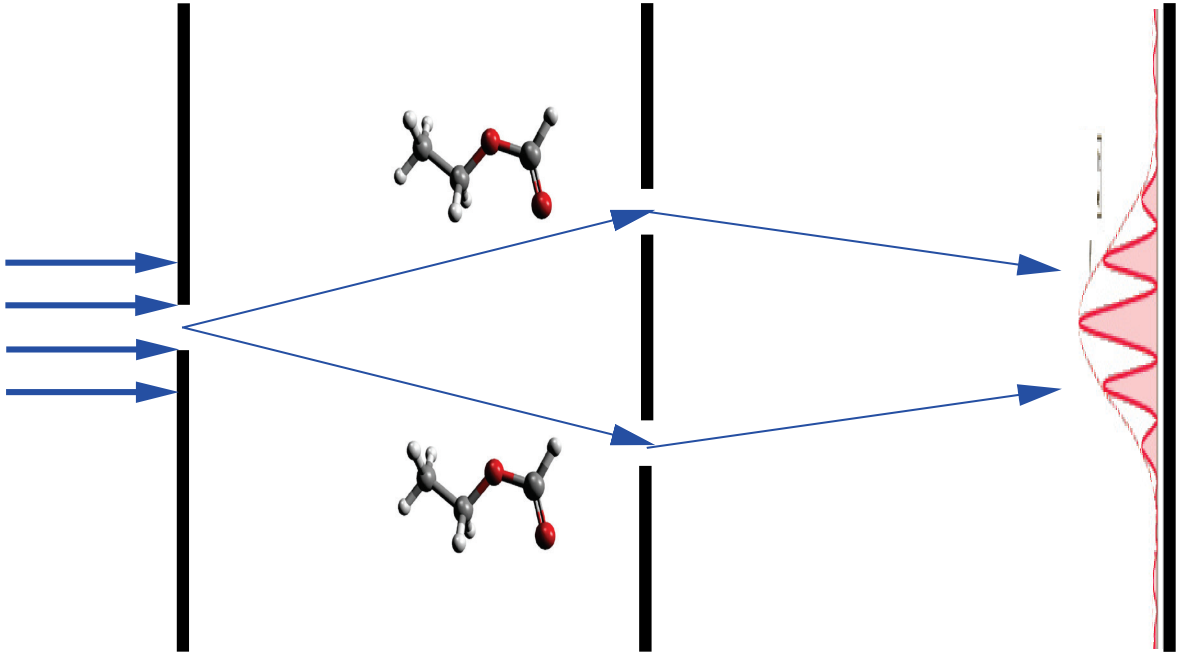
NONLOCAL INTERFEROMETERY USING SCHRODINGER CATS

Jim Franson
UMBC

Wigner Symposium

QUANTUM INTERFERENCE OF LARGE MOLCEULES

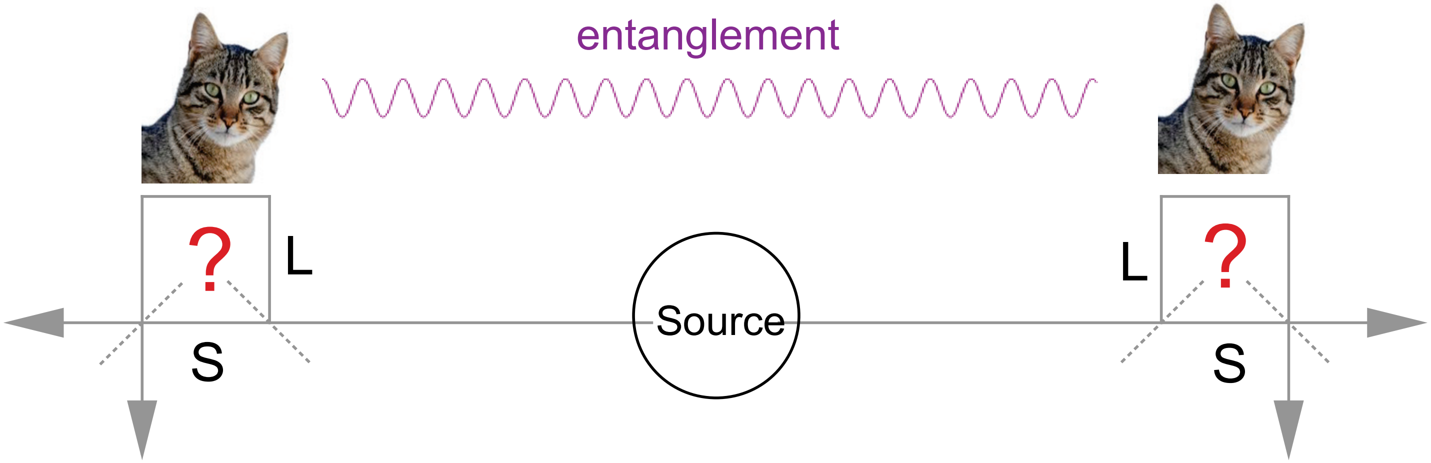
- Zeilinger’s group has demonstrated quantum interference using large molecules[1]



- How large is “macroscopic”?

1. M. Arndt, O. Nairz, J. Voss-Andreae, C. Keller, G. van der Zouw, A. Zeilinger, Nature **401**, 680 (1999).

NONLOCAL INTERFEROMETRY USING SCHRODINGER CATS



- Macroscopic Schrodinger cats propagate through one path or the other.
- Entanglement produces a violation of Bell's inequality.

COLLABORATORS

Todd Pittman
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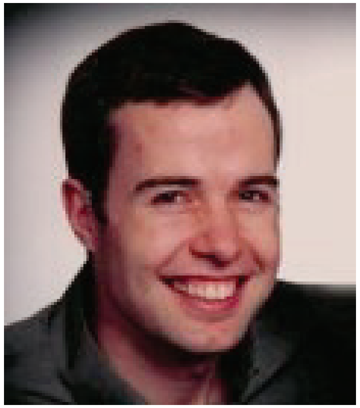

