Coloured condensates deep inside compact stars

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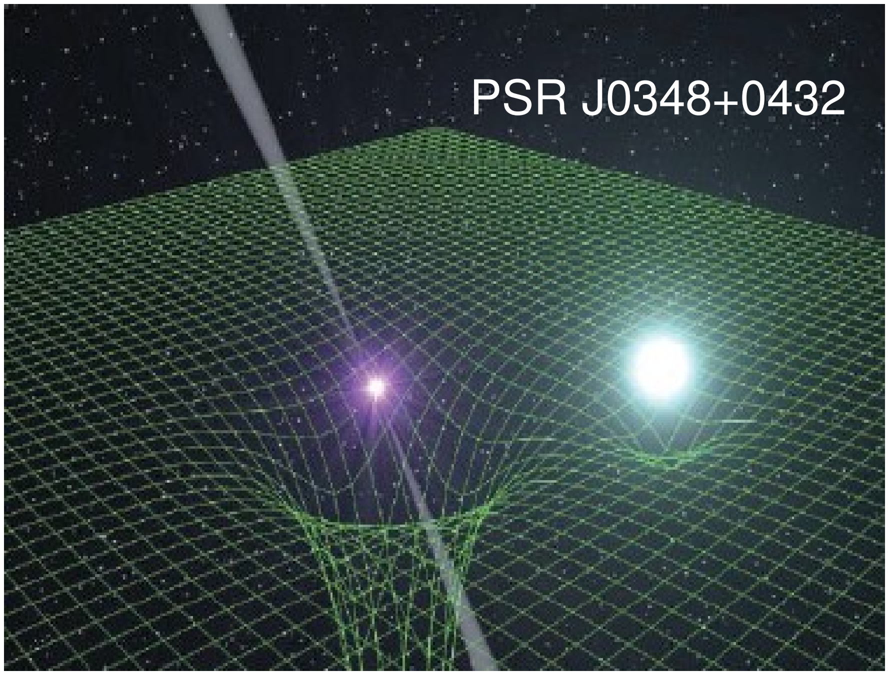
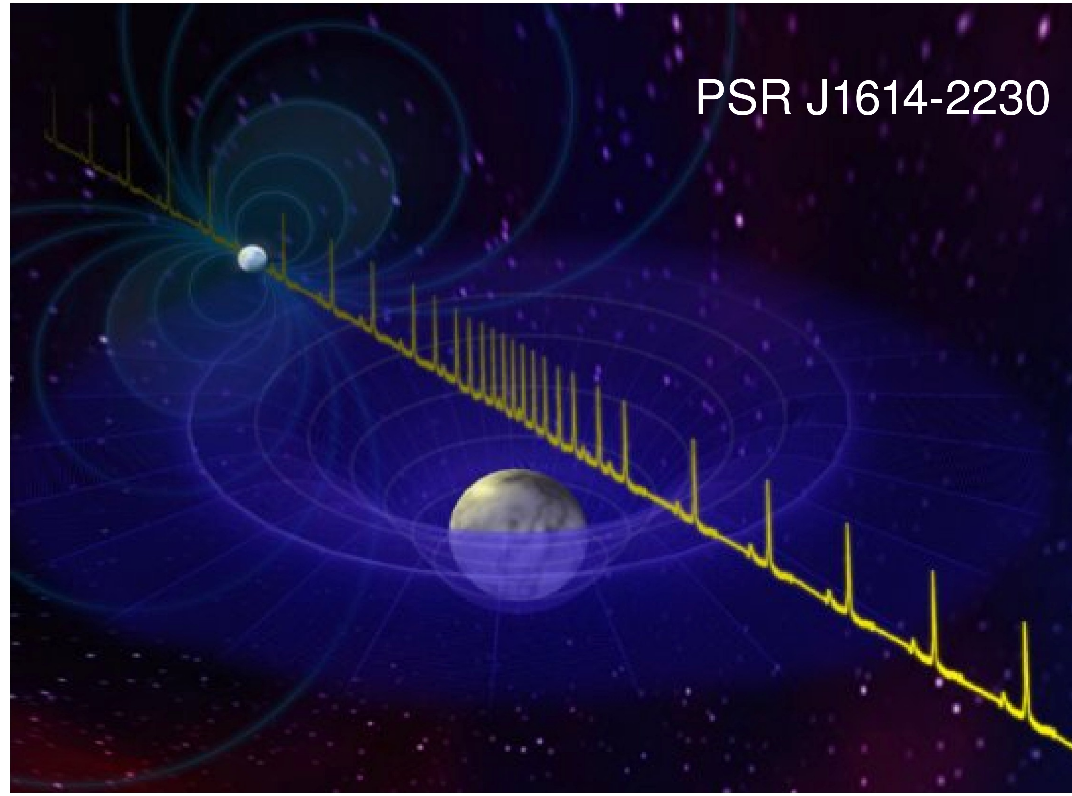


Antoniadis et al., Science 340 (2013) 448
Demorest et al., Nature 467 (2010) 1081



WIGNER – Colorful and Deep, Budapest, 12.11.13

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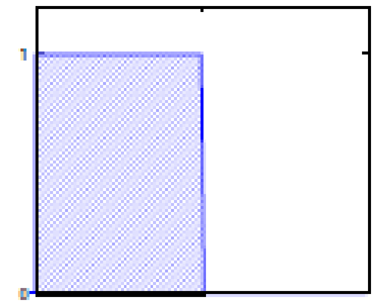
Coloured condensates (CC) - Deep inside compact stars?

- Cooper instability
- Quark condensates
- Symmetries and pairing patterns
- Two-flavor color superconductors → 2SC phase
- Three-flavor color superconductors → CFL phase
- NJL model and Nambu-Gorkov formalism
- Mean field gap equations and solutions
- Thermodynamic potential
- Phase diagram
- EoS and TOV equations – Hybrid stars with CC

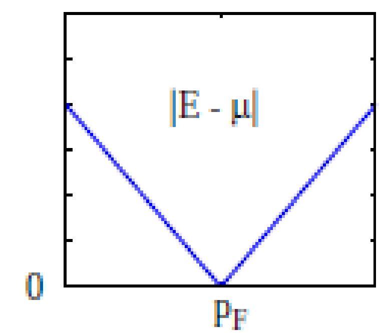
Cooper instabilities

- ideal Fermi gas:
  - ➜ pair creation @ Fermi surface
    with no free energy
- (arbitrarily small) attraction:
  - ➜ instability:
    condensation of Cooper pairs

occupation #



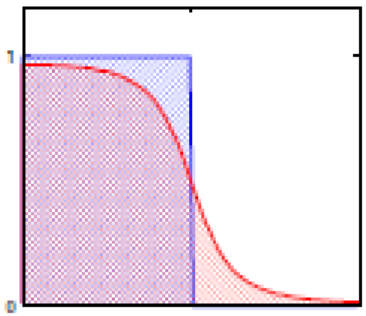
free energy



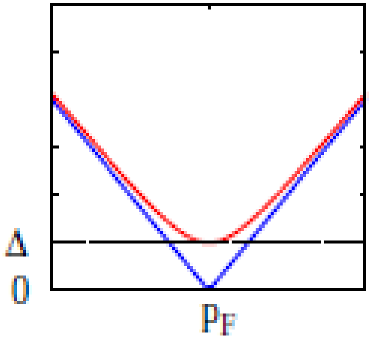
Cooper instabilities

- ideal Fermi gas:
  - ➜ pair creation @ Fermi surface with no free energy
- (arbitrarily small) attraction:
  - ➜ instability: condensation of Cooper pairs
  - ➜ reorganisation of the Fermi surface
  - ➜ gaps

occupation #



free energy



- QCD: attractive qq interaction ➜ diquark condensates

Field operators

- quark field operator: $q(x) = \begin{pmatrix} q_1(x) \\ \vdots \\ q_{4N_fN_c}(x) \end{pmatrix}$
  - 4 Dirac $\times N_f$ flavor $\times N_c$ color components
  - annihilates a quark or creates an antiquark
- transposed operator: $q^T = (q_1, \ldots, q_{4N_fN_c})$
- adjoint operator: $q^\dagger, \qquad \bar{q} = q^\dagger \gamma^0$
  - annihilates an antiquark or creates a quark

Quark-antiquark condensates

- quark-antiquark condensates: $\langle \bar{q}\, \hat{\mathcal{O}}\, q \rangle$
  - $\hat{\mathcal{O}}$ = operator in color, flavor, and Dirac space (including derivatives)

- examples:
  - "chiral condensate": $\langle \bar{q} q \rangle$
  - quark number density: $\langle \bar{q} \gamma^0 q \rangle = \langle q^\dagger q \rangle$
  - electric charge density:
    $\langle \bar{q}\, \hat{Q} \gamma^0\, q \rangle = \frac{2}{3} n_u - \frac{1}{3} n_d - \frac{1}{3} n_s, \qquad \hat{Q} = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}_f$
  - color charge densities

Diquark condensates

- diquark condensates: $\langle q^T \hat{O} q \rangle \;\equiv\; \langle g.s. | \, q^T \hat{O} \, q \, | g.s. \rangle$
  - qq annihilates two quarks

    ➜ baryon number (formally) not conserved!

