Wigner and the groups in classifying elementary particles and nuclear states

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I. Introduction

Wigner did a pioneer work in the application of group theory in physics.

E.P. Wigner,
Grouppentheorie und Ihre Anwendung auf die Quantenmechanik der Atomspektren, Braunschweig, F. Vieweg und Sohn, 1931.

Supermultiplet-theory
E.P. Wigner, Phys. Rev. 51, 106 (1937).

The representations of the inhomogeneous Lorentz group
E.P. Wigner, Ann. Math. 40, 149 (1939).

Content:

I. Introduction

II. Classification of elementary particles:
invariance principle

III. Unitary symmetries in particle and nuclear physics:
similarities and differences

IV. Classification of nuclear states:
relation of the basic structure models

V. Summary

II. Classification of elementary particles: invariance principle

E.P. Wigner, Ann. Math. 40, 149 (1939).
International Wigner symposium

J.P. Elliott, P.G. Dawber, Symmetry in Physics,
MacMillen Press Ltd. London, 1979.

Y.S. Kim, E.P. Wigner, J. Math. Phys. 28, 1175 (1987); 31, 55
(1990): geometrical interpretation.

II. Classification of elementary particles: invariance principle

Inhomogeneous Lorentz group (Poincare group):
SO(3,1) Lorentz group + translation in 4D space-time.

Spin and mass: irreps of the Poincare group.

A particle (fundamental or composite) transforms according to an irreducible representation (irrep).

II. Classification of elementary particles: invariance principle

Eigenvalue equation of the invariant operator for a specific irrep.
wave-equation

Mass	Spin	Equation
finite	0	Klein-Gordon
finite	1/2	Dirac
0	1/2	Weyl
0	1	Maxwell

II. Classification of elementary particles: invariance principle

Events > Natural laws > Invariance principles
E.P. Wigner, Nobel Lecture, 1963.

ISO(3,1): Exact symmetry of space-time,
general classification of particles.

(SO(4) in Kepler problem (classical and quantum)
Györgyi G., the 4D space in which SO(4) transform.
Hungarian translations of Wigner's works.)

III. Unitary symmetries in particle and nuclear physics: similarities and differences

Supermultiplet (spin-isospin) $U^{ST}(4)$ theory

E.P. Wigner, Phys. Rev. 51, 106 (1937);

F. Hund, Z. Phys. 105, 202 (1937).

Following Heisenberg's isospin $SU^{T}(2)$.

Z. f. Physik 77, 1 (1932).

Application in particle and nuclear physics.

Prototype of similar symmetries.

Instead of historical order: U(3), U(4), U(6).

III. Unitary symmetries in particle and nuclear physics: similarities and differences

U(3)

Nuclear physics:

J.P. Elliot, Proc. Roy. Soc. A245, 128; 562 (1958).

SU(3) shell model.

Space symmetry.

In fact $U^{ST}(4)xSU(3)$ shell model.

Harmonic oscillator potential: exact SU(3) symmetry.

III. Unitary symmetries in particle and nuclear physics: similarities and differences

$$H = nh\omega + kQ \cdot Q$$

$$H = aC^{(1)}_{U3} + bC^{(2)}_{SU3} + cC^{(2)}_{SO3}$$

$$U(3) \supset SU(3) \supset SO(3)$$

Dynamically broken symmetry.

Detailed spectroscopy of light nuclei.
Quadrupole deformation and rotation from spherical shell model.

U(3) in nuclear physics: dynamically broken space symmetry.

III. Unitary symmetries in particle and nuclear physics: similarities and differences

Dynamically broken symmetry.

SU(3) symmetries

Type	Operator	Eig.vect.	System
Exact	symm	symm	3D-HO
Dynam.br.	nonsymm	symm	Elliott

Symmetric (H) operator: scalar $[X_i,H] = 0$.
Simple example: spherical symm: $[J_i,H] = 0$.

Symmetric eigenvectors: transform according to an irrep.
Simple example: spherical symm:

$$J_{\pm}|J,M\rangle = [J(J+1)-M(M\pm 1)]^{\frac{1}{2}}|J,M\pm 1\rangle,$$

$$J_0|J,M\rangle = M|J,M\rangle.$$

III. Unitary symmetries in particle and nuclear physics: similarities and differences

Dynamical breaking:

$$G \supset G' \supset \ldots$$

$$H = aC_i(G) + bC_j(G') + \ldots$$

SU(2):

$$H = aJ^2 + bJ_z$$

$$H = aT^2 + bT_z$$

III. Unitary symmetries in particle and nuclear physics: similarities and differences

U(3)

Particle physics:

M. Gellmann, Phys. Rev. 125, 1067 (1962).

Y. Ne'eman, Nucl. Phys. 26, 222 (1962).

Eightfold way.

Selection rule: associated production.

Good quantum numbers of the states.

Dynamically broken symmetry.