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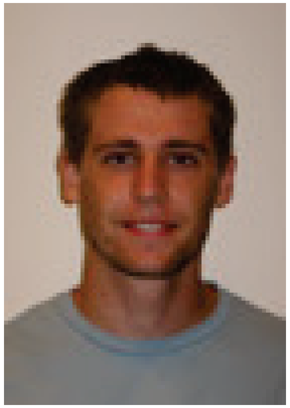
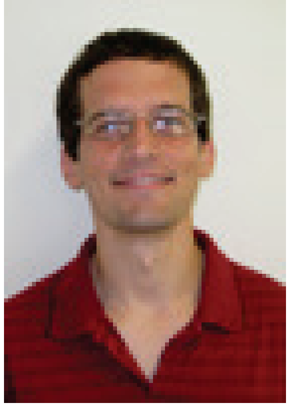
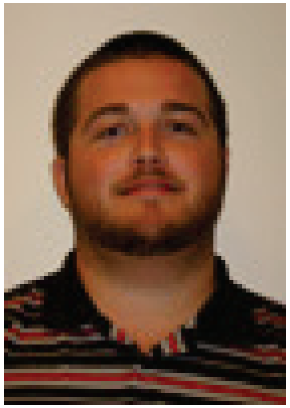


Brian Kirby
Theory



Garrett Hickman
Experiment



Dan Jones
Experiment

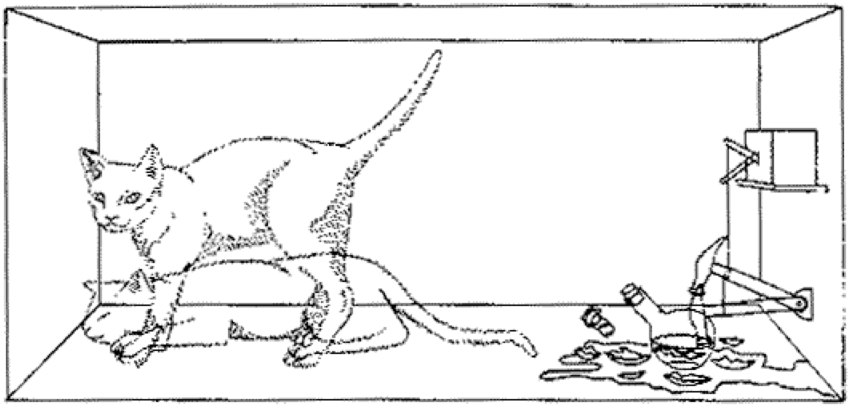
OUTLINE

- Brief review of Schrodinger cats and nonlocal interferometry.
- New approach for nonlocal interferometry using phase-entangled coherent states.[1]
- Effects of photon loss and decoherence.
- Experiment in progress at UMBC.

[1] B. T Kirby and J.D. Franson, Phys. Rev. A **87**, 053822 (2013).

SCHRODINGER CATS

- Schrodinger considered a random quantum process such as alpha decay.
  - At intermediate times, the quantum system is in a superposition of the original state and the final state.
- A detection of the alpha particle sets off a mechanism that kills a cat.
  - Is the system left in a superposition of a live and dead cat?



SCHRODINGER CATS

- This topic has received a great deal of interest:

“Don’t let the cat out of the box”

ENTANGLEMENT

- Schrodinger also considered a situation where two systems are correlated in a quantum-mechanical superposition state.

- For example, two photons with entangled polarization states:

$$\left|\psi\right\rangle = \left(\left|x_1\right\rangle\left|x_2\right\rangle + \left|y_1\right\rangle\left|y_2\right\rangle\right) / \sqrt{2}$$

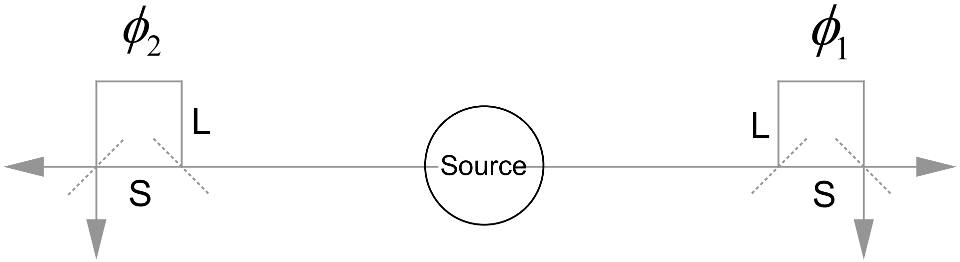
- We can also consider two photons known to have been emitted at the same time:

$$\left|\psi\right\rangle = \int f(t)\hat{E}_1{}^{(-)}(r_s,t)\hat{E}_2{}^{(-)}(r_s,t)dt$$

  - Energy-time entanglement.

NONLOCAL INTERFEROMETRY

- Suppose the two photons travel in opposite directions to two interferometers[1]:



- If we only accept events in which the photons arrive at the same time, there are two possibilities.
  - They both took the long path (L_1L_2) or the short paths (S_1S_2)
- There is no contribution from L_1S_2 or S_1L_2 .
- Interference between L_1L_2 and S_1S_2 gives a coincidence rate proportional to $\cos^2[(\phi_1+\phi_2)/2]$.
  - Violates Bell's inequality.

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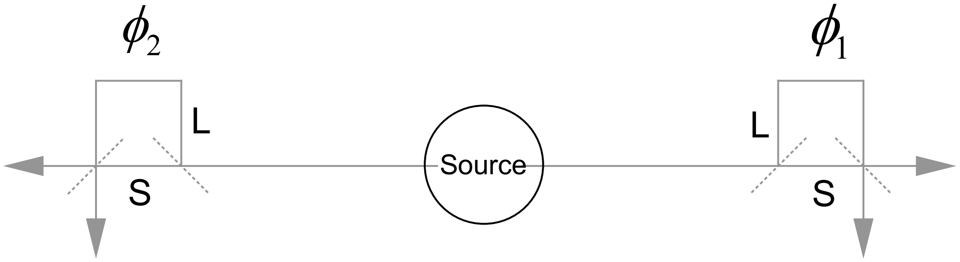
1. J. D. Franson, Phys. Rev. Lett. **62**, 2205-2208 (1989).

CONTROVERSIAL PREDICTION?



