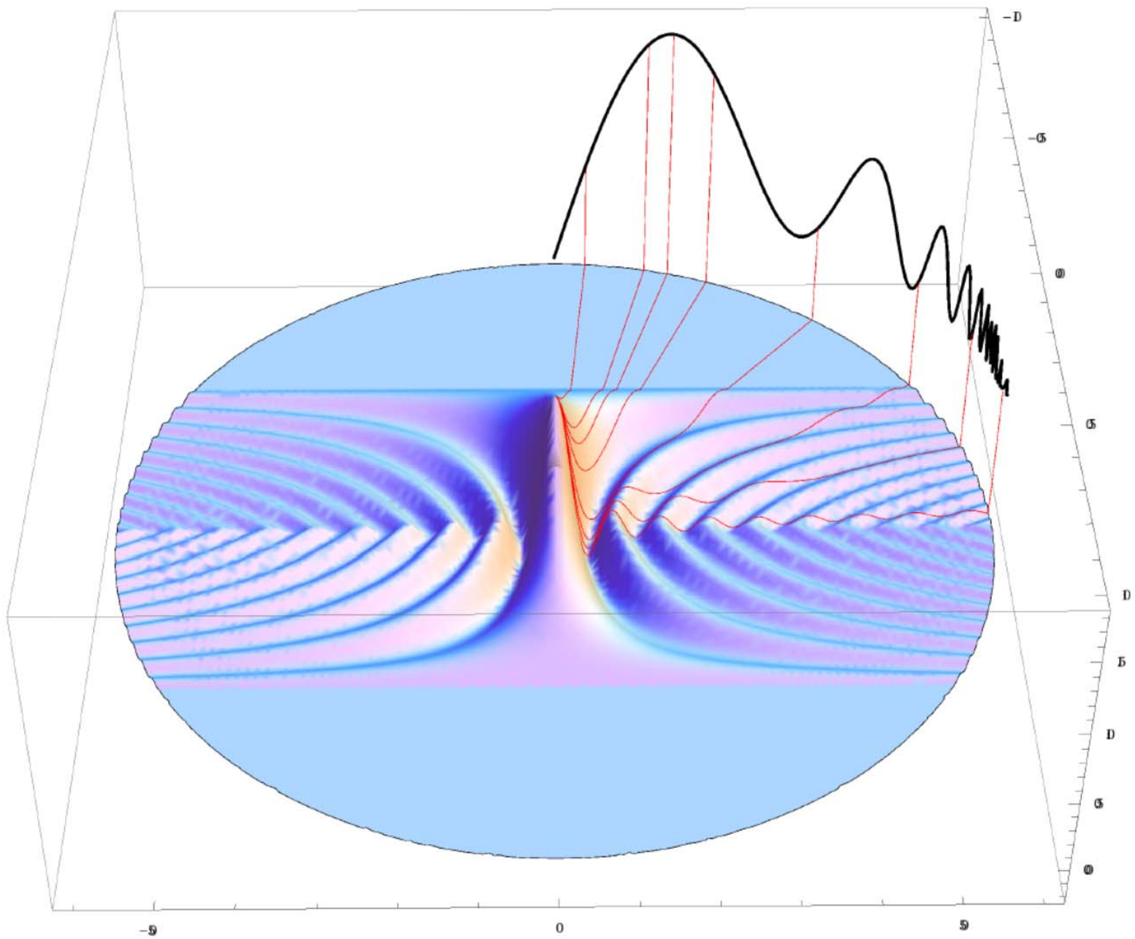
Diffraction from a single slit

$$W(x,p;t) = \int_{-\infty}^{\infty} dx_0 \int_{-\infty}^{\infty} dp_0 W_0(x_0,p_0)\delta\left[x - (x_0 + \frac{pt}{M})\right]\delta(p - p_0)$$

$$|\psi(x,t)|^2 = \int_{-\infty}^{\infty} dx W(x,p;t)$$

$$\begin{aligned}|\psi(x,t)|^2 &= \int_{-\infty}^{\infty} dp W_0\left(x - \frac{pt}{M}, p\right)\\ &= \frac{M}{t}\int_{-\infty}^{\infty} dy W_0\left(y, \frac{M(x-y)}{t}\right)\end{aligned}$$