    (ground state does not have fixed baryon number.)

- Bogoliubov rotation:

$$|g.s.\rangle \;=\; \prod_{\vec{p},s,c,c'} \left[\cos\theta_s^b(\vec{p}) + \varepsilon_{3cc'}\, e^{i\xi_s^b(\vec{p})}\, \sin\theta_s^b(\vec{p})\, b^\dagger(\vec{p},s,u,c)\, b^\dagger(-\vec{p},s,d,c')\right]$$
$$\left[\cos\theta_s^d(\vec{p}) + \varepsilon_{3cc'}\, e^{i\xi_s^d(\vec{p})}\, \sin\theta_s^d(\vec{p})\, d^\dagger(\vec{p},s,u,c)\, d^\dagger(-\vec{p},s,d,c')\right] |0\rangle$$

  - quasiparticles $=$ superpositions of particles and holes ($+$ antiparticles) which annihilate $|g.s.\rangle$

Diquark condensates

- diquark condensates: $\langle q^T \hat{\mathcal{O}} q \rangle$

- Pauli principle: $q_i q_j = -q_j q_i$

$$\Rightarrow q^T \hat{\mathcal{O}} q = q_i \hat{\mathcal{O}}_{ij} q_j = -q_j \hat{\mathcal{O}}_{ij} q_i = -q_j \hat{\mathcal{O}}^T_{ji} q_i = -q^T \hat{\mathcal{O}}^T q$$

➜ $\hat{\mathcal{O}}$ must be totally antisymmetric: $\hat{\mathcal{O}}^T = -\hat{\mathcal{O}}$

Operators in flavor and color space

- Pauli matrices (for two flavors):

$$\underbrace{\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},\ \tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},}_{\text{symmetric triplet}} \quad \underbrace{\tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}}_{\text{antisymm. singlet}}$$

- Gell-Mann matrices (for three flavors or colors):

$$\underbrace{\mathbb{1},\ \lambda_1,\ \lambda_3,\ \lambda_4,\ \lambda_6,\ \lambda_8,}_{\text{symmetric sextet}} \qquad \underbrace{\lambda_2,\ \lambda_5,\ \lambda_7}_{\text{antisymmetric antitriplet}}$$

  - antitriplet: The vector $\langle q^T \begin{pmatrix} \lambda_7 \\ -\lambda_5 \\ \lambda_2 \end{pmatrix} q \rangle$ transforms like an antiquark $\bar{q} = \begin{pmatrix} \bar{r} \\ \bar{g} \\ \bar{b} \end{pmatrix}$ under $SU(3)_c$.

Operators in Dirac space

- hermitean basis of 4×4 matrices: $\mathbb{1},\ i\gamma_5,\ \gamma^\mu,\ \gamma^\mu\gamma_5,\ \sigma^{\mu\nu}$
- charge conjugation matrix: $C = i\gamma^2\gamma^0$
  - properties: $C = C^* = -C^T = -C^\dagger = -C^{-1}$
- antisymmetric:
  - $C\gamma_5$ (scalar)
  - C (pseudoscalar)
  - $C\gamma^\mu\gamma_5$ (vector)
- symmetric:
  - $C\gamma^\mu$ (axial vector)
  - $C\sigma^{\mu\nu}$ (tensor)

Combined operators

	symmetric	antisymmetric
Dirac	$C\gamma^\mu$, $C\sigma^{\mu\nu}$ A T	C, $C\gamma_5$, $C\gamma_5\gamma^\mu$ P S V
$U(2)$	$\underbrace{\mathbb{1}, \tau_1, \tau_3}_{3}$	$\underbrace{\tau_2}_{1}$
$U(3)$	$\underbrace{\mathbb{1}, \lambda_1, \lambda_3, \lambda_4, \lambda_6, \lambda_8}_{6}$	$\underbrace{\lambda_2, \lambda_5, \lambda_7}_{\bar{3}}$

- combination: Dirac $\otimes$ flavor $\otimes$ color

$$\text{totally antisymmetric} = \begin{cases} (\text{antisymmetric})^3 \\ (\text{symmetric})^2 \times \text{antisymmetric} \end{cases}$$

→ many possibilities ...

Two-flavor color superconductors

- important example: $\langle q^T C\gamma_5 \tau_A \lambda_{A'} q \rangle$
  - spin 0, antisymmetric in color and flavor
  - 2 flavors: $q = \begin{pmatrix} u \\ d \end{pmatrix}, \quad \tau_A = \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$
  - 3 colors: $q = \begin{pmatrix} r \\ g \\ b \end{pmatrix}, \quad \lambda_{A'} \stackrel{e.g.}{=} \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$$\langle q^T C\gamma_5 \tau_2 \lambda_2 q \rangle \sim \underbrace{(\uparrow\downarrow - \downarrow\uparrow)}_{\text{spin}} \otimes \underbrace{(ud - du)}_{\text{flavor}} \otimes \underbrace{(r\,g - g\,r)}_{\text{color}}$$

Symmetry properties: color

- only red and green quarks are paired:

  $$\langle q^T \lambda_2 q \rangle \sim \langle rg - gr \rangle \hat{=} \langle \bar{b} \rangle$$ "antiblue"

  ➜ $SU(3)_c$ "spontaneously" broken to $SU(2)_c$

  ➜ 5 of 8 gluons receive a mass ("Meissner effect")

- more general ansatz?

  $$\langle q^T (\alpha\lambda_2 + \beta\lambda_5 + \gamma\lambda_7)\, q \rangle = \alpha \langle \bar{b} \rangle - \beta \langle \bar{g} \rangle + \gamma \langle \bar{r} \rangle \equiv \begin{pmatrix} \gamma \\ -\beta \\ \alpha \end{pmatrix}$$

  - can always be rotated into the "antiblue" direction by a global color transformation $q \to \exp(i\theta_a \cdot \frac{\lambda_a}{2})\, q$

  ➜ equivalent to the "simple" ansatz

Symmetry properties: global symmetries

- $\delta \equiv \langle q^T C\gamma_5 \tau_2 \lambda_2 q \rangle$

- baryon number:

  B: $\quad q \to e^{i\alpha} q \quad \Rightarrow \quad \delta \to e^{2i\alpha}\delta$ broken

but: There is a conserved *"modified baryon number"*:

$\tilde{B} \quad q \to e^{i\alpha(1-\sqrt{3}\lambda_8)} q \quad \Rightarrow \quad \delta \to \delta$

- chiral symmetry:

  - $SU(2)_V$: $\quad q \to e^{i\theta_a \frac{\tau_a}{2}} q \quad \Rightarrow \quad \delta \to \delta$
  - $SU(2)_A$: $\quad q \to e^{i\theta_a \frac{\tau_a}{2}\gamma_5} q \quad \Rightarrow \quad \delta \to \delta$ conserved

Three-flavor color superconductors

- scalar color-antitriplet condensates:
  - $s_{AA'} = \langle q^T C\gamma_5 \tau_A \lambda_{A'} q \rangle$
  - notation:
    - $\tau_A =$ antisymmetric flavor generator
    - $\lambda_{A'} =$ antisymmetric color generator
- two flavors, three colors:
  - $\tau_A = \tau_2, \quad A' \in \{2,5,7\} \quad \Rightarrow \quad \vec{s} = (s_{22}, s_{25}, s_{27})$
  - can always be rotated into “antiblue” direction:

    $$\vec{s} \to \vec{s}\,' = \vec{s}\,U = (s'_{22}, 0, 0), \quad U \in SU(3)_c$$
- three flavors, three colors:
  - $A, A' \in \{2,5,7\} \quad \Rightarrow \quad s = (s_{AA'}) = \begin{pmatrix} s_{22} & s_{25} & s_{27} \\ s_{52} & s_{55} & s_{57} \\ s_{72} & s_{75} & s_{77} \end{pmatrix}$