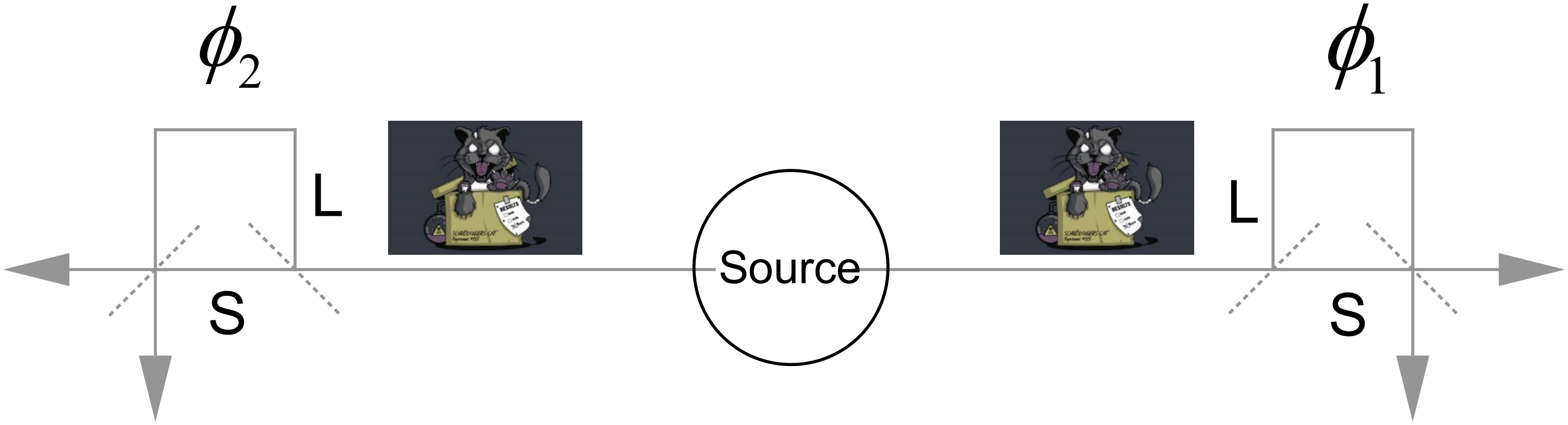
- This predicted effect was initially very controversial:
  - Classically, the output of interferometer 1 cannot depend on the setting of the distant phase shift ϕ_2 .
  - The difference in the path lengths is much larger than the (first-order) coherence length.
    - The second-order coherence length is much longer.

PHASE-ENTANGLED COHERENT STATES[1]

1. B.C. Sanders, Phys. Rev. A **45,** 6811 (1992).
2. C.C. Gerry, Phys. Rev. A **59**, 4095 (1999).

SCHRODINGER CATS AND NONLOCAL INTERFEROMETRY

Basic idea:



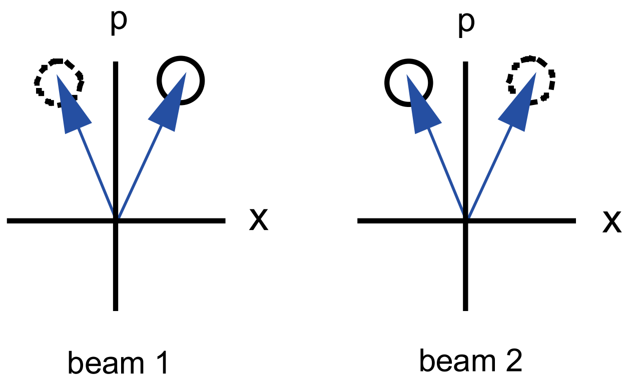
The Schrodinger cats will be macroscopic phase-entangled coherent states (laser beams).

How large is macroscopic? (Visible spot on a wall.)

ENTANGLED PHASE STATES

- We would like to generalize the nonlocal interferometer to use macroscopic phase-entangled states.
  - Two laser beams with anti-correlated phase shifts:

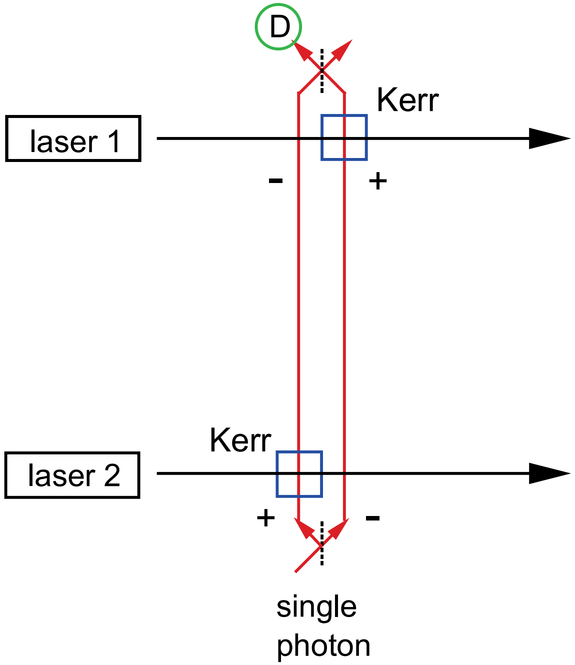
Wigner distributions:



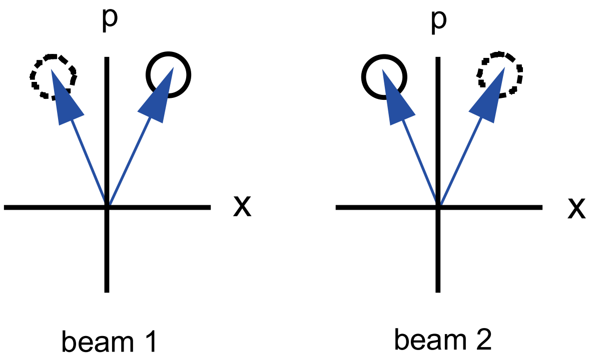
- We may expect macroscopic coherent states to be relatively robust against loss.
  - A coherent state subjected to loss remains coherent.
  - The only concern is “which-path” information left along the way.

GENERATION OF ENTANGLED PHASE STATES

- Munro et al.[1] have noted that a single photon can produce a significant phase shift in a coherent state:
  - Sufficient to produce orthogonal states.



- This can be used to produce an entangled state with anti-correlated phase shifts[2].
  - Post-select on a photon in detector D to get a superposition state with a well-defined relative phase.