Intensity distribution from tomographic cuts of Wigner function



Chemical Physics 375 (2010) 133–143

Contents lists available at ScienceDirect

Chemical Physics

journal homepage: www.elsevier.com/locate/chemphys

Optimally focusing wave packets ☆

K. Vogel [a,*], F. Gleisberg [a], N.L. Harshman [a,b], P. Kazemi [a], R. Mack [a], L. Plimak [a], W.P. Schleich [a]

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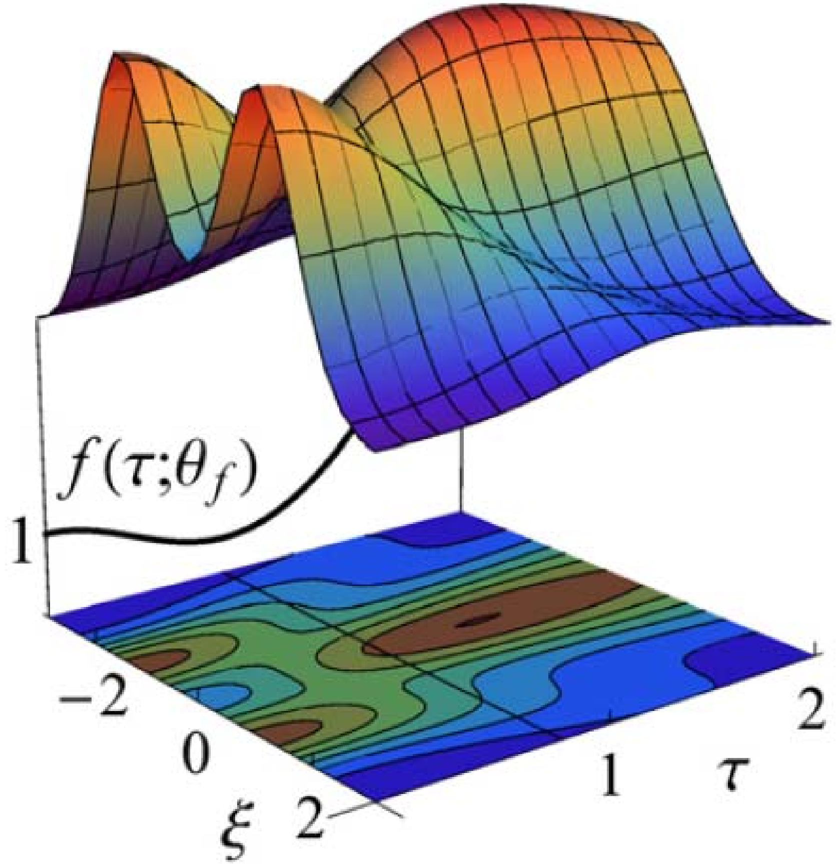
[b] *Department of Physics, American University, 4400 Massachusetts Ave. NW, Washington, DC 20016-8058, USA*

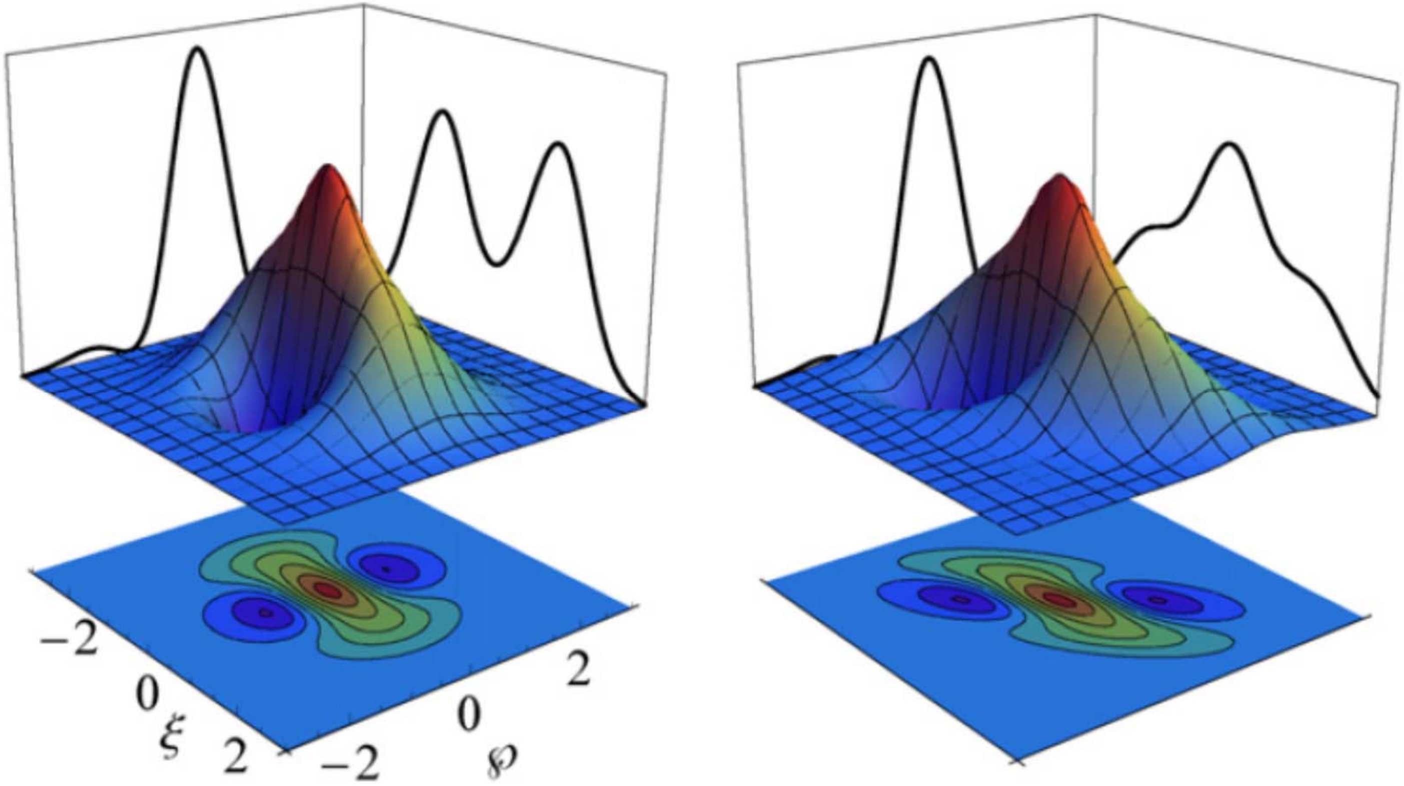
$$\psi(x, t=0) = \cos\theta\, u_0(x) + \sin\theta\, u_2(x)$$

$$u_0, u_2 = \begin{pmatrix} \text{energy wave function} \\ \text{of harmonic oscillator} \end{pmatrix}$$