III. Unitary symmetries in particle and nuclear physics: similarities and differences

Gellmann-Okubo mass formula

M. Gell-Mann, Cal. Inst. Techn. Rep. CTSL-20 (1961).

S. Okubo, Progr. Theor. Phys. 27, 949 (1962).

$$M = a + bY + c\left[I(I+1) - \frac{1}{4} Y^2 \right]$$

$$M = a + bC^{(1)}(U1) + c\left[C^{(2)}(SU2) - \left(\frac{1}{4} C^{(1)}(U1) \right)^2 \right]$$

$$U(3) \supset SU(2) \times U(1)$$

U(3) in particle physics: dynamically broken internal (flavour) symmetry.

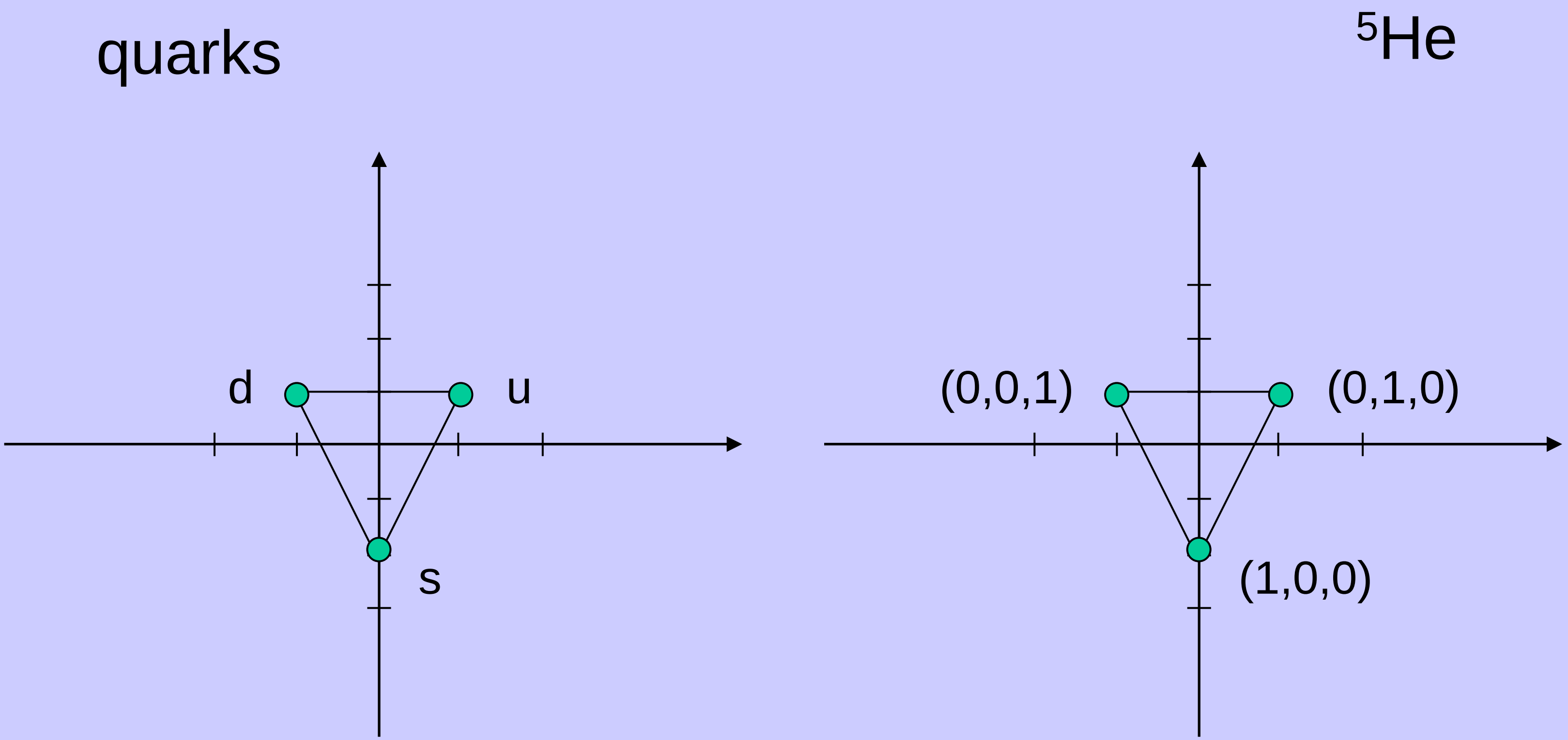
III. Unitary symmetries in particle and nuclear physics: similarities and differences

U(3) weight diagrams

In particle physics: usual,
in nuclear physics: possible.

In nuclear physics the weight diagrams are unusual, because:
1. the dimensions are large, e.g. ground state of ^{24}Mg dim =180,
2. the physical subgroup is SO(3), not SU(2).