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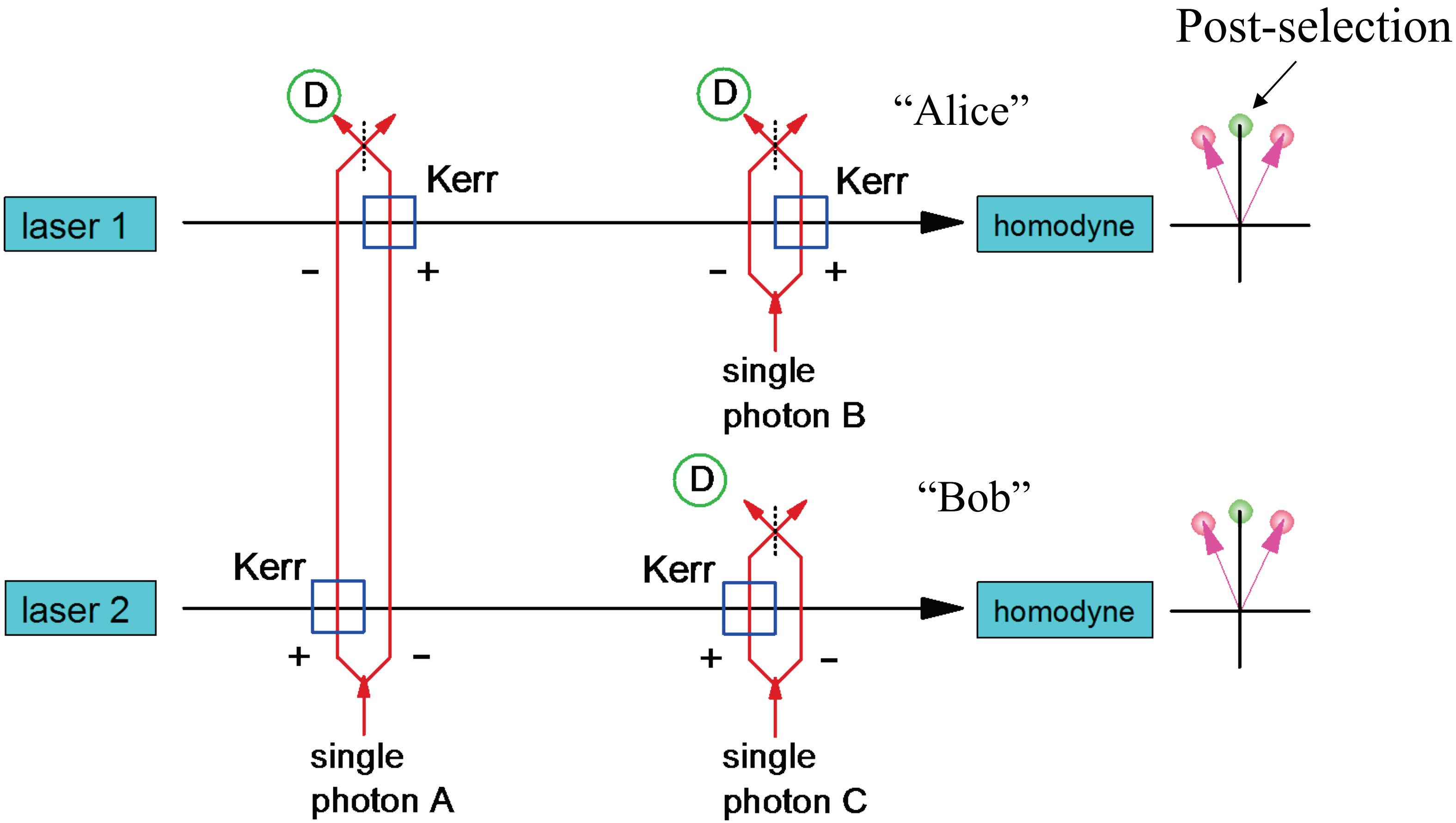
1. W.J. Munro, K. Nemoto, and T.P. Spiller, New J. Phys. 7, 137 (2005).
2. B. T Kirby and J.D. Franson, Phys. Rev. A **87**, 053822 (2013).

NONLOCAL INTERFEROMETETRY USING MACROSCOPIC COHERENT STATES

- Phase entanglement of coherent states can be used to implement a nonlocal interferometer:



VIOLATION OF BELL'S INEQUALITY

- After post-selection on the three single-photon detectors, the state of the system is

$$
\begin{aligned}
|\psi\rangle = \frac{1}{2\sqrt{2}}[& e^{i\sigma_2}\,|\alpha_{++}\rangle|\beta_{--}\rangle - |\alpha_{++}\rangle|\beta_{-+}\rangle \\
& - e^{i(\sigma_1+\sigma_2)}\,|\alpha_{+-}\rangle|\beta_{--}\rangle| + e^{i\sigma_1}\,|\alpha_{+-}\rangle|\beta_{-+}\rangle \\
& - e^{i\sigma_2}\,|\alpha_{-+}\rangle|\beta_{+-}\rangle + |\alpha_{-+}\rangle|\beta_{++}\rangle \\
& + e^{i(\sigma_1+\sigma_2)}\,|\alpha_{--}\rangle|\beta_{+-}\rangle - e^{i\sigma_1}\,|\alpha_{--}\rangle|\beta_{++}\rangle] \\
& \times|1\rangle_1\,|0\rangle_2\,|1\rangle_3\,|0\rangle_4\,|1\rangle_5\,|0\rangle_6 .
\end{aligned}
$$

- The projection onto a state in which there is no net phase shift gives

single-photon phase shifts

$$
\begin{aligned}
|p\rangle = \frac{1}{2\sqrt{2}}[& e^{i\sigma_1}\,|\alpha_{+-}\rangle|\beta_{-+}\rangle - e^{i\sigma_2}\,|\alpha_{-+}\rangle|\beta_{+-}\rangle] \\
& \times|1\rangle_1\,|0\rangle_2\,|1\rangle_3\,|0\rangle_4\,|1\rangle_5\,|0\rangle_6 .
\end{aligned}
$$

Analogous to the long-long and short-short paths in the original interferometer.

NONLOCAL INTERFERENCE

- Interference between these two terms gives a coincidence counting rate proportional to

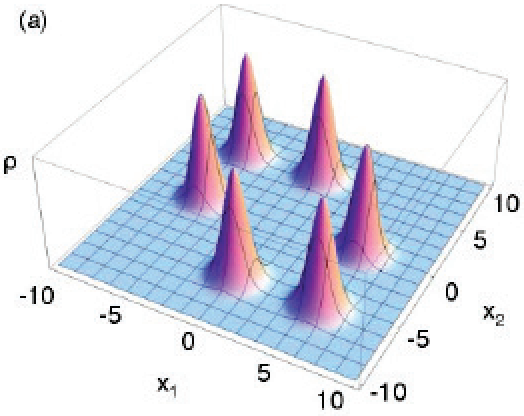
$$R_C = \frac{1}{2}\left[\sin^2\left(\frac{\sigma_1 - \sigma_2}{2}\right)\right].$$

  - In the absence of any decoherence.