Three-flavor color superconductors

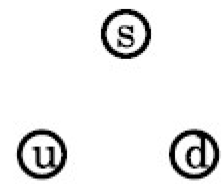
- three flavors, three colors:
  - $A, A' \in \{2,5,7\} \quad \Rightarrow \quad s = (s_{AA'}) = \begin{pmatrix} s_{22} & s_{25} & s_{27} \\ s_{52} & s_{55} & s_{57} \\ s_{72} & s_{75} & s_{77} \end{pmatrix}$
  - $SU(3)_c$ rotation: $\quad \rightarrow s' = s\,U \quad = \begin{pmatrix} s_{22} & s_{25} & s_{27} \\ 0 & s_{55} & s_{57} \\ 0 & 0 & s_{77} \end{pmatrix}$
  - In general, that's all we can do ...
- three degenerate flavors: $M_u = M_d = M_s$
  - ➜ $SU(3)_f$-symmetric
  - ➜ diagonalization by combined color and flavor rotations:

$$s \rightarrow s' = V\,s\,U = \begin{pmatrix} s_{22} & 0 & 0 \\ 0 & s_{55} & 0 \\ 0 & 0 & s_{77} \end{pmatrix}, \quad U \in SU(3)_c,\ V \in SU(3)_f$$

Pairing patterns

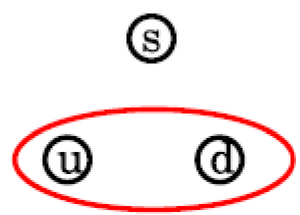
- eight possible phases:

normal quark matter (NQ)
$s_{22} = s_{55} = s_{77} = 0$



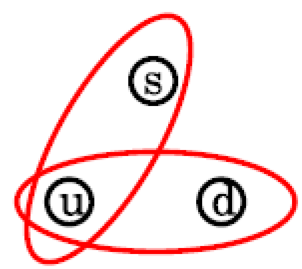
2SC phase
$s_{22} \neq 0, \quad s_{55} = s_{77} = 0$

+ two more phases of this kind

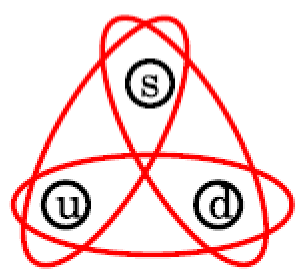


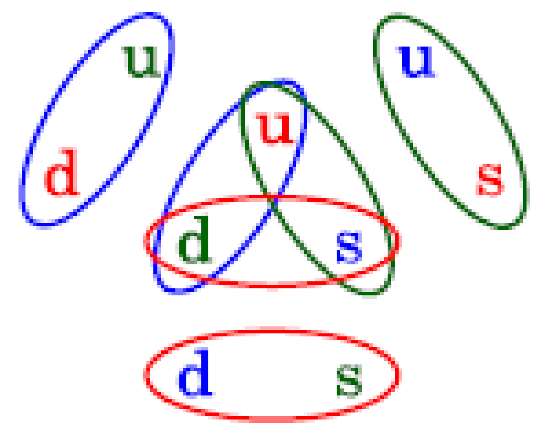
uSC phase
$s_{22}, \ s_{55} \neq 0, \quad s_{77} = 0$

+ two more phases of this kind



CFL phase
$s_{22}, \ s_{55}, \ s_{77} \neq 0$





- CFL pairing pattern (more explicitly):

$$(\uparrow\downarrow - \downarrow\uparrow) \otimes \Big(\Delta_2\,(ud - du) \otimes (r\,g - g\,r) + \Delta_5\,(ds - sd) \otimes (g\,b - b\,g) + \Delta_7\,(su - us) \otimes (b\,r - r\,b) \Big)$$

Color-flavor locking

- symmetries:
  - color: $SU(3)_c$ completely broken ➜ 8 massive gluons
  - chiral: $SU(3)_A$ " ➜ 8 Goldstone bosons

    $SU(3)_V$ "

  but: symm. under "locked" color-flavor rotations $q \to e^{i\theta_a(\tau_a - \lambda_a^T)}q$

  - baryon #: broken ➜ 1 scalar Goldstone boson

- electromagnetism:
  - invariant under (local) $q \to \exp(i\alpha\tilde{Q})q$

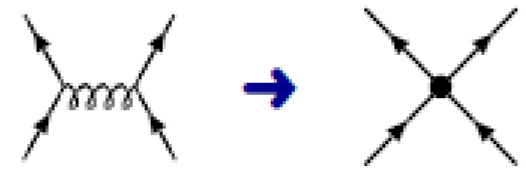
    $\tilde{Q} = Q - \frac{\lambda_3}{2} - \frac{\lambda_8}{2\sqrt{3}} = \mathrm{diag}_f(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}) - \mathrm{diag}_c(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$
  - "rotated photon" $=\cos\varphi$ photon $+\sin\varphi$ gluon

  ➜ no electromagnetic Meissner effect!

  - all quarks carry integer $\tilde{Q}$ charge

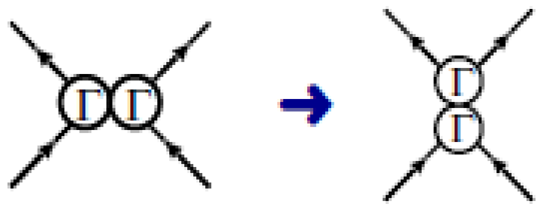
NJL model for color superconductivity

- "color-current interaction"
  - replace gluon exchange by point interactions:



$$\mathcal{L}_{int}(x) \overset{e.g.}{=} -g\,(\bar{q}(x)\,\gamma^{\mu}\lambda_a\,q(x))^2$$

- Fierz transformation
  - identically rewrite particle-antiparticle interactions as particle particle interactions:



$$(\bar{q}\,\Gamma^{(I)}\,q)^2 \;=\; \sum_D d_D^I\,(\bar{q}\Gamma^{(D)}C\bar{q}^T)(q^T C\Gamma^{(D)}q)$$

- toy model (two flavors):

$$\mathcal{L}_{int} \;=\; H \sum_{A=2,5,7} (\bar{q}\,i\gamma_5\tau_2\lambda_A\,C\bar{q}^T)(q^T C\,i\gamma_5\tau_2\lambda_A\,q) \qquad (H = \tfrac{N_c+1}{2N_c}\,g)$$

Nambu-Gorkov formalism

- interaction Lagrangian:

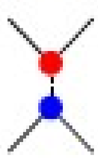
$$\mathcal{L}_{int} = H \sum_{A=2,5,7} (\bar{q}\, i\gamma_5\tau_2\lambda_A\, C\bar{q}^T)(q^T C\, i\gamma_5\tau_2\lambda_A\, q)$$

- artificially double # d.o.f. *(Nambu-Gorkov spinors)*:

$$\Psi = \frac{1}{\sqrt{2}}\begin{pmatrix} q \\ C\bar{q}^T \end{pmatrix}, \qquad \bar{\Psi} = \frac{1}{\sqrt{2}}\left(\bar{q}, q^T C\right)$$

➜ $$\mathcal{L}_{int} = 4H \sum_{A=2,5,7} \bar{\Psi}\begin{pmatrix} 0 & i\gamma_5\tau_2\lambda_A \\ 0 & 0 \end{pmatrix}\Psi\, \bar{\Psi}\begin{pmatrix} 0 & 0 \\ i\gamma_5\tau_2\lambda_A & 0 \end{pmatrix}\Psi$$

➜ vertices:



$$= 4Hi\,\Gamma_A^{\uparrow}\otimes\Gamma_A^{\downarrow} = 4Hi\begin{pmatrix} 0 & i\gamma_5\tau_2\lambda_A \\ 0 & 0 \end{pmatrix}\otimes\begin{pmatrix} 0 & 0 \\ i\gamma_5\tau_2\lambda_A & 0 \end{pmatrix}$$