Optimal wave packet: time evolution



Optimal wave packet: Wigner function



Quantum dynamics in phase space

$$i\hbar \frac{\partial}{\partial t} \left| \psi(t) \right\rangle = \hat{H} \left| \psi(t) \right\rangle \qquad \hat{H} \equiv \frac{\hat{p}^2}{2m} + V(\hat{z})$$

$$\left(\frac{\partial}{\partial t} + \frac{p}{m} \frac{\partial}{\partial z} - \frac{\partial V}{\partial z} \frac{\partial}{\partial p} - \hat{\mathcal{L}}_o \right) W(z, p; t) = 0$$

$$\hat{\mathcal{L}}_o \equiv \sum_{l=1}^{\infty} \frac{(-1)^l}{(2l+1)!} \left(\frac{\hbar}{2} \right)^{2l} \frac{\partial^{2l+1} V(z)}{\partial z^{2l+1}} \frac{\partial^{2l+1}}{\partial p^{2l+1}}$$

Quantum kinematics in phase space

$$\hat{H}|E\rangle = E|E\rangle$$

$$\left(\frac{p}{m}\frac{\partial}{\partial z} - \frac{\partial V}{\partial z}\frac{\partial}{\partial p} - \hat{\mathcal{L}}_{\mathrm{o}}\right) W_E = 0$$

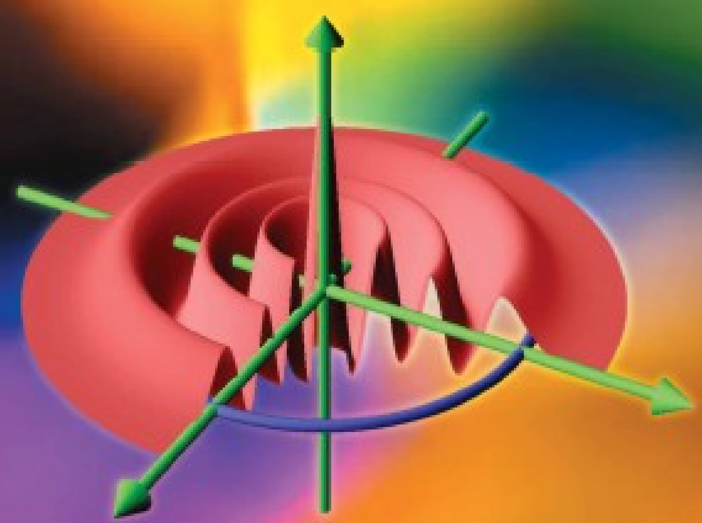
$$\left(-\frac{\hbar^2}{8m}\frac{\partial^2}{\partial z^2} + \frac{p^2}{2m} + V(z) + \hat{\mathcal{L}}_{\mathrm{e}}\right) W_E = E W_E$$

$$\hat{\mathcal{L}}_{\mathrm{e}} \equiv \sum_{l=1}^{\infty} \frac{(-1)^l}{(2l)!}\left(\frac{\hbar}{2}\right)^{2l} \frac{\partial^{2l} V(z)}{\partial z^{2l}} \frac{\partial^{2l}}{\partial p^{2l}}$$

Area in phase space

$$2\pi\hbar \leq \left[\int_{-\infty}^{\infty} \mathrm{d}z \int_{-\infty}^{\infty} \mathrm{d}p\, W_0^2(z, p)\right]^{-1}$$

opn
Vol. 3 No. 1, October 2003
TRENDS



The Nature of Light
What Is a Photon?

OSA
Supplement to Optics & Photonics News

Physics Letters A 377 (2013) 1822–1825

Contents lists available at SciVerse ScienceDirect

Physics Letters A

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Tunneling of an energy eigenstate through a parabolic barrier viewed from Wigner phase space