- These states are relatively insensitive to photon loss.
  - Including beam splitter loss.

- Small nonlinear phase shifts can be produced in several ways.
  - We plan to use a Kerr interaction in a resonant cavity.

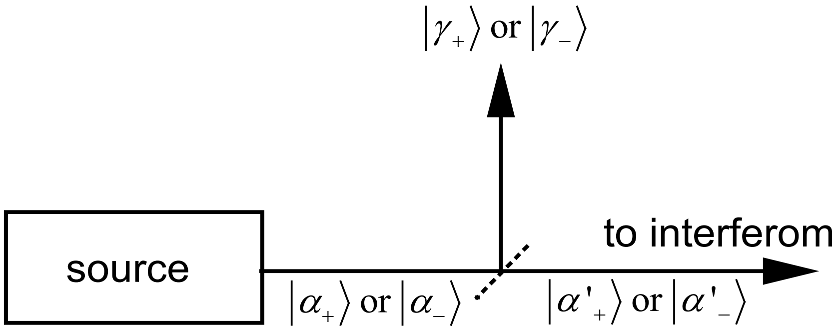
DECOHERENCE AND LOSS[1-4]



1. A. Mecozzi and P. Tombesi, Phys. Rev. Lett. **58**, 1055 (1987).
2. S.J. van Enk and O. Hirota, Phys. Rev. A. **64**, 022313 (2001).
3. H. Jeong, Phys. Rev. A **72**, 034305 (2005).
4. C. Stroud, Phys. Rev. A **81**, 052304 (2010).

DECOHERENCE DUE TO BEAM SPLITTERS

- Suppose a beam splitter with reflectivity r is inserted into the path to Alice:



- Slightly different coherent states are created in the output port of the beam splitters:

$$|\gamma_+\rangle = \left|r\alpha_0 e^{i\phi}\right\rangle \qquad |\alpha_+\rangle \rightarrow |\alpha'_+\rangle|\gamma_+\rangle$$

$$|\gamma_-\rangle = \left|r\alpha_0 e^{-i\phi}\right\rangle \qquad |\alpha_-\rangle \rightarrow |\alpha'_-\rangle|\gamma_-\rangle.$$

which-path information depending on ϕ

ENTANGLEMENT WITH THE ENVIRONMENT

- The coherent states become entangled with the beam splitter outputs.

$$|p\rangle = \frac{1}{2^3}[e^{i\sigma_1} |\alpha'_{+-}\rangle |\beta'_{-+}\rangle |\gamma_+\rangle |\delta_-\rangle$$

$$-e^{i\sigma_2} |\alpha'_{-+}\rangle |\beta'_{+-}\rangle |\gamma_-\rangle |\delta_+\rangle] |1\rangle_1 |0\rangle_2 |1\rangle_3 |0\rangle_4 |1\rangle_5 |0\rangle_6$$

- The cross-terms that give nonlocal interference are reduced by a factor of f^2:

$$f = \langle \gamma_- | \gamma_+ \rangle = \langle \delta_- | \delta_+ \rangle .$$

$$|f|^2 = |\langle \gamma_+ | \gamma_- \rangle|^2 = \exp[-|\gamma_+ - \gamma_-|^2]$$

$\phi << 1$

$$|f| = \exp[-2(r\alpha_0\phi)^2] = \exp[-2N_L\phi^2]$$

Number of photons lost

REDUCED VISIBILITY

- This which-path information reduces the visibility:

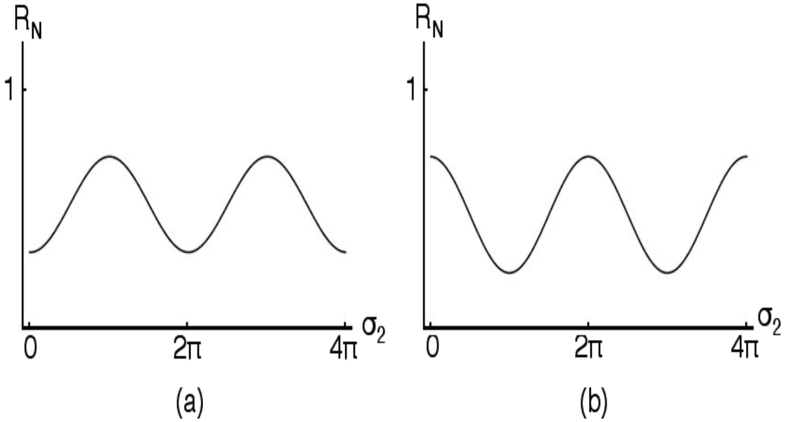
$$v = \frac{R_{\max} - R_{\min}}{R_{\max} + R_{\min}} = | f |^2 = \exp[-4 N_L \phi^2]$$

- Note that the decoherence is bounded regardless of the distance:

$$N_L = g N_0 = g \alpha_0^2$$

$$| f | = \exp[-2g(\alpha_0 \phi)^2]$$

$$g = \text{fractional loss} \leq 1$$

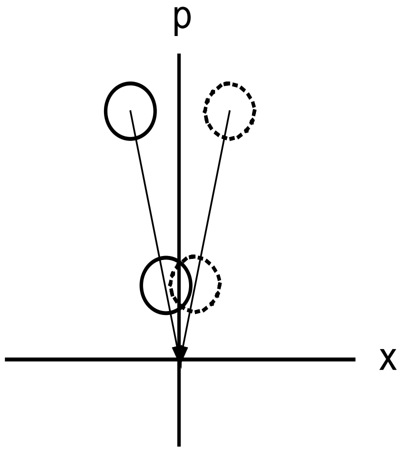


Same as for atomic absorption

For a loss of 4000 photons

OVERLAP OF COHERENT STATES

- Photon loss will also decrease the amplitude of the coherent states:



- The homodyne measurements can no longer completely resolve the phase shifts.
  - This will also reduce the visibility.