Nambu-Gorkov propagator

- kinetic term + chemical potential:

$$
\begin{aligned}
\mathcal{L}_{kin} + \mu\, q^\dagger q &= \bar{q}(i\partial\!\!\!/ + \mu\gamma^0)q \\
&= \frac{1}{2}\left[\,\bar{q}(i\partial\!\!\!/ + \mu\gamma^0)q \;-\; q^T C(i\overleftarrow{\partial\!\!\!/} + \mu\gamma^0)C\bar{q}^T\,\right] \\
&= \bar{\Psi}\begin{pmatrix} i\partial\!\!\!/ + \mu\gamma^0 & 0 \\ 0 & -i\overleftarrow{\partial\!\!\!/} - \mu\gamma^0 \end{pmatrix}\Psi \\
&= \bar{\Psi}(x)\, S_0^{-1}(x)\, \Psi(x)
\end{aligned}
$$

➜ inverse bare propagator in momentum space:

$$
S_0^{-1}(p) = \begin{pmatrix} p\!\!\!/ + \mu\gamma^0 & 0 \\ 0 & p\!\!\!/ - \mu\gamma^0 \end{pmatrix}
$$

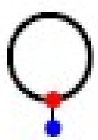
Selfconsistency problem

- dressed propagator (Hartree approximation):

$$iS(p) = iS_0(p) + iS_0(p)\,(-i\Sigma)\,iS(p)$$

$$\Leftrightarrow\ S^{-1}(p) = S_0^{-1}(p) - \Sigma$$

- self-energy:

$$-i\Sigma =$$

$$= 4iH \sum_A \Big\{ \Gamma_A^{\uparrow} \int \frac{d^4k}{(2\pi)^4} \left(-\tfrac{1}{2}\right) \mathrm{Tr}[\Gamma_A^{\downarrow}\, iS(k)] + \Gamma_A^{\downarrow} \int \frac{d^4k}{(2\pi)^4} \left(-\tfrac{1}{2}\right) \mathrm{Tr}[\Gamma_A^{\uparrow}\, iS(k)] \Big\}$$

➜ selfconsistency problem!

Gap equation

- selfconsistency problem: $S^{-1} = S_0^{-1} - \Sigma[S]$
- ansatz:

$$\Sigma = \begin{pmatrix} 0 & -\Delta\,\gamma_5\tau_2\lambda_2 \\ \Delta^*\,\gamma_5\tau_2\lambda_2 & 0 \end{pmatrix} \Rightarrow S^{-1} = \begin{pmatrix} \not{p} + \mu\gamma^0 & \Delta\,\gamma_5\tau_2\lambda_2 \\ -\Delta^*\,\gamma_5\tau_2\lambda_2 & \not{p} - \mu\gamma^0 \end{pmatrix}$$

- strategy:

  invert S^{-1} ➜ calculate $\Sigma[S]$ ➜ compare with ansatz

- result:

$$\Delta = 16H\,\Delta\,i \int \frac{d^4k}{(2\pi)^4}\Big(\frac{1}{k_0^2-\omega_-^2}+\frac{1}{k_0^2-\omega_+^2}\Big)$$ "gap equation"

quasiparticle dispersion laws: $\omega_\mp = \sqrt{(|\vec{k}| \mp \mu)^2 + |\Delta|^2}$

Propagator

- dressed propagator: $S = \begin{pmatrix} \not{p} + \mu\gamma^0 & \Delta\,\gamma_5\tau_2\lambda_2 \\ -\Delta^*\,\gamma_5\tau_2\lambda_2 & \not{p} - \mu\gamma^0 \end{pmatrix}^{-1}$
  - dimension: $2 \times 4 \times N_f \times N_c$

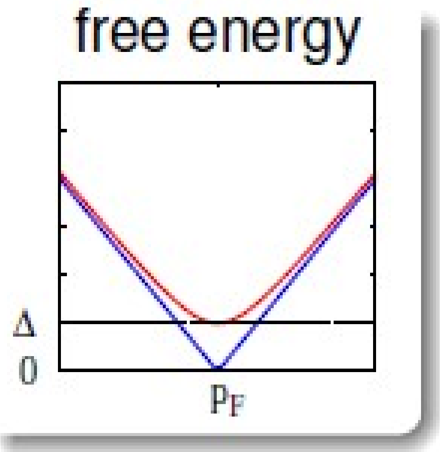
    → 48×48 matrix for $N_f = 2, N_c = 3$
  - inversion straight forward, but some work required . . .

- diagonalization:

$$S(p^0, \vec{p}) = U(\vec{p}) \begin{pmatrix} \frac{1}{p^0 - \lambda_1(\vec{p})} & & \\ & \ddots & \\ & & \frac{1}{p^0 - \lambda_{48}(\vec{p})} \end{pmatrix} U^\dagger(\vec{p})\,\gamma^0$$

  - $U(\vec{p})$ = unitary matrix, does not depend on p^0 !

Dispersion relations



- 48 eigenvalues

  $= 24$ quasiparticle dispersion relations:

  - $\omega_-(\vec{p}) = \sqrt{(|\vec{p}| - \mu)^2 + |\Delta|^2}$ (8-fold) red and green quarks
  - $\omega_+(\vec{p}) = \sqrt{(|\vec{p}| + \mu)^2 + |\Delta|^2}$ (8-fold) " antiquarks
  - $\epsilon_-(\vec{p}) = \big|\, |\vec{p}| - \mu \,\big|$ (4-fold) blue quarks
  - $\epsilon_+(\vec{p}) = \big|\, |\vec{p}| + \mu \,\big|$ (4-fold) " antiquarks

- $+ 24$ quasiholes: $-\omega_\mp(\vec{p}),\ -\epsilon_\mp(\vec{p})$

Dispersion relations (CFL)

- 72 eigenvalues

  = 36 quasiparticle dispersion relations:

• $\omega_{8,-}(\vec{p}) = \sqrt{(|\vec{p}| - \mu)^2 + |\Delta|^2}$	(16-fold)	quark octet × spin
• $\omega_{8,+}(\vec{p}) = \sqrt{(|\vec{p}| + \mu)^2 + |\Delta|^2}$	(16-fold)	antiquark "
• $\omega_{1,-}(\vec{p}) = \sqrt{(|\vec{p}| - \mu)^2 + |2\Delta|^2}$	(2-fold)	quark singlet × spin
• $\omega_{1,+}(\vec{p}) = \sqrt{(|\vec{p}| + \mu)^2 + |2\Delta|^2}$	(2-fold)	antiquark "

- + 36 quasiholes: $-\omega_{8,\mp}(\vec{p}),\ -\omega_{1,\mp}(\vec{p})$

Gap equation: solutions

- gap equation: $\Delta = 16H\,\Delta\, i \int \frac{d^4k}{(2\pi)^4}\left(\frac{1}{k_0^2-\omega_-^2} + \frac{1}{k_0^2-\omega_+^2}\right)$

Gap equation: solutions

- gap equation: $\Delta = 16H\,\Delta \int \frac{d^3k}{(2\pi)^3}\, T\sum_n \left(\frac{1}{\omega_n^2+\omega_-^2} + \frac{1}{\omega_n^2+\omega_+^2}\right)$
  - $\omega_n = (2n+1)\pi T$ fermionic Matsubara frequencies

Gap equation: solutions

- gap equation: $\Delta = 16H\,\Delta \int \frac{d^3k}{(2\pi)^3}\, T \sum_n \left(\frac{1}{\omega_n^2+\omega_-^2} + \frac{1}{\omega_n^2+\omega_+^2}\right)$
  - $\omega_n = (2n+1)\pi T$ fermionic Matsubara frequencies
- turning out the sum, $T \to 0$:

$$\Delta = \frac{4H}{\pi^2}\,\Delta \int k^2\,dk \left\{\frac{1}{\omega_-(k)} + \frac{1}{\omega_+(k)}\right\}$$

- solutions:
  - trivial solution: $\Delta = 0$
  - other solutions? $\int k^2\,dk \left\{\frac{1}{\omega_-(k)} + \frac{1}{\omega_+(k)}\right\} \stackrel{!}{=} \frac{\pi^2}{4H}$

    $\Delta \to 0 \quad \Rightarrow \quad \frac{1}{\omega_-(k)} \to \frac{1}{|k-\mu|} \quad \Rightarrow \quad \int \cdots \to \infty$

    → nontrivial solutions always exist for $H > 0$!

