GROUP THEORY ORIGIN OF THE HIGHER SPIN/CFT DUALITY

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DUALITY: CFT_d/AdS_{d+1}

- Understanding of Gravity,and StringTheory (in terms of QFT: 't Hoofts 1/N Expansion)

- 4d SUSY Yang-Mills (Integrable Spin chain in the Large N,Planar limit)

- Matrix Models (earlier noncritical strings)

- HIGHER SPIN GRAVITY in AdS :

CONTENT

- Emergence of AdS   HS Gravity :Collective Phenomena
- ADS4 Higher Spins: Collective Dipole: Conformal Group
- AdS3 /CFT2: W_N Symmetry
- Dynamics:"Field Theory of Characters"

Higher Spin Gravity/ CFT

- Vector Model/Higher Spin Duality [Klebanov & Polyakov 02, Sezgin & Sundell]

3d Critical O(N) Field Theories[Wilson] in the Large N Limit manifest themselves as :

4D Higher Spin Theories in AdS (of the kind developed by M. Vasiliev 1985-1997)

- Three-point Correlators(Giombi-Yin 2010)
- Operator Correspondence[K.JIN, R. DE MELLO KOCH, J.P. RODRIGUES,AJ ,2011]

Construction of HS :O(N) Vector Model

Collective Invariants:Bi-local (single trace) ops

$$\Psi(t;\vec{x},\vec{y}) = \sum_a \phi^a(t,\vec{x}) \cdot \phi^a(t,\vec{y})$$

Conjugate momenta:

$$\Pi(\vec{x},\vec{y}) = -i\frac{\delta}{\delta\Psi(\vec{x},\vec{y})}$$

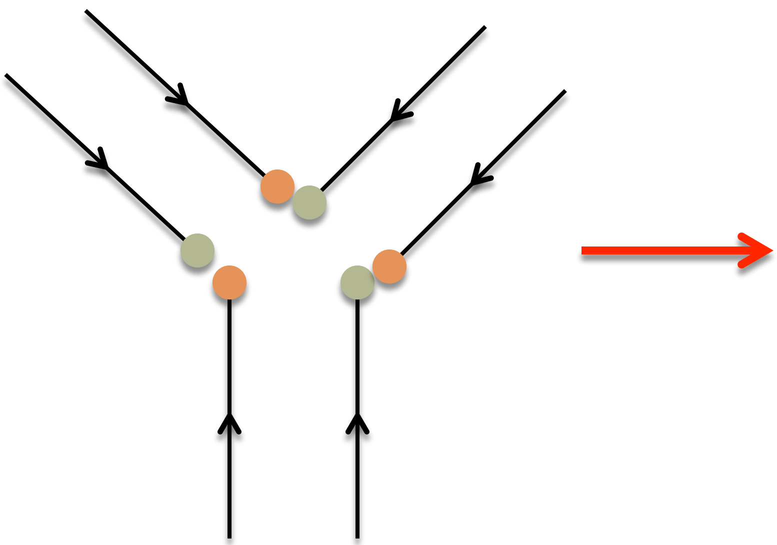
The Hamiltonian

$$H = 2\mathrm{Tr}(\Pi\Psi\Pi) + \frac{1}{2}\int[-\nabla_x^2\Psi(\tilde{x},\tilde{y})|_{\tilde{x}=\tilde{y}}] + \frac{N^2}{8}\mathrm{Tr}\Psi^{-1}$$

- Generates the 1/N to all orders

- Interactions: H3 , H4 ,…..

$$G = \frac{1}{\sqrt{N}}$$



Cubic Vertex in AdS_4

- Need to give a Map(from Bi-locals to AdS Higher Spin Fields)

Higher Spins in terms of Bi-local Fields

- Light-Cone [de Mello Koch & Jin& Rodrigues & AJ, 2010]
- One to One Map

Extra bulk coordinate

$$\Phi\left(\tau, \left(x_1^-, x_1\right), \left(x_2^-, x_2\right)\right) \longleftrightarrow H\left(\tau; x^-, x, z, \theta\right)$$

Spin

- Such that

$$H\left(\tau; x^-, x, z, \theta\right) \overset{z \longrightarrow 0}{\sim} z^{\Delta} \mathcal{O}\left(\tau, x\right)_{\mathrm{CFT}}$$

- Conformal Symmetry CFT → Symmetry of AdS_4

Map:SO(2,3)

- **Identifying symmetries of the dipole with the SO(2,3) symmetry of HS/AdS4: gives 10 equations of 2×4=8 canonical variables**

$$x^- = \frac{x_1^- p_1^+ + x_2^- p_2^+}{p_1^+ + p_2^+},$$
$$p^+ = p_1^+ + p_2^+,$$
$$x = \frac{x_1 p_1^+ + x_2 p_2^+}{p_1^+ + p_2^+},$$
$$p^x = p_1 + p_2.$$

$$z = \frac{(x_1 - x_2)\sqrt{p_1^+ p_2^+}}{p_1^+ + p_2^+},$$

$$\theta = 2\arctan\sqrt{\frac{p_2^+}{p_1^+}}.$$

- **Spin**

$$p^\theta = \sqrt{p_1^+ p_2^+}(x_1^- - x_2^-) + \frac{x_1 - x_2}{2}\left(\sqrt{\frac{p_2^+}{p_1^+}}p_1 + \sqrt{\frac{p_1^+}{p_2^+}}p_2\right).$$