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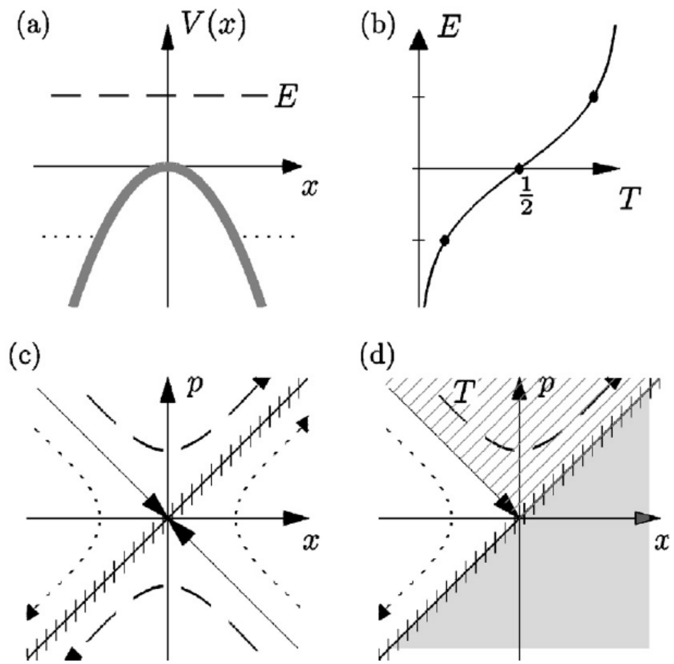
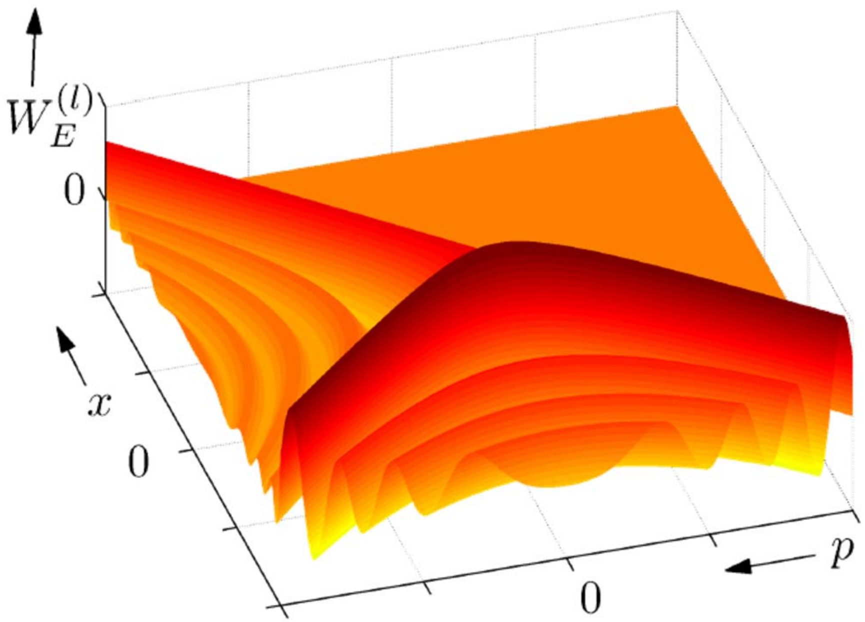
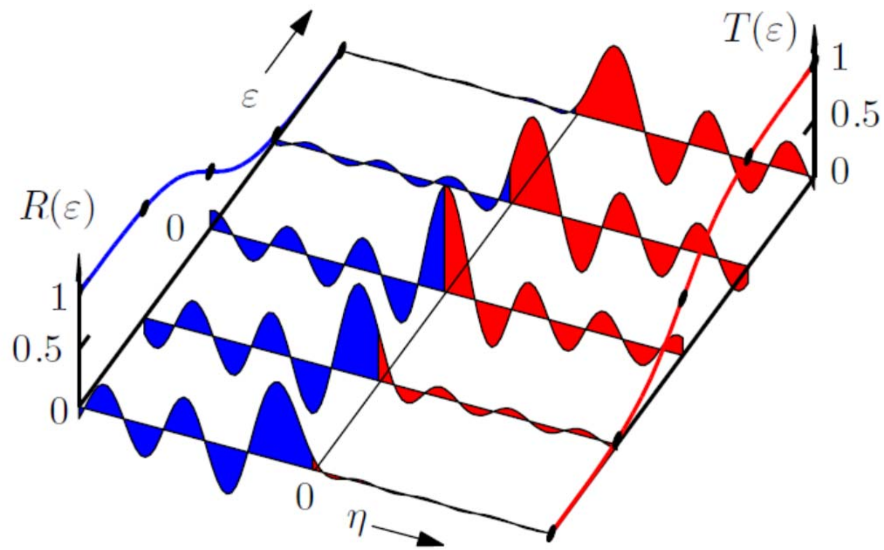
[b] Information Directorate, Air Force Research Laboratory, Rome, NY 13441, USA
[c] Chemical Physics, Department of Chemistry, Technical University of Denmark, DTU 207, DK-2800 Kgs. Lyngby, Denmark
[d] Wigner Research Centre for Physics, Hungarian Academy of Sciences, Institute for Solid State Physics and Optics, 1525 Budapest, Hungary



$$\left[\frac{p}{M}\frac{\partial}{\partial x} + M\Omega^2 x\frac{\partial}{\partial p}\right] W_E(x,p) = 0$$

$$\left\{\left[\frac{p^2}{2M} - \frac{1}{2}M\Omega^2 x^2\right] - \frac{\hbar^2}{8}\left[\frac{1}{M}\frac{\partial^2}{\partial x^2} - M\Omega^2\frac{\partial^2}{\partial p^2}\right]\right\}$$
$$\times W_E(x,p) = EW_E(x,p)$$