HARMONIC OSCILLATORS

- A single mode of the EM field is mathematically equivalent to an harmonic oscillator.
- In the coordinate (quadrature) representation:

$$\psi(q) = \langle q | \alpha \rangle = e^{|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} \langle q | n \rangle$$

$$= \left(\frac{\omega}{\pi \hbar} \right)^{1/4} e^{|\alpha|^2/2} e^{-\omega q^2/2\hbar} \sum_{n=0}^{\infty} \frac{\alpha^n}{n!} \frac{1}{2^{n/2}} H_n \left[\left(\frac{\omega}{\hbar} \right)^{1/2} q \right]$$

- This can be simplified using the generating function for the Hermite polynomials

$$\sum_{k=0}^{\infty} \frac{H_k(z) \nu^k}{k!} = e^{2z\nu - \nu^2}$$

COORDINATE REPRESENTATION

- The use of the generating function gives a gaussian:

$$\psi_{\alpha}(x)=\left(\frac{1}{\pi}\right)^{1/4}\exp\left\{-\frac{x^{2}}{2}+\frac{2x\alpha}{\sqrt{2}}-\frac{1}{2}|\alpha|^{2}-\frac{1}{2}\alpha^{2}\right\}$$

$$x=\sqrt{\omega/\hbar q}$$

- The probability density of obtaining a quadrature measurement with a value of x is then

$$\rho(x)=\psi^{*}(x)\psi(x)$$

- This is for a single homodyne measurement on a single coherent state.

GENERALIZATION TO ENTANGLED STATES

- This can be generalized to homodyne measurements on two entangled beams:

single photons

$$\psi(x_1,x_2)=\langle x_1,x_2;1,3,5|\Psi\rangle$$

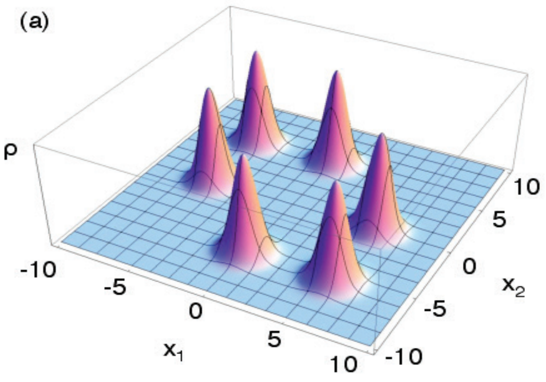
$$\rho(x_1,x_2)=\psi^*(x_1,x_2)\psi(x_1,x_2)$$

- All eight terms must now be retained

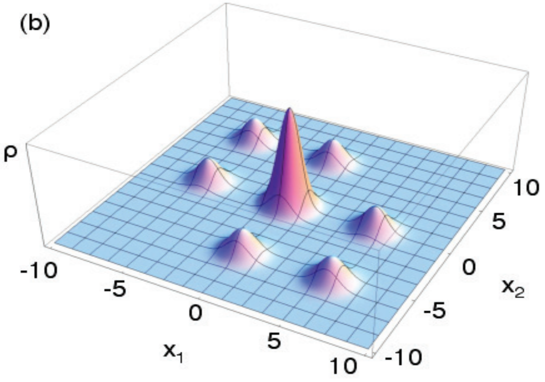
$$\begin{aligned}\psi(x_1,x_2)=\frac{1}{2^3}[&e^{i\sigma_2}\psi_{\alpha++}(x_1)\psi_{\beta--}(x_2)-\psi_{\alpha++}(x_1)\psi_{\beta-+}(x_2)\\&-e^{i(\sigma_1+\sigma_2)}\psi_{\alpha+-}(x_1)\psi_{\beta--}(x_2)+e^{i\sigma_1}\psi_{\alpha+-}(x_1)\psi_{\beta-+}(x_2)\\&-e^{i\sigma_2}\psi_{\alpha-+}(x_1)\psi_{\beta+-}(x_2)+\psi_{\alpha+-}(x_1)\psi_{\beta++}(x_2)\\&-e^{i(\sigma_1+\sigma_2)}\psi_{\alpha--}(x_1)\psi_{\beta+-}(x_2)-e^{i\sigma_1}\psi_{\alpha--}(x_1)\psi_{\beta++}(x_2)]\end{aligned}$$

EFFECTS OF OVERLAP

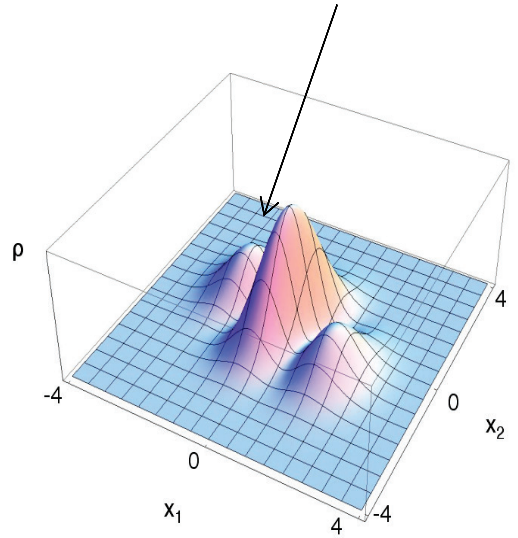
$$\sigma_1 - \sigma_2 = 0$$



$$\sigma_1 - \sigma_2 = \pi$$



quantum interference



$$\sigma_1 - \sigma_2 = \pi$$

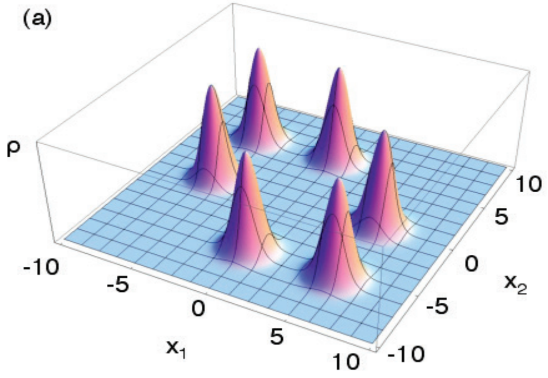
no overlap

large overlap

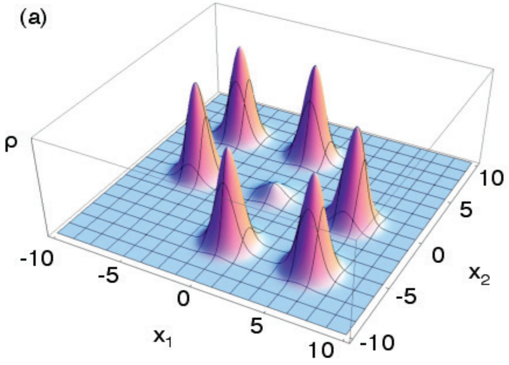
DECOHERENCE AND OVERLAP

- Here the effects of decoherence are combined with the overlap of the coherent states.