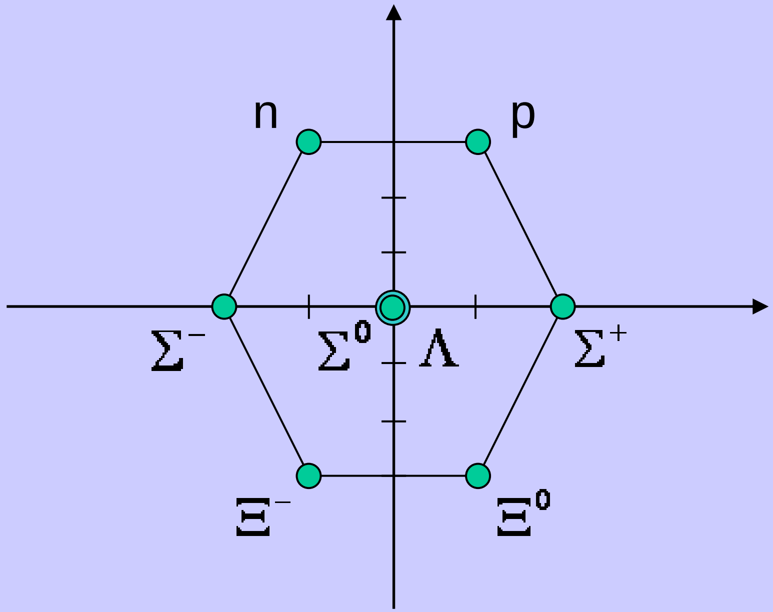
(1,0)
(triplet)

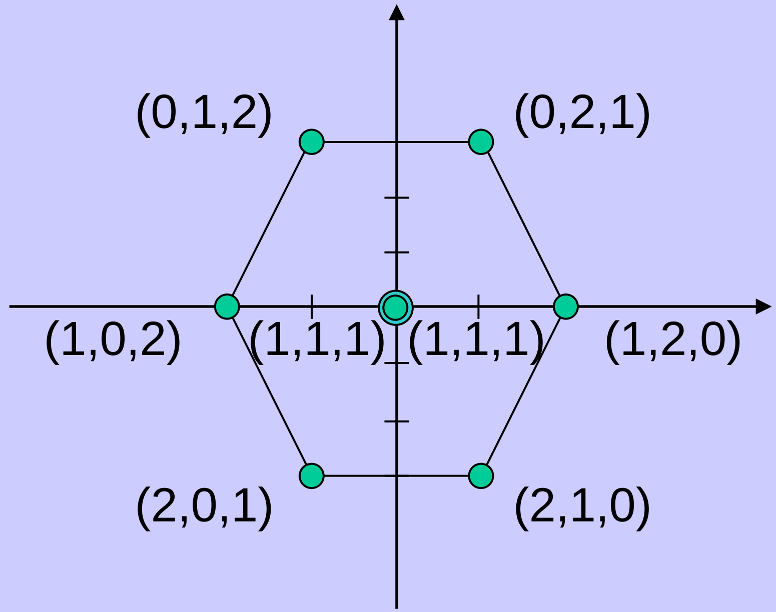


(1,1)
(octet)