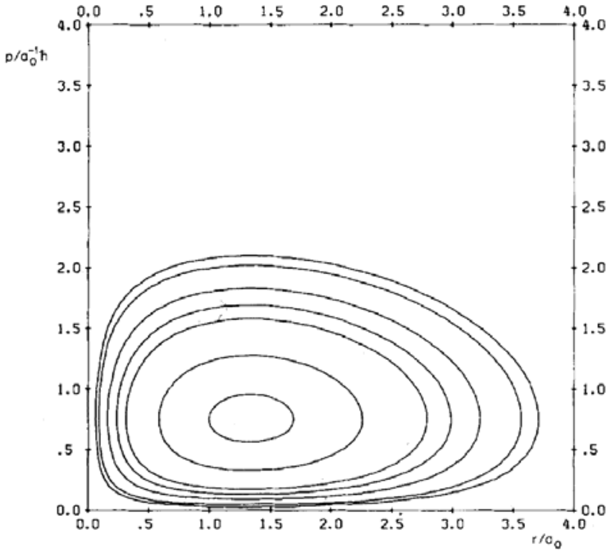


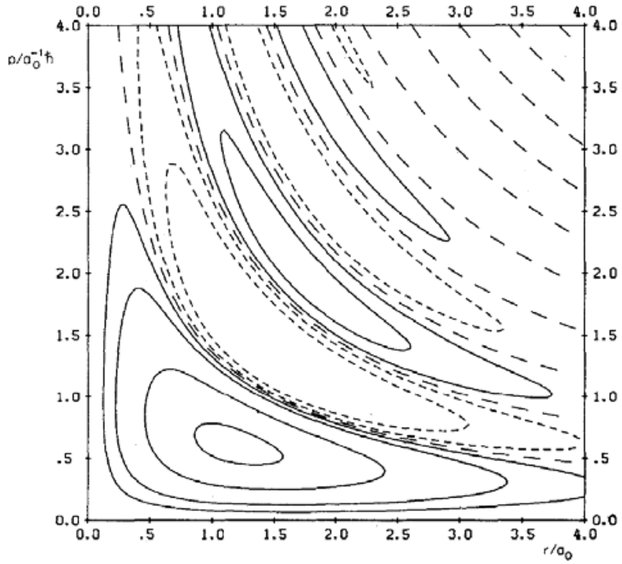


MOLECULAR PHYSICS, 1982, VOL. 47, NO. 5, 1001–1019

Wigner's phase space function and atomic structure
I. The hydrogen atom ground state

by JENS PEDER DAHL and MICHAEL SPRINGBORG
Department of Chemical Physics, Technical University of Denmark,
DTH 301, DK-2800 Lyngby, Denmark





PHYSICAL REVIEW A **75**, 052107 (2007)

Wigner functions of s waves

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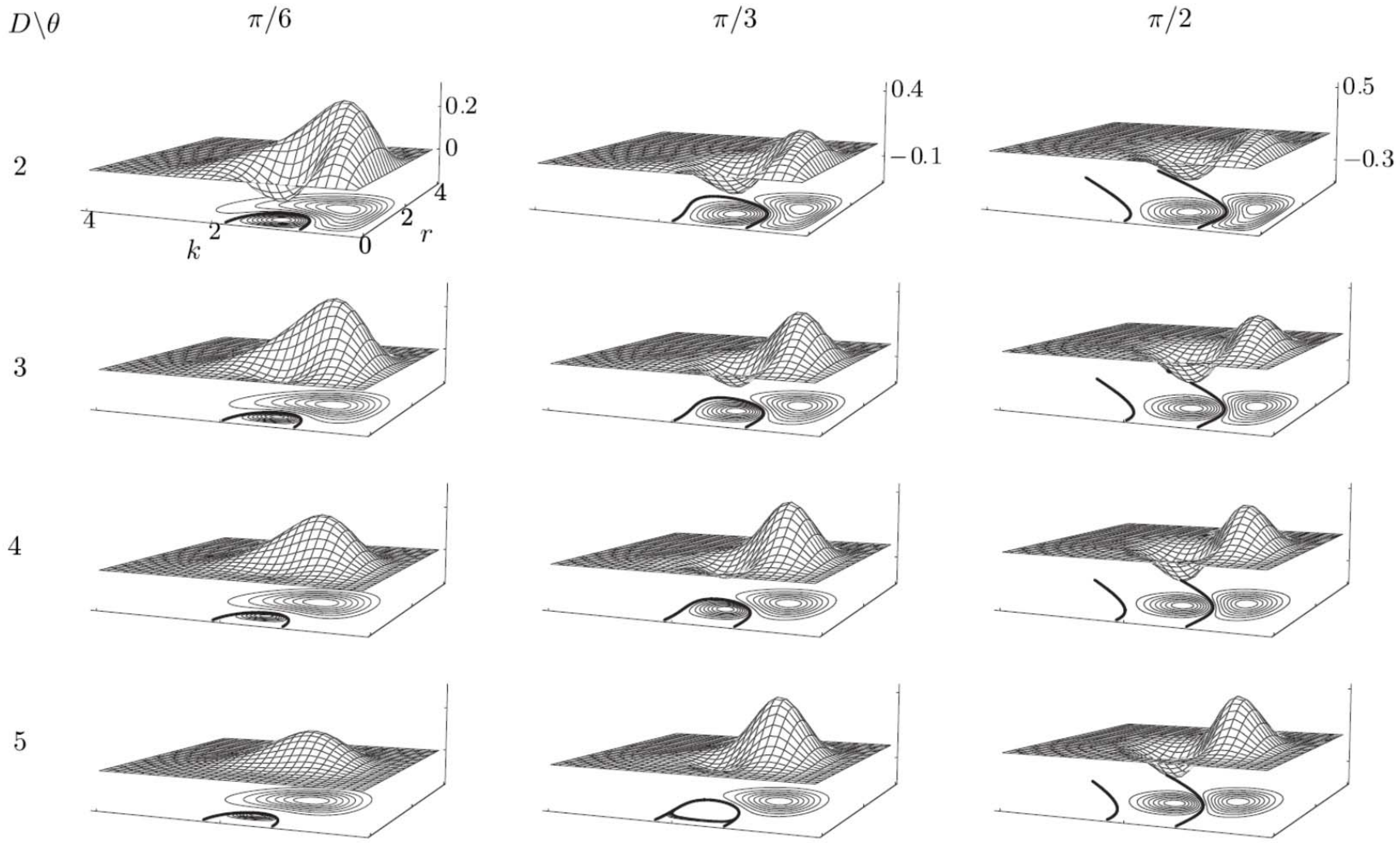