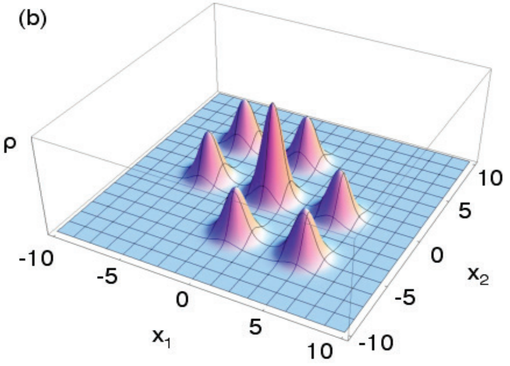
$$\sigma_1 - \sigma_2 = 0 \text{ for all cases}$$



no loss



100 photons

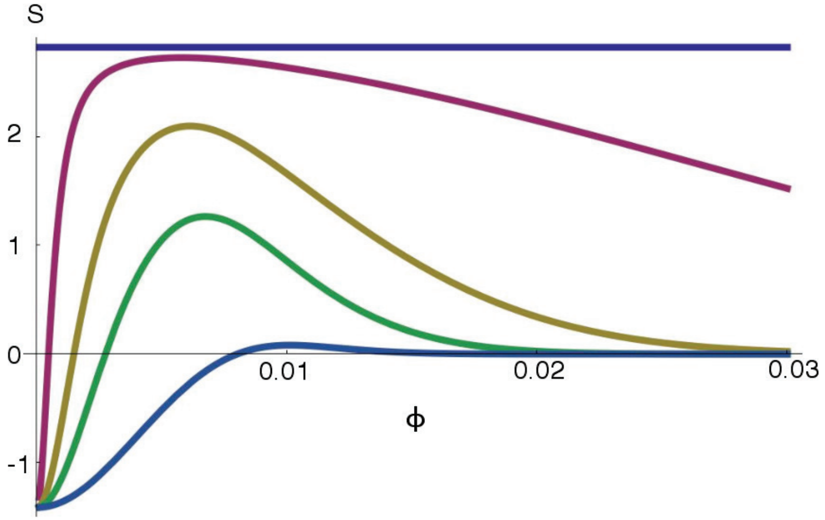


5800 photons

OPTIMIZED PERFORMANCE IN OPTICAL FIBER

- Here we took $\alpha = 100$ and varied ϕ .
- s is the parameter in the CHSH form of Bell's inequality.

$|s| > 2$ violates local realism.

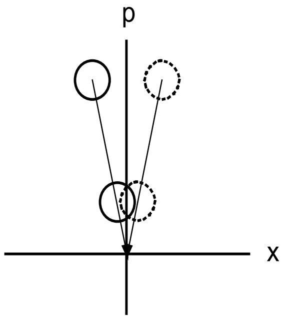


purple	1 km
yellow	8.2 km
green	20 km
blue	50 km

- Bell's inequality is violated up to 8.2 km separation.
- Coherent effects persist to much larger distances.

STATE-VECTOR DISCRIMINATION

- Loss tends to make the coherent states overlap, giving errors in the homodyne measurements.



- This problem can be avoided using state-vector discrimination.
  - We can determine the total phase shift with certainty, but only some fraction of the time.

- This should allow Bell’s inequality to be violated over a length of 400 km of commercially-available optical fiber.

EXPERIMENT
IN PROGRESS
AT UMBC

SMALL NONLINEAR PHASE SHIFTS

- A single photon is required to produce a small nonlinear phase shift (10^{-2} to 10^{-4} rad).
  - This is the main technical challenge.

- Nonlinear phase shifts can be produced using:
  - Natural resonances in atomic vapors.
  - Electromagnetically-induced transparency (EIT).
  - Nonlinear materials in wave guides.
  - Kerr effect in optical fibers.
  - …

- UMBC is investigating the use of high-finesse cavities for this purpose.

ADVANTAGES OF USING A CAVITY

- A small mode volume increases the magnitude of the nonlinear phase shift[1].
  - Nonlinear effects are typically inversely proportional to the mode volume.

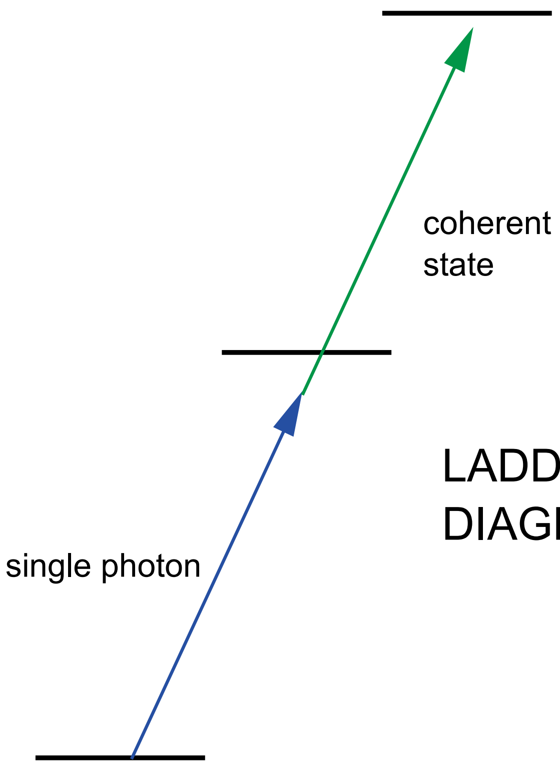
- A single-mode cavity avoids the fundamental decoherence associated with the Kerr effect in a propagating beam.[2]
  - These effects are not significant if there is a single cavity mode in the bandwidth of the medium.
  - The effects are small for small phase shifts.
  - Our approach is relatively robust against loss.

[1] Q.A. Turchette, C.J. Hood, W. Lange, H. Mabuchi, and H.J. Kimble, Phys. Rev. A **75**, 4710 (1995).

[2] J.H. Shapiro and M. Razavi, New J. Phys. **9**, 16 (2007).

EXPERIMENT AT UMBC

- We will concentrate on nonlinear phase shifts using natural resonances in hot atomic vapors.
- Results are expected to be comparable to experiments using single atoms[1].
  - For the same finesse.
- EIT may be used to further enhance the nonlinearity.



LADDER DIAGRAM

On resonance gives two-photon absorption.
Off resonance gives a nonlinear phase shift.