$\frac{1}{2}^{+}$ barions

^{7}He

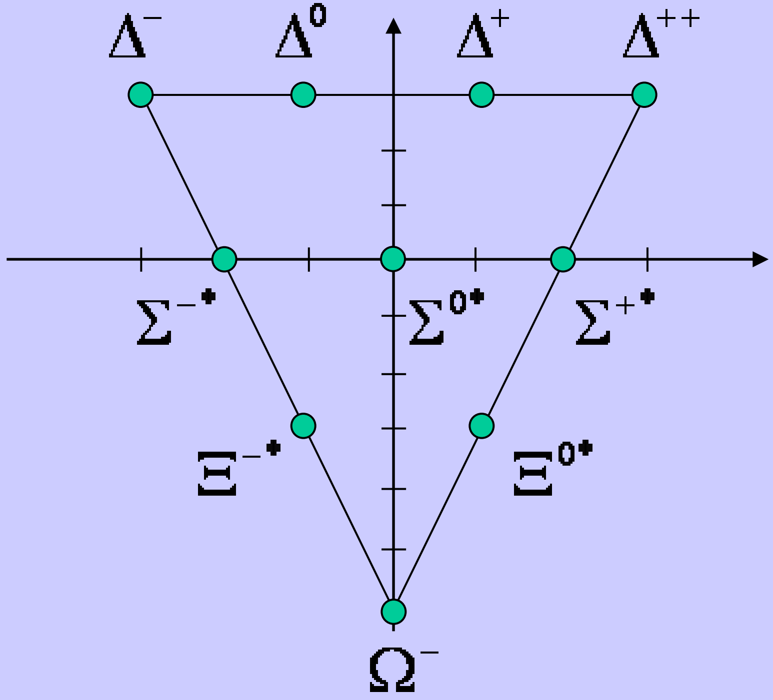


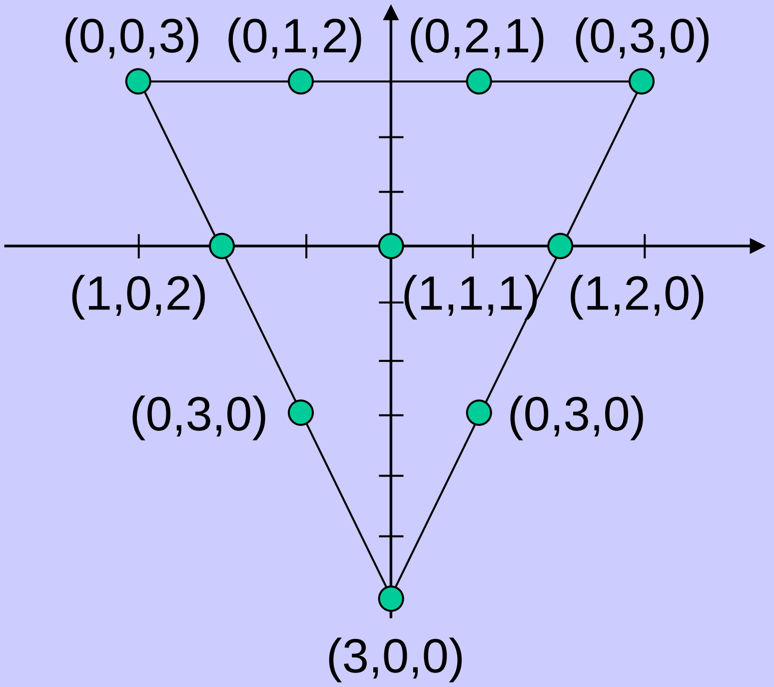


(3,0)
(decuplet)

$\frac{3}{2}^{+}$ barions

^{41}Ca





III. Unitary symmetries in particle and nuclear physics: similarities and differences

$U^{ST}(4)$

In nuclear physics:

E.P. Wigner, Phys. Rev. 51, 106 (1937);
F. Hund, Z. Phys. 105, 202 (1937);

L.E. Eisenbud, G.T. Garvey, E.P. Wigner, Genereal principles of nuclear strucure, Mc Graw-Hill Book Co., N.Y. 1967.

Relation of the spin and isospin multiplets

E.g. in the $U^{ST}(4)$xSU(3) shell model: distribution of the antisymmetrization.

Nowadays: ab initio shell model, computational group theory
T. Dytrych, K.D. Shviracheva, J.P. Draayer, C. Bahri, J.P. Vary, J. Phys. G 35, 123001 (2008).

Selection rule: Gamow-Teller beta-decay can take place between states of a SU(4) irrep.

III. Unitary symmetries in particle and nuclear physics: similarities and differences

Mass formula

Self-conjugate (N=Z) nuclei are tightly bound. An extra term in the semi-empirical mass formula is required to deal with this: Wigner-energy.

E.P. Wigner, Phys. Rev 51, 947 (1937).

D.D. Warner, M.A. Bentley, P. Van Isacker,
Nature physics 2, 311 (2006).

$$B = a_v A - a_s A^{\frac{2}{3}} - a_c \frac{Z^2}{A^{\frac{1}{3}}} - a_a \frac{(N-Z)^2}{A} + a_m \frac{\langle M \rangle}{A^{\gamma_M}}$$

$$M = -\frac{1}{8}\left(A(A-16) + C^{(2)}_{SU4}\right)$$

Supermultiplets in particle physics: more SU(6).

$U^{ST}(4)$: dynamically broken internal symmetry.

III. Unitary symmetries in particle and nuclear physics: similarities and differences

U(4) as a space group

In nuclear and particle physics:
- Dynamical algebra of the two-body problem
- F. Iachello, Phys. Rev. C 23, 2778 (1981).

Dynamical algebra: symmetry and spectrum generating algebra.

III. Unitary symmetries in particle and nuclear physics: similarities and differences

Hadron physics: meson spectrum

F. Iachello, N.C. Mukhopadhyay, L. Zhang, Phys. Rev. D 44, 898 (1991).

F. Iachello, D. Kusnezov, Phys. Rev. D 45, 4156 (1992).

Nuclear physics: clusterization

Phenomenological: H. Daley, F. Iachello, Ann. Phys. (NY) 167, 73 (1986).

Semimicroscopical: J. Cseh, Phys. Lett. B 281, 173 (1992).

J. Cseh, G. Lévai, Ann. Phys. (NY) 230, 165 (1994).

Molecular physics (most extensive)

F. Iachello, R.D. Levine, Algebraic Theory of molecules Oxford Univ. Press, Oxford, 1995.

A. Frank, P. Van Isacker, Algebraic methods in molecular and nuclear structure physics John Wiley and Sons, New York, 1994.

III. Unitary symmetries in particle and nuclear physics: similarities and differences

Comparison of applications in molecular, nuclear and particle physics

in the Wigner-volume of the Acta Physica Hungarica:

F. Iachello, J. Cseh, G. Lévai, APH NS Heavy Ion Physics, 1, 91 (1995).

U(4) space group in particle and nuclear physics:

Dynamically broken dynamical group (incl. spectr. gen.).

III. Unitary symmetries in particle and nuclear physics: similarities and differences

Supermultiplets in particle physics

SU(6) > SU(3)xSU(2)

F. Gürsey, L.A. Radicatti, Phys. Rev. Lett. 13, 173 (1964).

Relation of the spin and flavour multiplets

E.g. magnetic moments of the barions: octet+decuplet

SU(3) model: in terms of 3 parameters

SU(6) sup-mult. in terms of a single parameter

mm(p)/mm(n) = -1.5, exp: -1.46.

Selection rule

III. Unitary symmetries in particle and nuclear physics: similarities and differences

Mass formula: in particle physics.