In- and Outbound Spreading of a Free-Particle *s*-Wave

I. Białynicki-Birula,[1,2] M. A. Cirone,[2] J. P. Dahl,[2,3] M. Fedorov,[2,4] and W. P. Schleich[2]

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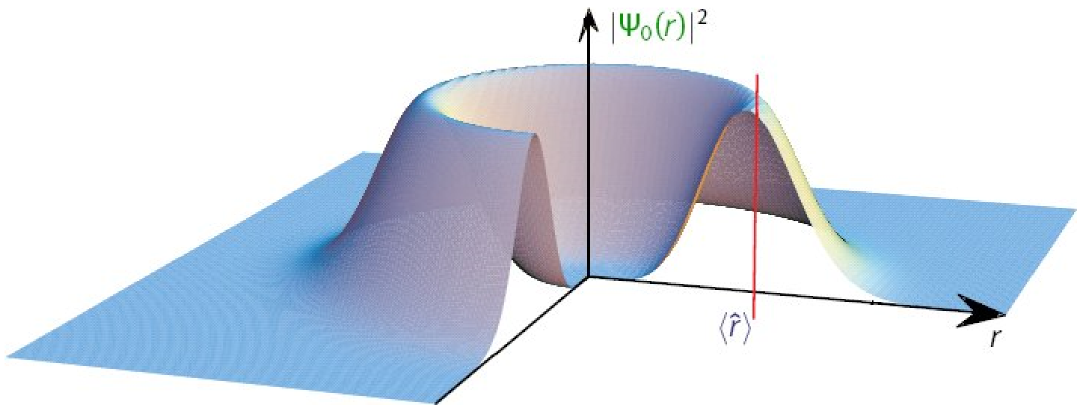
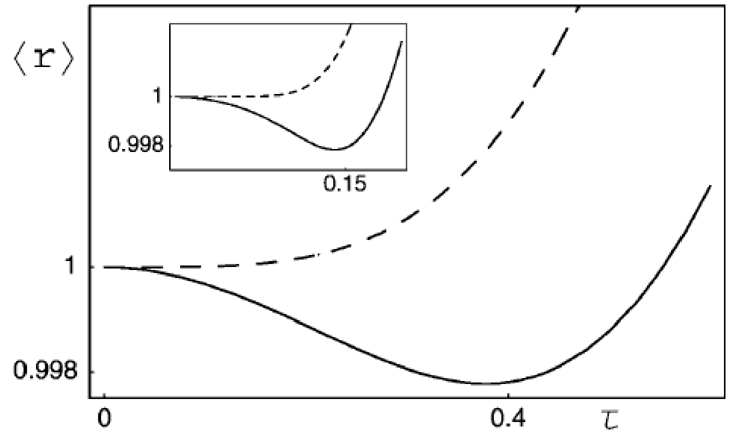
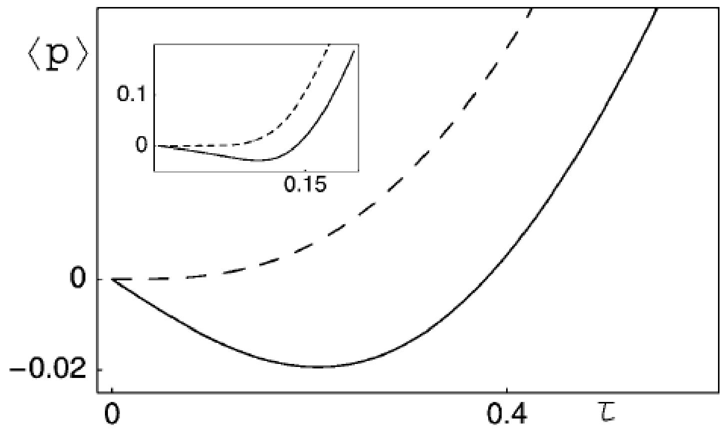
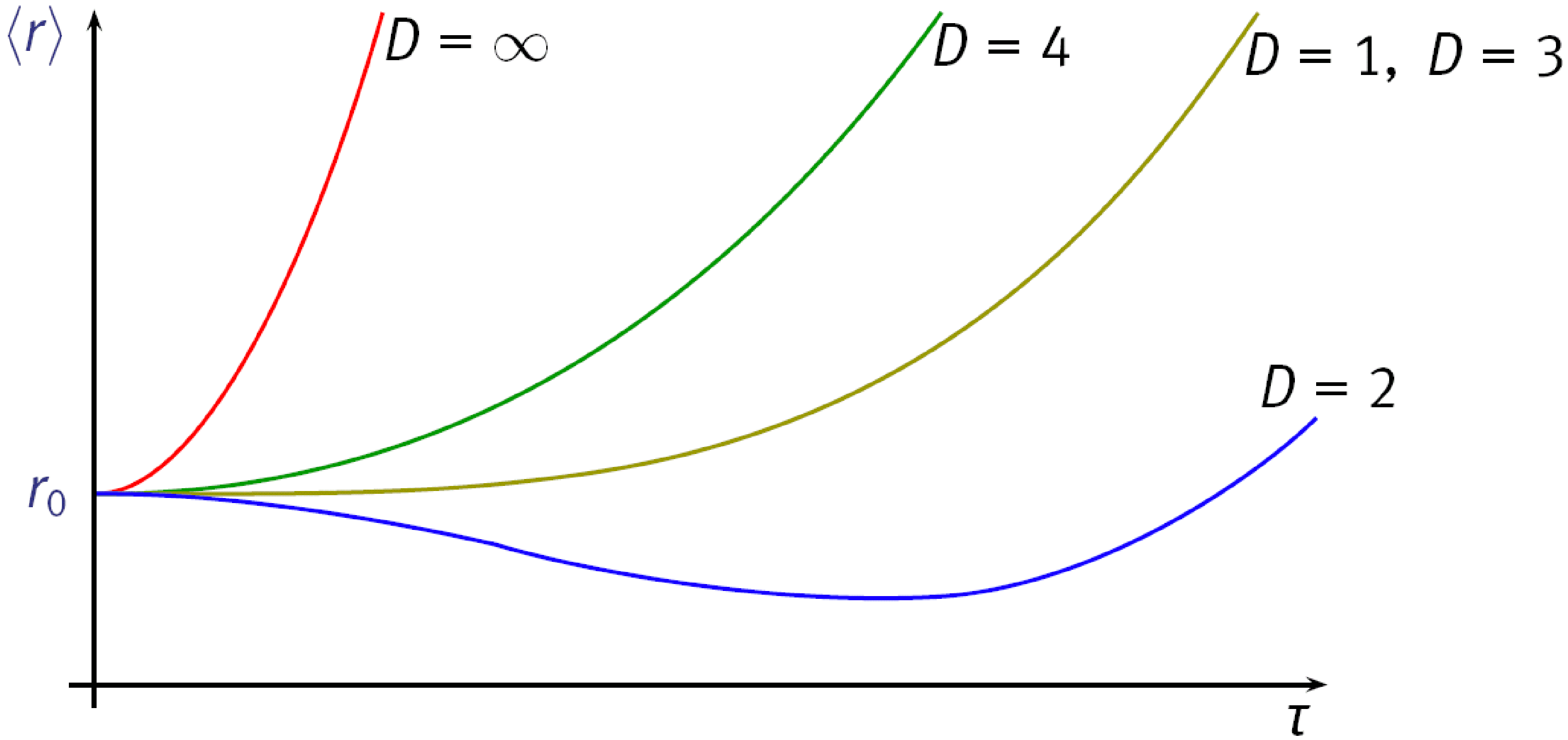