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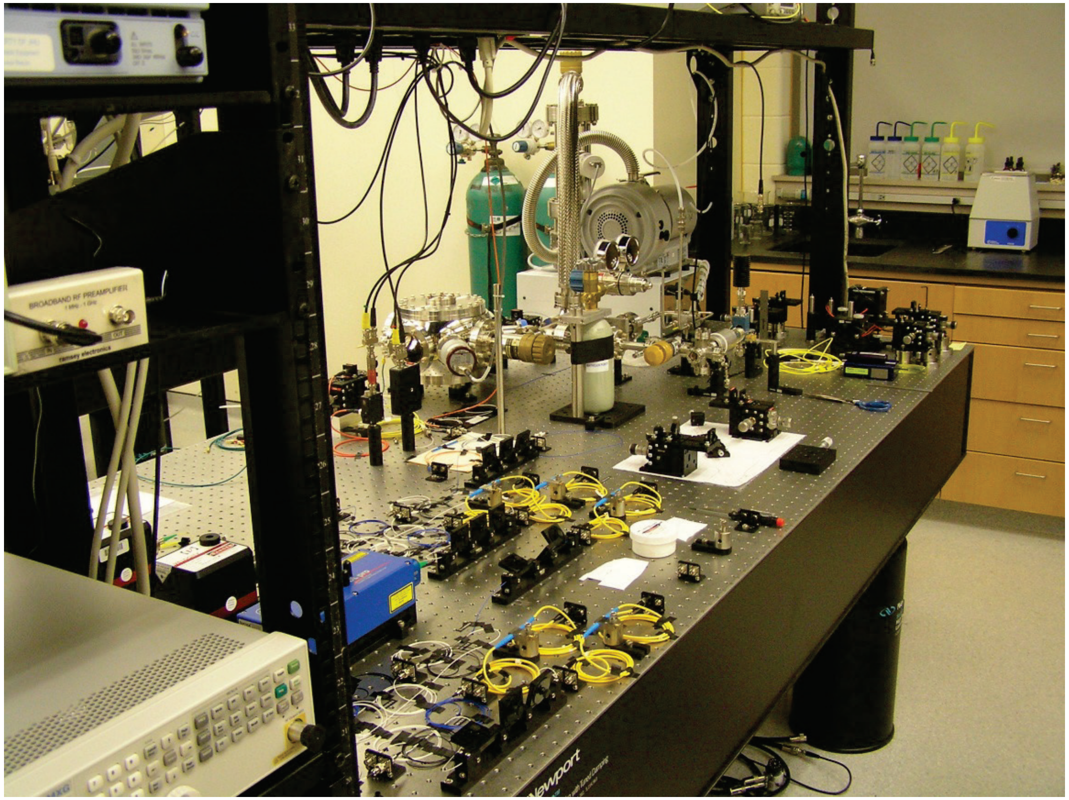
[1] H. You and J.D. Franson, Quantum Information Processing **11**, 1627 (2012).

EXPERIMENT IN PROGRESS AT UMBC



Enclosures for Alice and Bob

SUMMARY

- Schrodinger cats can be used to test the boundary between the quantum and classical worlds.
  - Nonlocal interferometry can demonstrate that the cats are really in a coherent superposition state.
  - Not already “alive” or “dead”.

- We are investigating the use of phase-entangled coherent states for this purpose.
  - May also be useful for QKD.

- This is part of a collaborative effort with U. Rochester, Boston U., and UMBC.

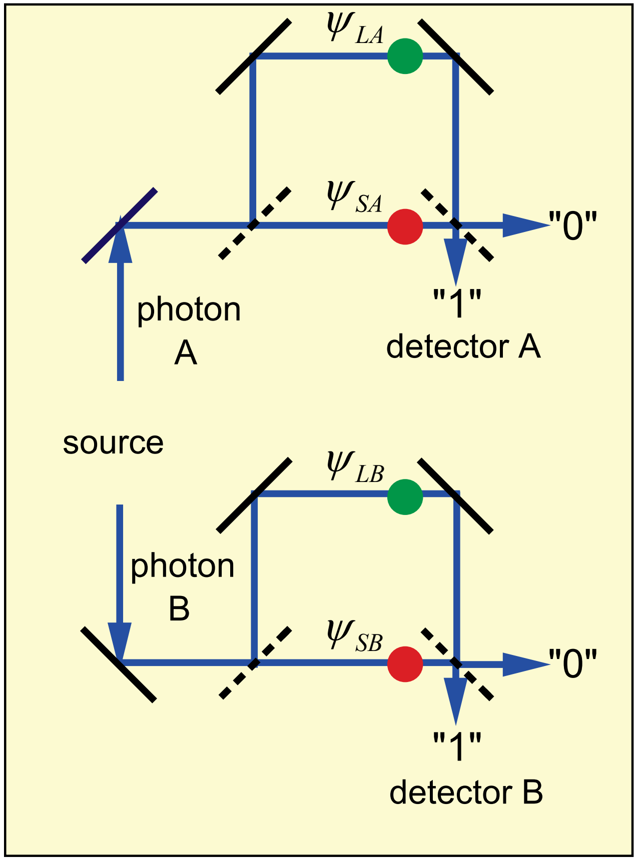
POST-SELECTION

- It has been suggested[1] that post-selection may allow hidden variable theories to agree with the results of interferometer experiments of this kind.
- That can be avoided using an “effective source”[2] in which the homodyne measurement is performed first.
- The output of the single-photon interferometers will then be in a pure Bell state.
  - They are allowed to propagate away from the post-selection region.
- Bell’s inequality can then be violated with no further post-selection.

[1] S. Aerts, P. Kwiat, J.-A. Larsson, and M. Zukowski, Phys. Rev. Lett. **83**, 2872 (1999).
[2] J.D. Franson, Phys. Rev. A **61**, 012105 (1999).

QUANTUM CRYPTOGRAPHY USING ENTANGLED PHOTONS

- Entangled photons produce correlated outputs from two distant interferometers.
- A secret code can be generated if we assign "0" and "1" bit values.
- This can be used to encode and decode secure messages.
- There is no information for an eavesdropper to intercept.



QKD PROTOCOL

- Some fraction of the pulses pass through the second set of interferometers.
  - A violation of Bell’s inequality rules out the possibility of an eavesdropper (Ekert protocol).
- The remainder of the pulses do not pass through the second set of interferometers.
  - Homodyne measurements directly read the correlated phase shifts.
  - A positive phase shift represents a bit value of “1”, a negative phase shift represents a bit value of “0”.
- Note that the actual bits in the key are classical.
  - Robust against loss and noise.
- Security proofs in general do not need a violation of Bell’s inequality.