Nuclear: natural isotopes + lambda-hyperons

G. Lévai, J. Cseh, P. Van Isacker, O. Juliett,

Phys. Lett. B 433, 250 (1998).

$$B = a_v A + a_s A^{\frac{2}{3}} + a_c \frac{Z^2}{A^{\frac{1}{3}}} - a_a \frac{(N-Z)^2}{A} + a_y \frac{S}{A^{\gamma_M}} + a_m \frac{\langle M \rangle}{A^{\gamma_M}},$$

$$M = -\frac{1}{12}\left(A(A-36) + C^{(2)}_{SU6}\right)$$

U(6) in particle physics: dynamically broken internal symmetry.

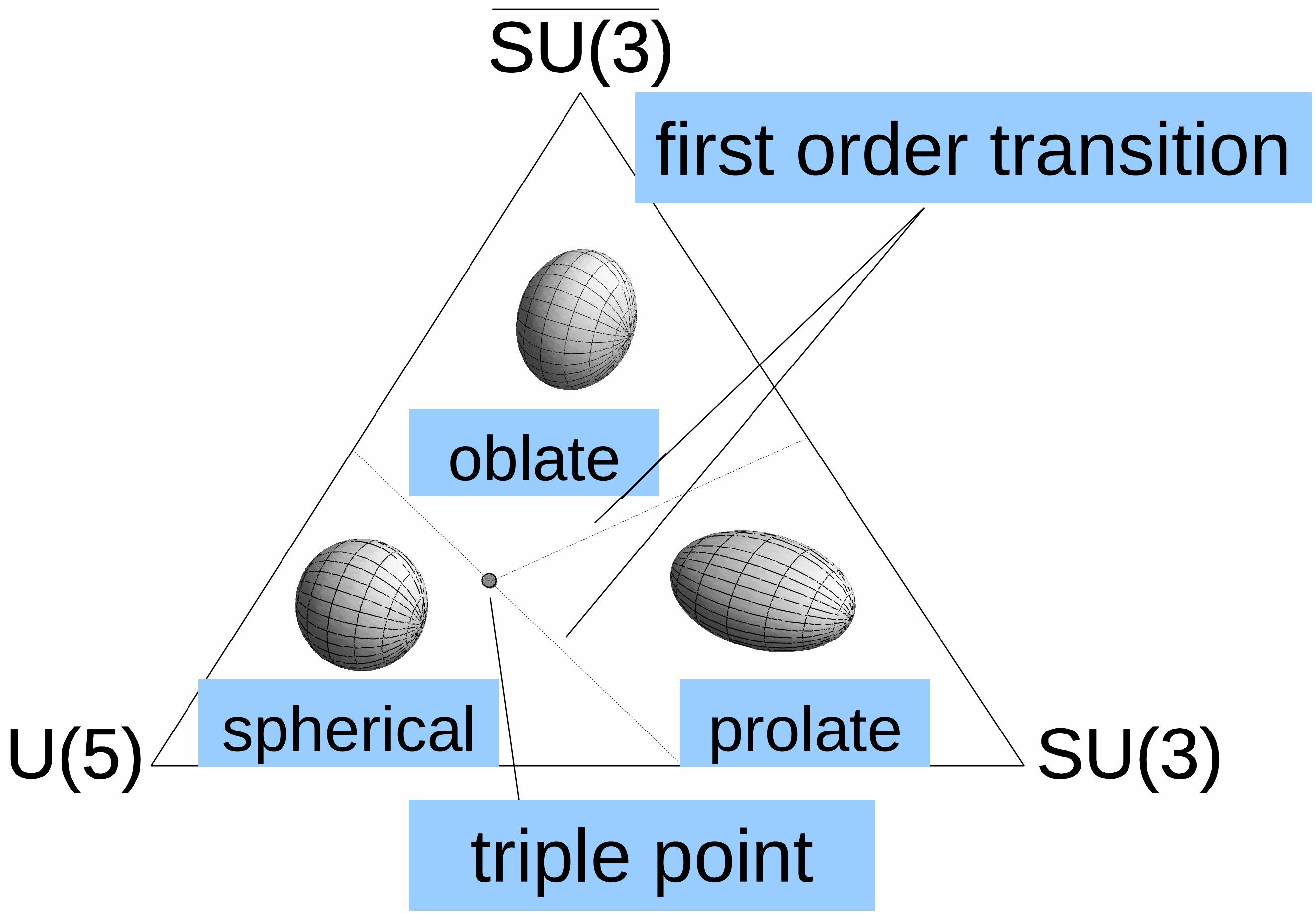
III. Unitary symmetries in particle and nuclear physics: similarities and differences

U(6) as a space group

In nuclear physics:

Dynamical algebra of the quadrupole collectivity
F. Iachello, A. Arima, The Interacting Boson Model
Cambridge Univ. Press, Cambridge, 1987.

Extensive applications for the description of the collective spectra of medium and heavy nuclei.
Classification of collective motion: rotation, vibration,...
Recently: in terms of cold quantum phases.
D. Warner, Nature 420, 614 (2002).