Dependence on dimensions



New Journal of Physics
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A representation-free description of the Kasevich–Chu interferometer: a resolution of the redshift controversy

Wolfgang P Schleich[1], **Daniel M Greenberger**[2] **and Ernst M Rasel**[3]

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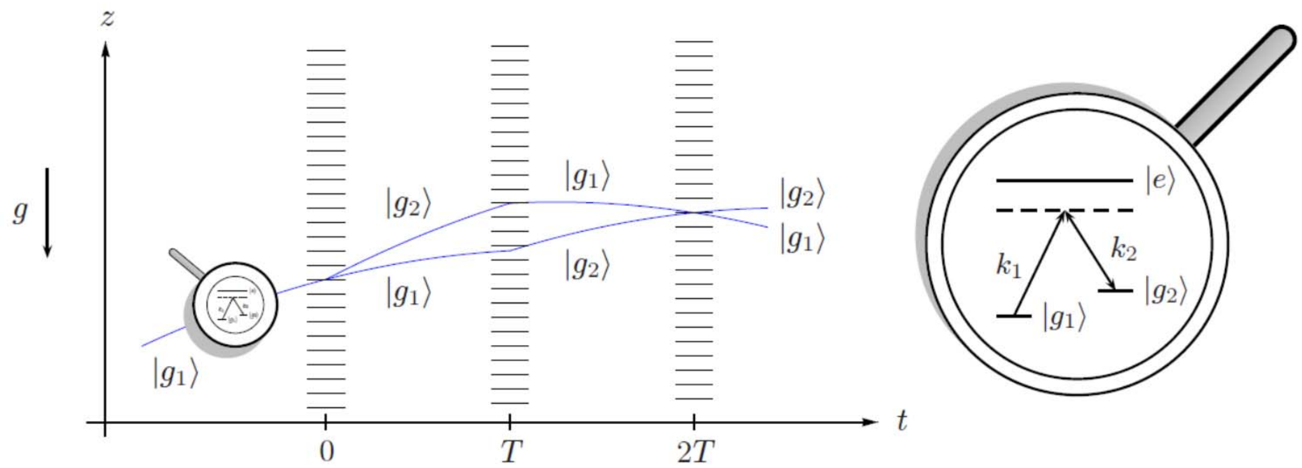
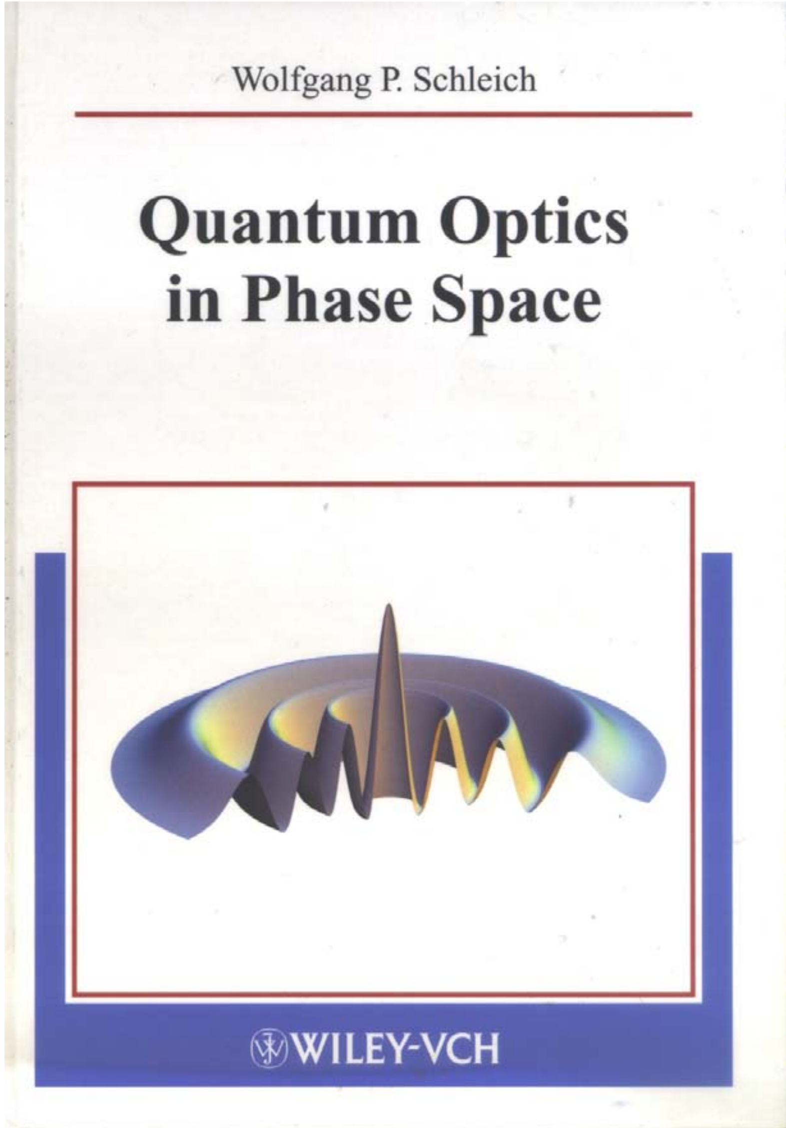
New Journal of Physics **15** (2013) 013007 (48pp)