Thermodynamic potential

- back to our Lagrangian:

$$\hat{\mathcal{L}} = \bar{q}(i\partial\!\!\!/ + \mu\gamma^0)q + H \sum_{A=2,5,7} (q^T C \gamma_5 \tau_2 \lambda_A q)^\dagger (q^T C \gamma_5 \tau_2 \lambda_A q)$$

- linearize $\mathcal{L}_{int}$ around $\Delta = -2H\langle q^T C \gamma_5 \tau_2 \lambda_A q \rangle$

  and use Nambu-Gorkov spinors to get :

$$\mathcal{L}_{MF} = \bar{\Psi} \begin{pmatrix} i\partial\!\!\!/ + \mu\gamma^0 & \Delta \gamma_5 \tau_2 \lambda_2 \\ -\Delta^* \gamma_5 \tau_2 \lambda_2 & -i\overleftarrow{\partial\!\!\!/} - \mu\gamma^0 \end{pmatrix} \Psi - \frac{|\Delta|^2}{4H} \equiv \bar{\Psi} S^{-1} \Psi - \mathcal{V}$$

- thermodynamic potential:

$$\Omega(T,\mu) = -T \sum_n \int \frac{d^3k}{(2\pi)^3} \frac{1}{2} \mathrm{Tr} \ln \left(\frac{1}{T} S^{-1}(i\omega_n, \vec{k}) \right) + \mathcal{V}$$

  - $\ln A = \ln\big((1-(1-A)\big) = \sum_{n=1}^{\infty} \frac{1}{n}(1-A)^n$
  - useful formula: $\mathrm{Tr}\,\ln A = \ln \mathrm{Det} A$

Thermodynamic potential

- result after Matsubara summation:

$$\Omega(T,\mu) = -\int \frac{d^3p}{(2\pi)^3}\Big\{ \; 8\left(\frac{\omega_-}{2} + T\ln(1+e^{-\omega_-/T}) + \frac{\omega_+}{2} + T\ln(1+e^{-\omega_+/T})\right) + 4\left(\frac{\epsilon_-}{2} + T\ln(1+e^{-\epsilon_-/T}) + \frac{\epsilon_+}{2} + T\ln(1+e^{-\epsilon_+/T})\right)\Big\} + \frac{|\Delta|^2}{4H}$$

Thermodynamic quantities

- standard thermodynamic relations:
  - pressure: $p = -\Omega$
  - density: $n = -\frac{\partial \Omega}{\partial \mu}$
  - entropy density: $s = -\frac{\partial \Omega}{\partial T}$
  - energy density: $\varepsilon = -p + Ts + \mu n$

Minima

- thermodynamic potential:

$$\Omega(T,\mu) \;=\; -T\sum_n \int \frac{d^3k}{(2\pi)^3}\,\frac{1}{2}\,\mathrm{Tr}\,\ln\left(\frac{1}{T}\,S^{-1}(i\omega_n,\vec{k})\right) \;+\; \frac{|\Delta|^2}{4H}$$

- stable solutions = minima

$$\rightarrow\; \frac{\partial\Omega}{\partial\Delta^*} \;=\; -T\sum_n \int \frac{d^3k}{(2\pi)^3}\,\frac{1}{2}\,\mathrm{Tr}\left(S\,\frac{\partial S^{-1}}{\partial\Delta^*}\right) \;+\; \frac{\Delta}{4H} \stackrel{!}{=} 0$$

- inverse propagator:

$$S^{-1}(p) = \begin{pmatrix} \not{p} + \mu\gamma^0 & \Delta\,\gamma_5\tau_2\lambda_2 \\ -\Delta^*\,\gamma_5\tau_2\lambda_2 & \not{p} - \mu\gamma^0 \end{pmatrix}$$

$$\Rightarrow\quad \frac{\partial S^{-1}}{\partial\Delta^*} \;=\; \begin{pmatrix} 0 & 0 \\ -\gamma_5\tau_2\lambda_2 & 0 \end{pmatrix} \;=\; i\Gamma_2^{\downarrow}$$

$$\rightarrow\; \Delta \;=\; 4H\,T\sum_n \int \frac{d^3k}{(2\pi)^3}\,\frac{1}{2}\,\mathrm{Tr}\,[\,iS\,\Gamma_2^{\downarrow}\,]$$ gap equation!

Condensation energy

- free energy gain: $\delta\Omega = \Omega(\Delta) - \Omega(0)$
- simplifications: neglect antiparticles, $T = 0$

➜ $\Omega(\Delta) \;=\; -\frac{1}{2\pi^2}\int p^2\,dp\,\sqrt{(p-\mu)^2 + |\Delta|^2} \;+\; \frac{|\Delta|^2}{4H}$

- gap equation:

$$\frac{\partial\Omega}{\partial\Delta^*} \;=\; -\frac{1}{4\pi^2}\int p^2\,dp\,\frac{\Delta}{\sqrt{(p-\mu)^2+|\Delta|^2}} \;+\; \frac{\Delta}{4H} \;=\; 0$$

➜ $\Omega(\Delta) \;=\; -\frac{1}{2\pi^2}\int p^2\,dp\,\frac{(p-\mu)^2+\frac{1}{2}|\Delta|^2}{\sqrt{(p-\mu)^2+|\Delta|^2}}$

- integrand strongly peaked at $|\vec{p}| = \mu$ ➜ $\int p^2\,dp \approx \mu^2 \int dp$
- Taylor expansion of the remaining integral in Δ

➜ $-\delta\Omega \propto \mu^2\Delta^2$

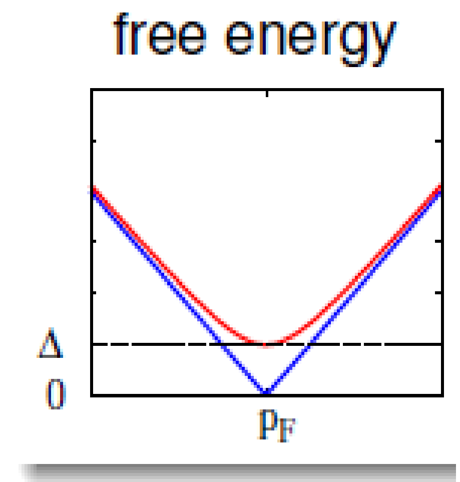
CFL pairing in the bag model

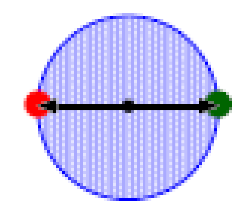
- bag-model pressure for unpaired quark matter at $T = 0$:
  - $\Omega_{BM}(\mu, \mu_Q) \;=\; \frac{3}{\pi^2} \sum_f \int_0^{p_F^f} dp\, p^2 \left(\sqrt{p^2 + m_f^2} - \mu_f\right) \;+\; B$
  - $\mu_u = \mu + \frac{2}{3}\mu_Q, \quad \mu_d = \mu_s = \mu - \frac{1}{3}\mu_Q, \quad p_F^f = \sqrt{\mu_f^2 - m_f^2}$
- effects of BCS pairing:
  - equalize Fermi momenta
  - pairing energy (expressed through the gap)
- CFL phase:
  - $\Omega_{BM}^{CFL}(\mu) \;=\; \frac{3}{\pi^2} \sum_f \int_0^{p_F^{common}} dp\, p^2 \left(\sqrt{p^2 + m_f^2} - \mu\right) \;-\; \frac{3\Delta^2\mu^2}{\pi^2} \;+\; B$
  - $p_F^{common} = 2\mu - \sqrt{\mu^2 + \frac{m_s^2}{3}}$ (for $m_u = m_d = 0$)

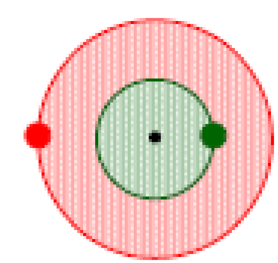
➜ parameters: masses, B, Δ

Realistic masses

- realistic quark masses: $M_u, M_d \ll M_s < \infty$
  - ➜ unequal Fermi momenta, $p_F^a = \sqrt{\mu^2 - M_a^2}$
- recall argument about Cooper instability:
  - pairing close to the Fermi surface
    (no free energy cost for pair creation)
- Cooper pairs in BCS theory:
  - opposite momenta
- unequal Fermi momenta: $p_F^{a,b} = \bar{p}_F \pm \delta p_F$