$\overline{SU(3)}$
first order transition
oblate
spherical
prolate
U(5)
SU(3)
triple point

III. Unitary symmetries in particle and nuclear physics: similarities and differences

Unitary groups U(3), U(4), U(6) in particle and nuclear physics

Basically: internal in particle and space in nuclear physics.
Each of them is dynamically broken.

Group	Particle	Nuclear
U(3)	flavour	space sym.
U(4)	sup.mult.	sup.mult.+
U(4)	dynam.gr.	dynam.gr.+
U(6)	sup.mult.+	sup.mult.
U(6)		dynam.gr.+

Here sup. mult. stands for supermultiplet, which refers to internal degrees of freedom. Dynam.gr. indicates dynamical group (containing both symmetry and spectrum-generating subgroups), which refers to space degrees of freedom. When a group is applied in a similar manner in the two disciplines, the sign + indicates where the more extensive applications are made.

IV. Classification of nuclear states:
relation of the basic structure models

Atomic nucleus is like

- a small atom (miniature solar system): shell model,
- a microscopic liquid drop: collective model
- a small molecule: cluster model.

IV. Classification of nuclear states: relation of the basic structure models

Common intersection 1958

Elliott: quadrupole deformation and collective rotation
from the spherical shell model ,
bands with definite SU(3) symmetry

K. Wildermuth, Th. Kanellopoulos, Nucl. Phys. 7, 150 (1958).
In HO approximation the Hamiltonian of the shell and
cluster models can be transformed into each other.

B.F. Bayman, A. Bohr, Nucl. Phys. 9, 596 (1958/59).
Cluster states are selected from the shell basis
by their specific SU(3) symmetry.

Common intersection: SU(3) symmetry.
Single major shell,
dynamically broken (shell-collective)
or exact (shell-cluster) symmetry.

IV. Classification of nuclear states: relation of the basic structure models

Multi-shell excitations

Shell model: Sp(3,R) symplectic model

G. Rosensteel, D.J. Rowe, Phys. Rev. Lett. 38, 10 (1977).

Electromagnetic transitions without effective charge.

Collective model: $\mathrm{U_b}(6) \times \mathrm{U_s}(3)$ model,

Castanos, Draayer, Nucl Phys. A 491, 349 (1989).

Contraction of the Sp(3,R) shell model,

large N limit of the multi-major-shell model.

Cluster model:

Fully microscopic (semi algebraic)

H. Horiuchi, K. Ikeda, K. Kato,

Progr. Theor. Phys. Suppl. 192, 1 (2012).

Semimicroscopic, fully algebraic

J. Cseh, Phys. Lett. B 281, 173 (1992),

J. Cseh, G. Lévai, Ann. Phys. (NY) 230, 165 (1994).

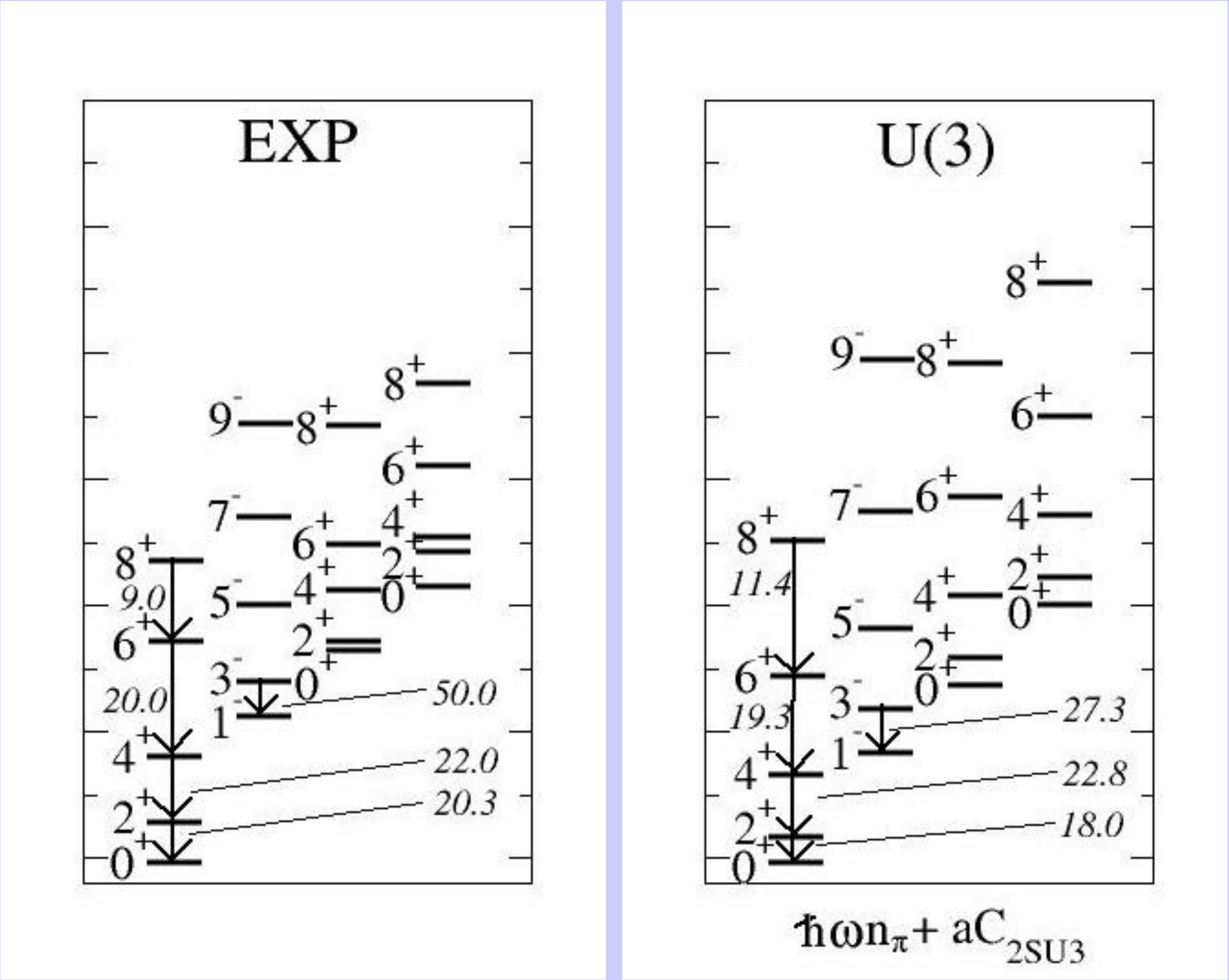
IV. Classification of nuclear states: relation of the basic structure models