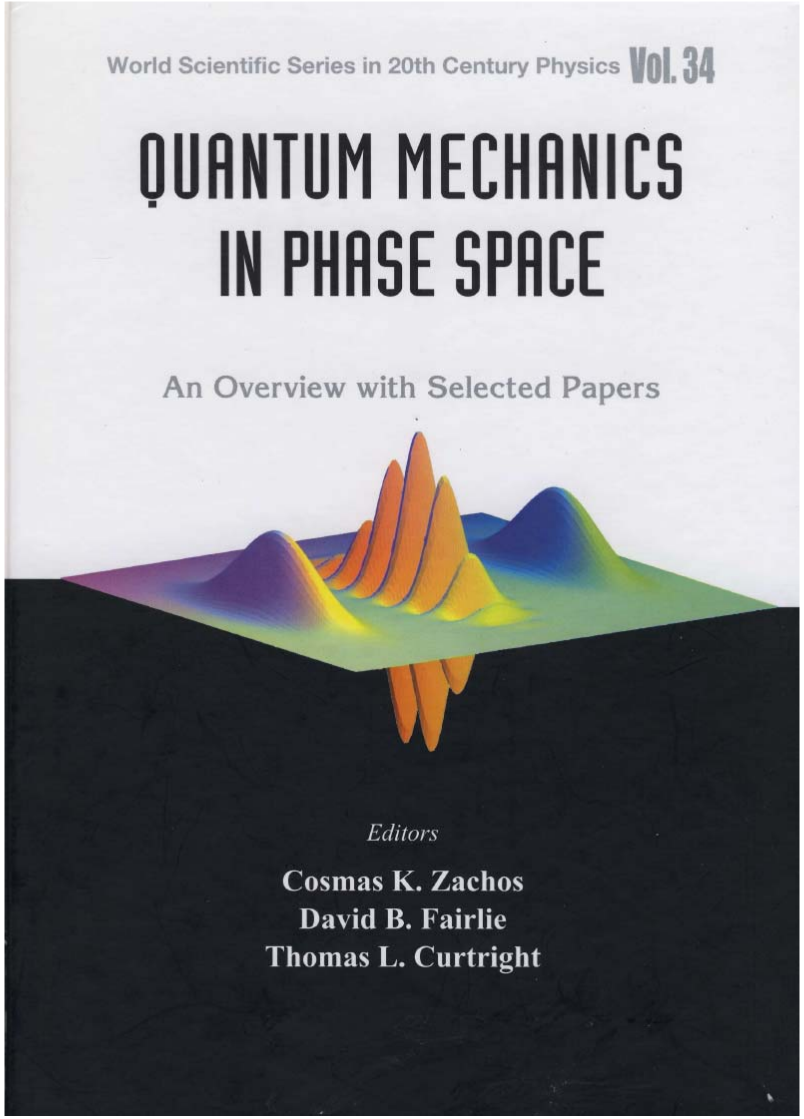


Summary

- Wigner function essentials
- focusing wave packets
- quantum dynamics and kinematics in phase space
- quantum tunneling
- Wigner function in higher dimensions



Wolfgang P. Schleich
Quantum Optics in Phase Space
WILEY-VCH



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