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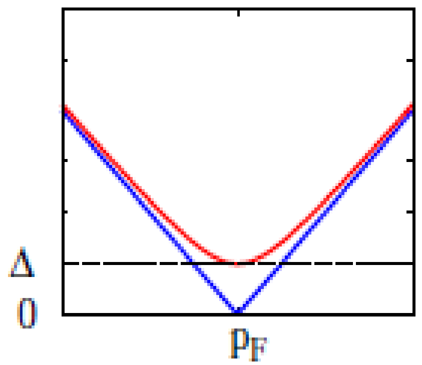
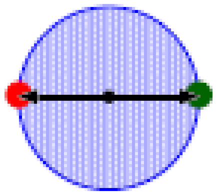
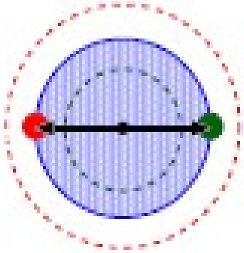
- Cooper pairs in BCS theory:
  - opposite momenta
- unequal Fermi momenta: $p_F^{a,b} = \bar{p}_F \pm \delta p_F$
  - BCS pairing favored if $E_{binding} > E_{pair\ creation}$
  - approximately: $\frac{\Delta}{\sqrt{2}} \gtrsim \delta p_F$

free energy



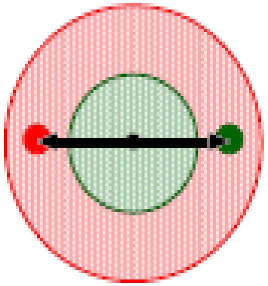




Which phase is favored ?

- precondition for standard BCS pairing:

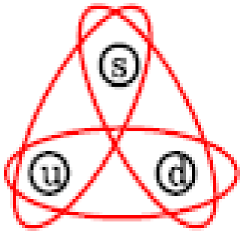
$$|p_F^a - p_F^b| \lesssim \sqrt{2}\Delta_{ab}$$



  - Fermi momenta: $p_F = \sqrt{\mu^2 - M^2}$
  - realistic quark masses: $M_s \gg M_d \approx M_u$

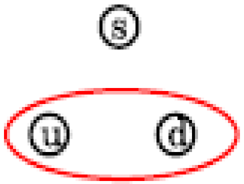
- <u>case 1:</u> *high densities*

$$\mu \gg M_s \;\rightarrow\; p_F^s \approx p_F^d \approx p_F^u \;\rightarrow\; \text{CFL}$$



- <u>case 2:</u> *moderate densities*

$$M_s \gtrsim \mu \gg M_{u,d} \;\rightarrow\; p_F^s \ll p_F^d \approx p_F^u \;\rightarrow\; \text{2SC}$$



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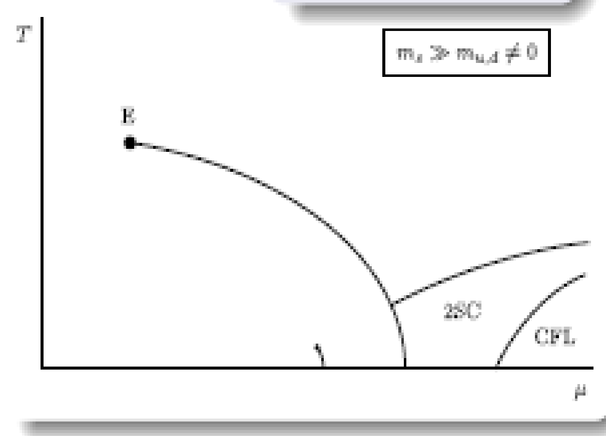
K. Rajagopal (1999)



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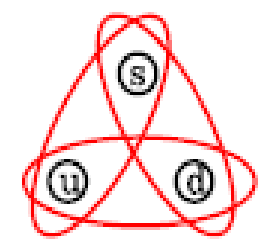

- case 2: *moderate densities*

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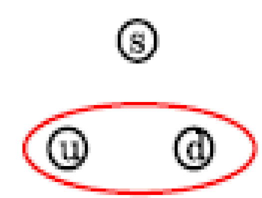


3-flavor NJL model

- Lagrangian: $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\bar{q}q} + \mathcal{L}_{qq}$
  - free part: $\mathcal{L}_0 = \bar{q}(i\partial\!\!\!/ - \hat{m})q$, $\hat{m} = diag_f(m_u, m_d, m_s)$
  - quark-antiquark interaction (as used earlier):

$$\mathcal{L}_{\bar{q}q} = G\left\{(\bar{q}\tau^a q)^2 + (\bar{q}i\gamma_5\tau^a q)^2\right\} - K\left\{\det_f\left(\bar{q}(1+\gamma_5)q\right) + \det_f\left(\bar{q}(1-\gamma_5)q\right)\right\}$$

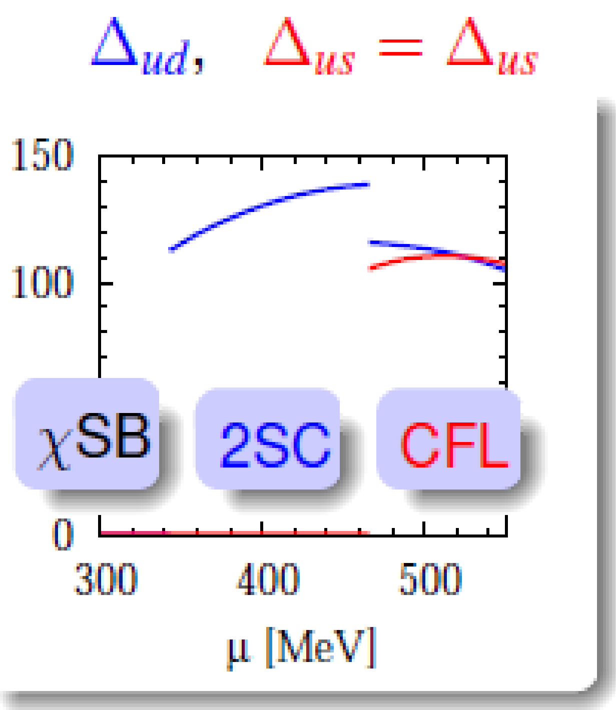
  - quark-quark interaction:

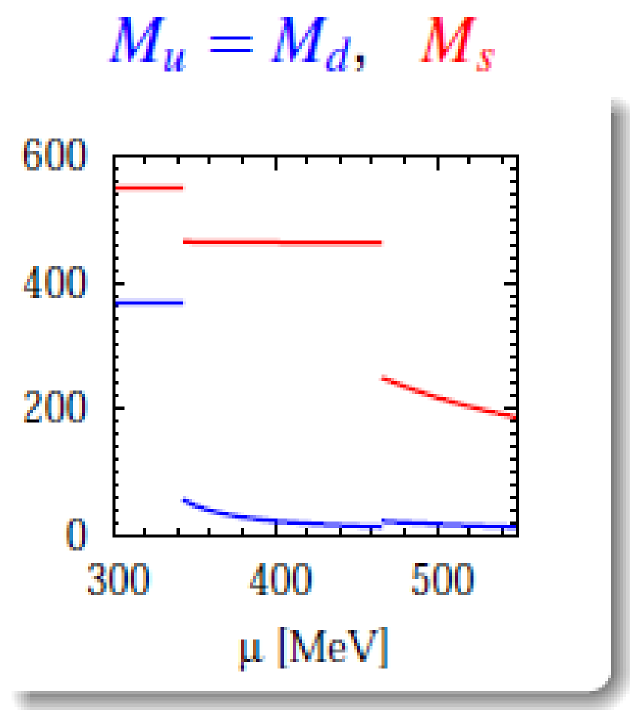
$$\mathcal{L}_{qq} = H\,(\bar{q}\,i\gamma_5\tau_A\lambda_{A'}\,C\bar{q}^T)(q^T C\,i\gamma_5\tau_A\lambda_{A'}\,q)$$