Each of them have an $U_e(3) \times U_s(3) \supset U(3)$ basis

Common intersection: $U_e(3) \times U_s(3) \supset U(3)$ dynamically broken symmetry.

(Recent proposal: J. Cseh, in preparation.)

^{20}Ne spectrum



V. Summary

The application of the group representations in physics, initiated by Wigner, proved to be very successful.

In this contribution we have considered some applications in classifying elementary particles and nuclear states.

In particular, the exact space-time symmetry of the Poincare group was mentioned, which serves as a general classification scheme for elementary particles.

V. Summary

Dynamically broken unitary symmetries proved to be important in both disciplines. They appear either as space symmetries, or internal symmetries, or both. Such a symmetry seems to be able to bridge the fundamental nuclear structure models, and classify their states, too.

These symmetries were partly invented by Wigner, or they have the same nature: they are global symmetries, described by Lie-groups.

V. Summary

In the meantime other kind of new symmetries became important as well. We mention here two of them.

Local symmetries, in particular local gauge symmetries turned out to be the guiding principle in the theory of fundamental interactions. The standard model, describing the electroweak and strong interaction has an U(1)xSU(2)xSU(3) local gauge invariance. The last year victimed the experimental discovery of its basic particle, the Higgs boson, and this year its theoretical inventors received the Nobel prize for it. Local gauge symmetry is essential in particle physics, but not so much in nuclear physics.

V. Summary

Another new symmetry is the supersymmetry, transforming bosons and fermions into each other. It is described by graded Lie-groups. This seems to be relevant both in particle and in nuclear physics, though in somewhat different form.

F. Iachello, AIP Conf. Proc. 1488, 402 (2012).

In particle physics the gauge supersymmetry may have important consequences: to the existing particles (bosons or fermions) correspond their superpartner (fermions or bosons).

J. Wess, B. Zumino, Nucl. Phys. B 70, 39 (1974).

At present we have no experimental evidence for such superpartners.

V. Summary

In nuclear physics the dynamical supersymmetry provides a unified description of the spectra of neighbouring nuclei with even and odd mass number.

F. Iachello, Phys. Rev. Lett. 44, 772 (1980).

In this scheme the bosons are fonons of the quadrupole vibrations, and the fermions are nucleons. There is experimental evidence for the appearance of this kind of supersymmetry in nuclear spectroscopy.

A. Frank, J. Jolie, P. Van Isacker, Symmetry in nuclear physics, Springer-Verlag

(It was conjectured, that similar supersymmetry may exist in relation with the dipole vibration of the clusters as well, but this question has not been investigated very extensively.