- mean-field approximation:
  - $\bar{q}q$-condensates: $\langle\bar{u}u\rangle$, $\langle\bar{d}d\rangle$, $\langle\bar{s}s\rangle$ $\leftrightarrow$ *dynamical masses*
  - qq-condensates: $\langle ud\rangle$, $\langle us\rangle$, $\langle ds\rangle$ $\leftrightarrow$ *diquark gaps*

Results for T=0

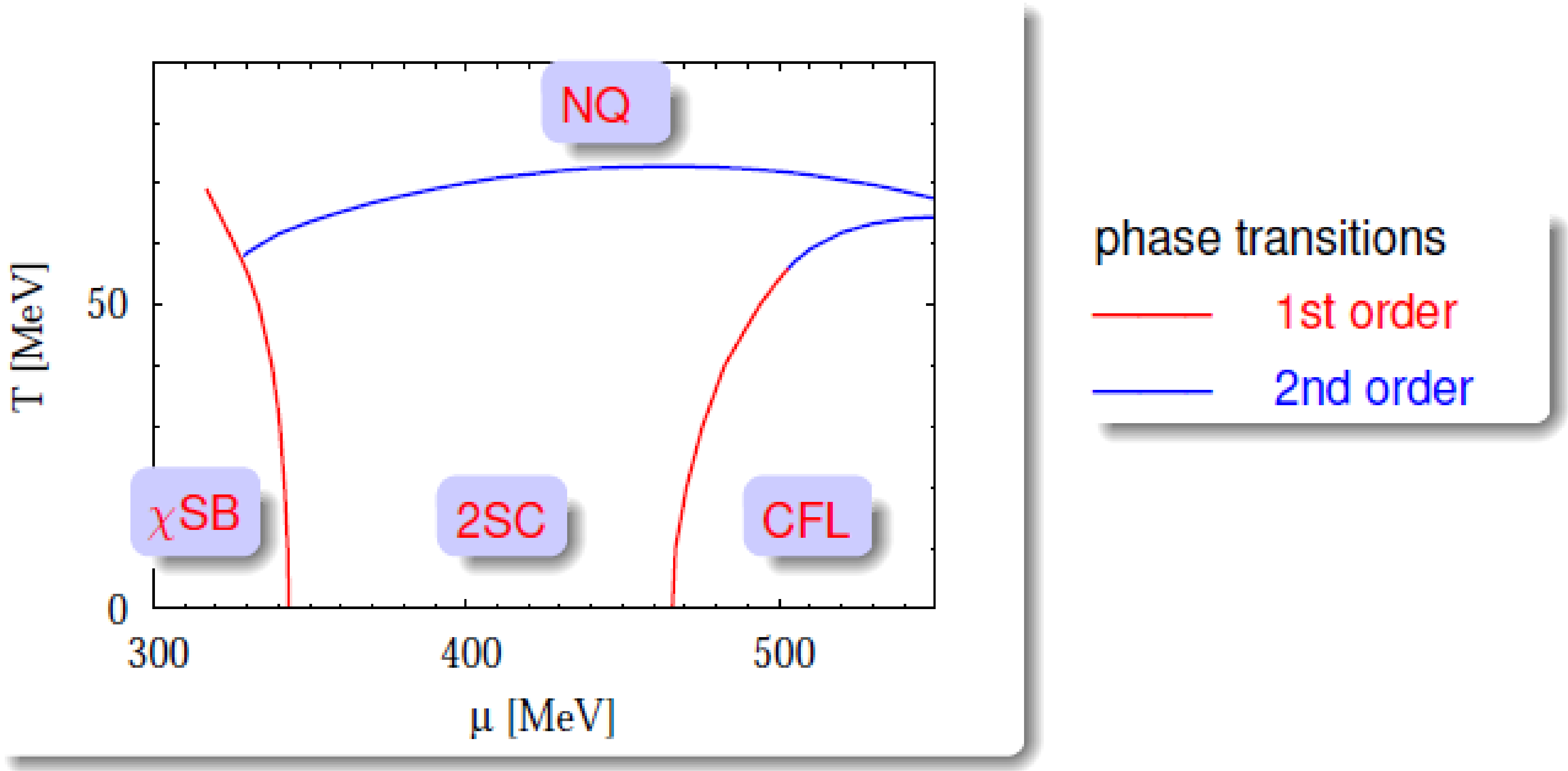
- “realistic” parameters
- isospin symmetry





→ strong interdependencies between dynamical masses and diquark gaps

Phase diagram



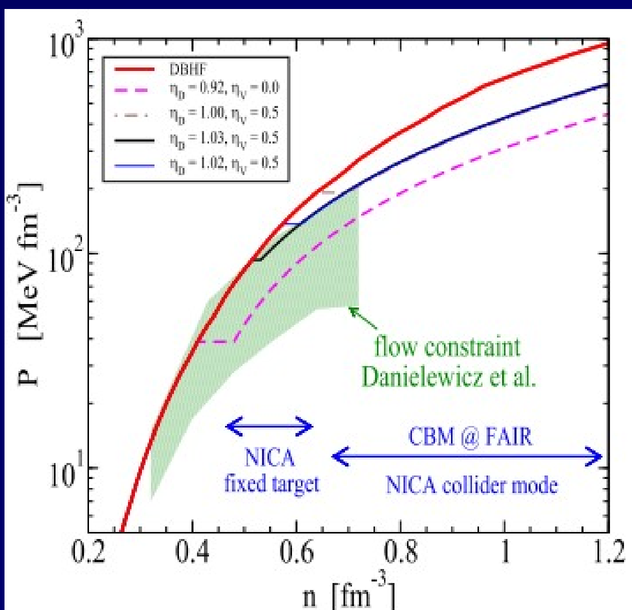
S. Ruester et al. Phys. Rev. D 72 (2005) 034004
D. Blaschke et al. Phys. Rev. D 72 (2005) 065020

Exploring hybrid star matter at NICA

T.Klähn (1), D.Blaschke (1,2), F.Weber (3)
(1) Institute for Theoretical Physics, University of Wroclaw, Poland
(2) Joint Institute for Nuclear Research, Dubna
(3) Department of Physics, San Diego State University, USA

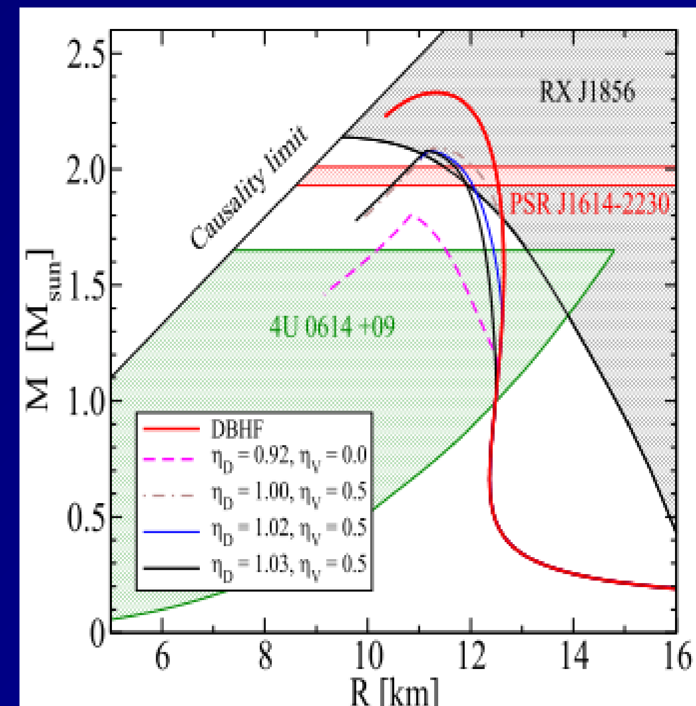
Heavy-Ion Collisions

Compact Stars



- stiff EoS (at flow limit)
- low n_{crit} (at NICA fixT)
- soft EoS (dashed line)

- high M_{max} (J1614-2230)
- low M_{onset} (all NS hybrid)
- excluded (J1614-2230)

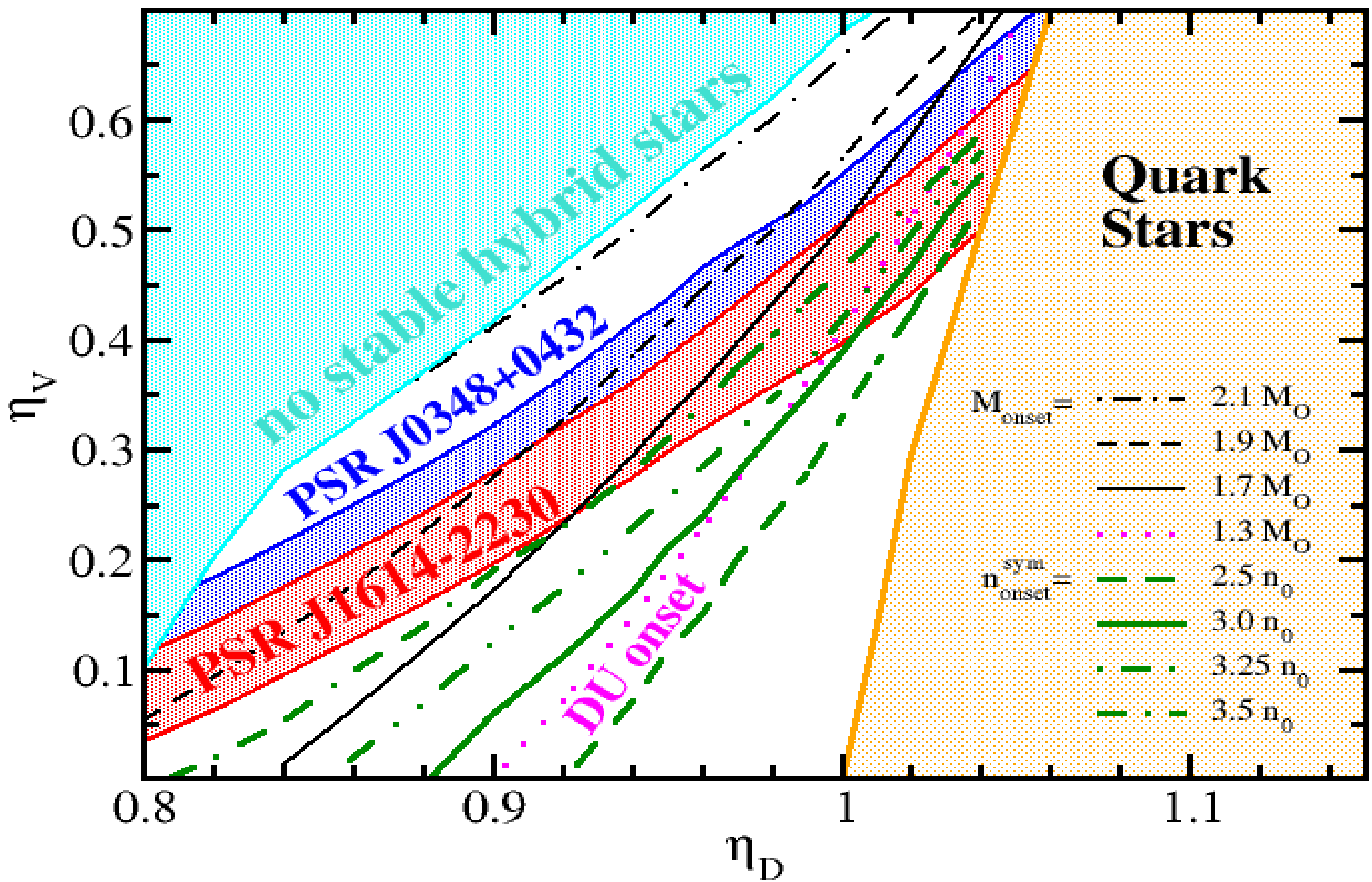


Proposal:

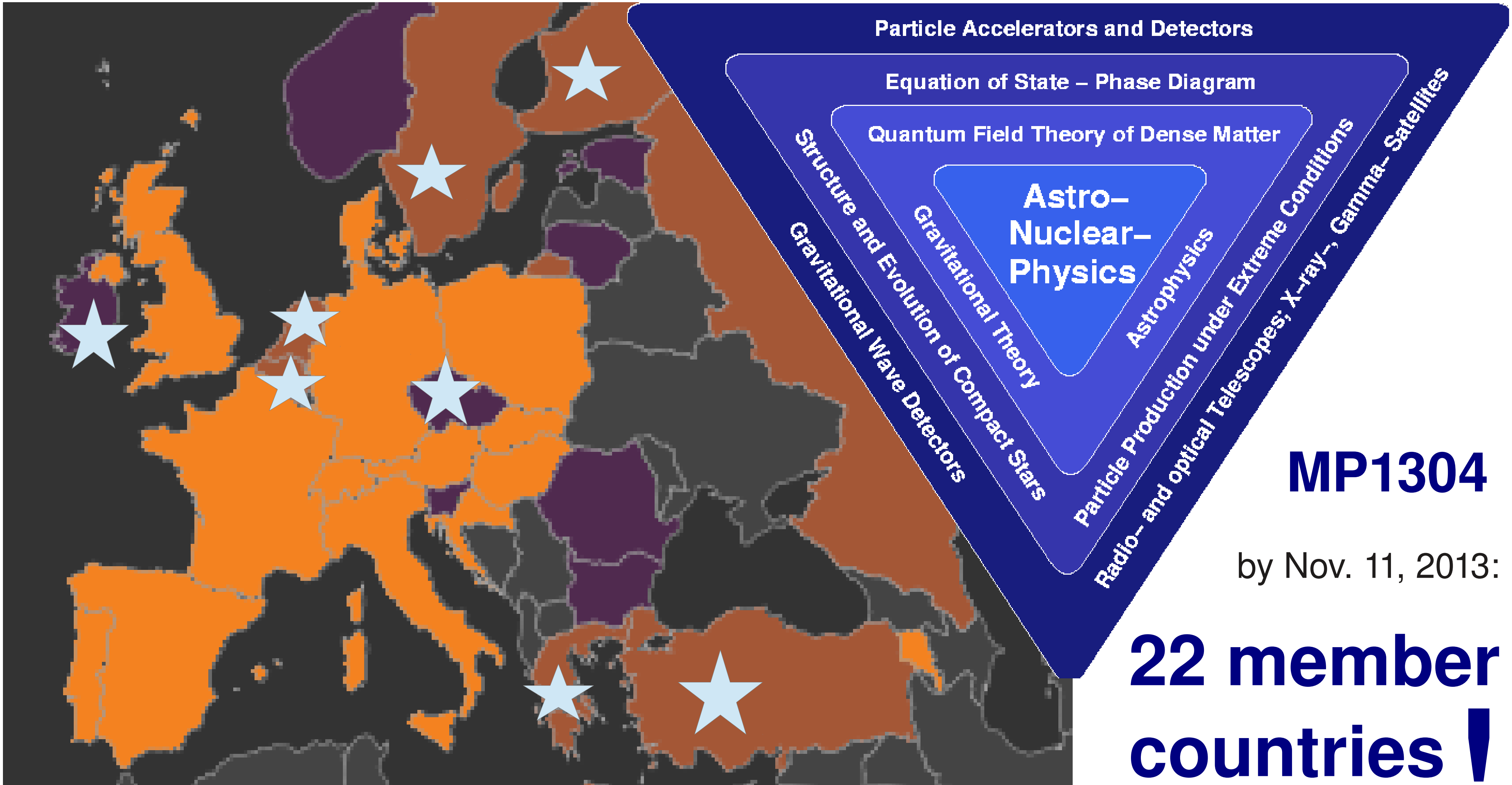
1. Measure transverse and elliptic flow for a wide range of energies (densities) at NICA and perform Danielewicz's flow data analysis ---> constrain stiffness of high density EoS
2. Provide lower bound for onset of mixed phase ---> constrain QM onset in hybrid stars

„The CBM Physics Book“, Springer LNP 841 (2011), pp.158-181
NICA White Paper, http://theor.jinr.ru → BLTP TWikipages

Quark matter in $2M_{sun}$ neutron stars?
→ only color superconducting + vector int.



T. Klahn et al., PRD 88 (2013) 085001; arxiv:1307.696



MP1304

by Nov. 11, 2013:

22 member countries !

New compstar !

COST
EUROPEAN COOPERATION
IN SCIENCE AND TECHNOLOGY

Kick-off: Brussels, November 25, 2013