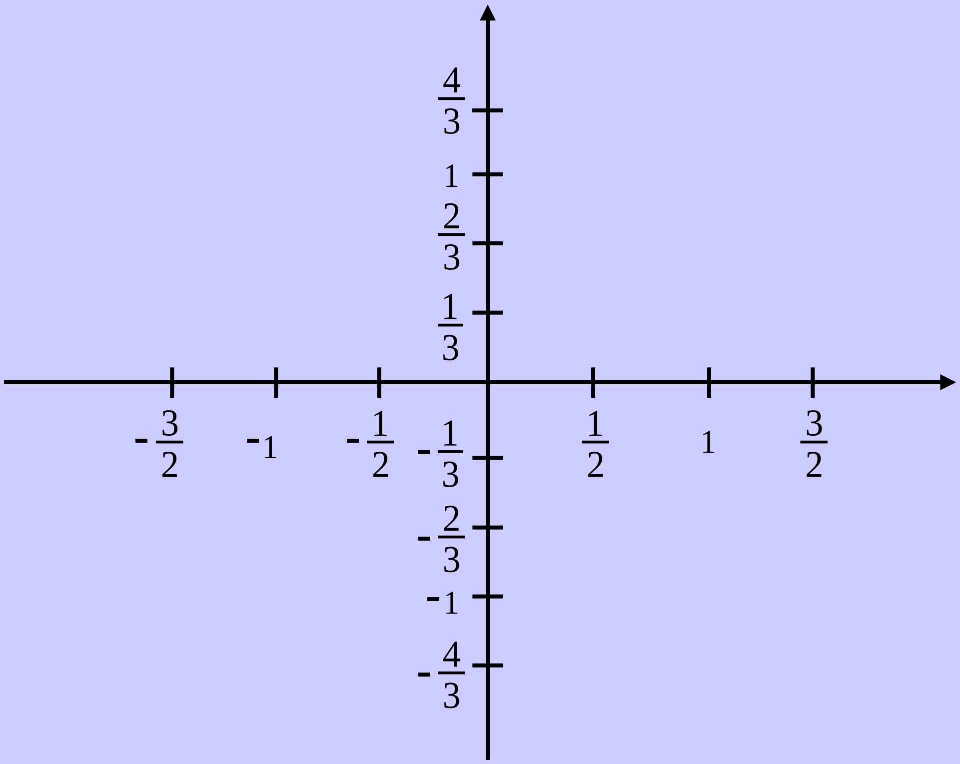
G. Lévai, J. Cseh, P. Van Isacker,

Eur. Phys. J. A 12, 305 (2001).)

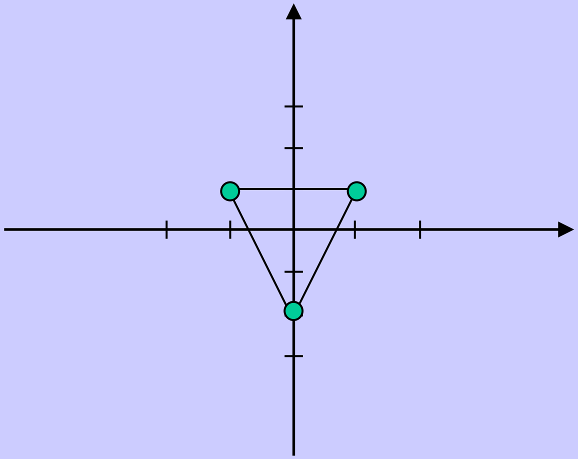
Conclusion

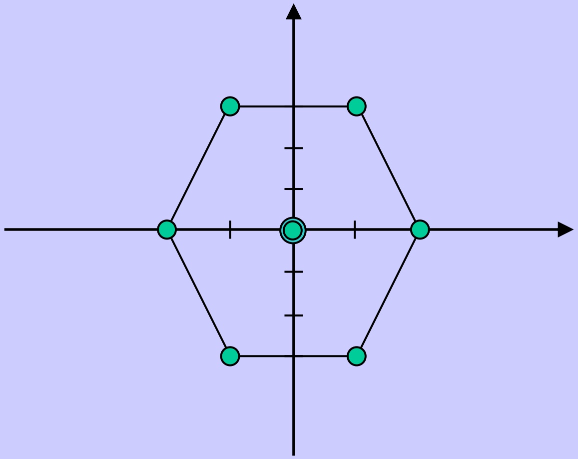
It turned out that "Gruppenpest", how the application of the group theoretical methods was called (after its invention by Wigner and others) is a funny desiese. It did not hurt the physics; on the contrary: group theory contributed a lot to the development of physics.

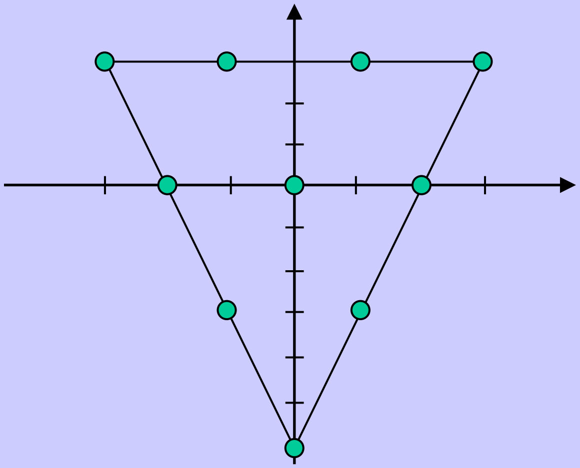
SU(3) weight diagram



	Particles		Nuclei	(n_z,n_x,n_y)
$M_T = T, T-1,\ldots$	M_T	x	$\frac{1}{2}\nu$	$\nu = n_x - n_y$
$Y = B + S$	Y	y	$-\frac{1}{3}\varepsilon$	$\varepsilon = \langle Q_0 \rangle - 2n_z - n_x - n_y$


triplet


octet


decuplet

III. Unitary symmetries in particle and nuclear physics: similarities and differences

IBM: U(6) in terms of intrashell operators,
i.e. single major shell.

U(6) in terms of intershell operators,
i.e. multi-major shells
O. Castanos, J.P. Draayer, Nucl. Phys. A. 491, 349 (1989).

U(6) space group in nuclear physics:
Dynamically broken dynamical group (incl. spectr. gen.).