Field Map

- Changing to AdS variables through a generalized Fourier transform

$$\begin{aligned}\Psi(x^-,x,z,\theta) \;=\; & \int dp^+ dp^x dp^z e^{i(x^- p^+ + x p^x + z p^z)} \\ & \int dp_1^+ dp_2^+ dp_1 dp_2 \delta(p_1^+ + p_2^+ - p^+)\delta(p_1 + p_2 - p^x) \\ & \delta\left(p_1\sqrt{p_2^+/p_1^+} - p_2\sqrt{p_1^+/p_2^+} - p^z\right) \\ & \delta(2\arctan\sqrt{p_2^+/p_1^+} - \theta)\tilde{\Phi}(p_1^+, p_2^+, p_1, p_2)\end{aligned}$$

Higher Spin Fields in AdS in terms of the Bi-local Map is 1-1

- 2d CFT:MINIMAL MODELS [Gaberdiel & Gopakumar, since 2011]

W_N Minimal Model CFT

Higher Spin theory on AdS_3

[Henneaux & Rey, 2010], [Campoleoni et al., 2010], [Gaberdiel, Hartman, 2011]

Asymptotic Symmetry Algebra of Higher Spin theory on AdS_3

Classical W_N or W_∞ symmetry Algebra

- Comparison of Large N Partition Function

[Gaberdiel ,Gopakumar,Raju ,Hartman, 2012, Perlmutter et al]

W_N Minimal Model

- Coset model $$\frac{su(N)_k \oplus su(N)_1}{su(N)_{k+1}}$$
- Central charge $$c = (N-1)\left(1 - \frac{N(N+1)}{(N+k)(N+k+1)}\right)$$
- The highest weight :primaries
  - Denoted by two Young tableaux : (Λ_+ ; Λ_-)
  - Λ_+ and Λ_- are representations of $su(N)_k$ and $su(N)_{k+1}$, respectively.
  - Conformal dimension

$$h(\Lambda_+;\Lambda_-) = \frac{1}{2p(p+1)}\left\{|(p+1)\Lambda_+ - p\Lambda_- + \rho|^2 - |\rho|^2\right\}$$

where $p = N + k$ and ρ is the Weyl vector

W Algebra

- Casimir Algebra :
  - Ext Sugawara :Balog,Feher,Forgacs,O'Raifeataigh ,Wipf
  - SU(N) WZW model conserved current J(z)

$$J^a(z)\,J^b(w) = \frac{k\delta^{ab}}{(z-w)^2} + f^{ab}{}_c \frac{J^c(w)}{z-w} + \cdots$$

  - Casimir operators

$$W^s = \frac{1}{s!}\eta^{(s)} \sum_{a,b,c,\cdots} d_{a,b,c,\cdots}\left(J^a\left(J^b\left(J^c\left(\cdots\right)\right)\right)\right)(z)$$

$d_{abc\cdots}$: totally symmetric traceless su(N) invariant tensor of rank s

$$W^2(z) = T(z) = \frac{1}{2(N+k)}\left(J^a J^a\right)(z)$$: Stress-energy tensor

$$W^3(z) = \frac{1}{6}\eta^{(3)} d_{abc}\left(J^a\left(J^b J^c\right)\right)(z)$$

W Algebra

- Commutation Relations

$$[W^2_m, W^2_n] = \frac{c}{12} m \left(m^2 - 1\right) \delta_{m,-n} + (m - n)\, W^2_{m+n}$$

$$[W^3_m, W^3_n] = \frac{c}{3 \cdot 5!} \left(m^2 - 2\right) \left(m^2 - 1\right) m \delta_{m,-n} + \frac{1}{30} (m - n) \left(2m^2 - mn + 2n^2 - 8\right) W^2_{n+m}$$
$$+ \frac{16}{22 + 5c} (m - n) \left(\sum_{p \in \mathbb{Z}} \left(W^2_{m+n+p} W^2_{-p}\right) + x_{m+n} W^2_{m+n} \right)$$

$$[W^2_m, W^3_n] = (2m - n)\, W^3_{m+n}$$

$$x_{n+m} = -\frac{3}{10} (n + m + 3)(n + m + 2)$$

Generator

- Dynamical generators : W^s_{-k} $\quad k \geqq s$
  - Corresponds to higher spin fields $\longrightarrow Z_{\mathrm{hs}}$
  - e.g. Virasoro algebra (s=2)

$$(L_{-n}\psi) = \frac{1}{(n-2)!}(\partial^{n-2}T\psi)$$

- Kinematical(Wedge) generators : W^s_{-k} $\quad k < s$
  - With primaries, correspond to scalar field
  - e.g. Virasoro algebra (s=2)

$$(L_{-1}\psi) = \partial\psi$$

  - Note : related to derivatives only.

OPERATOR CONSTRUCTION

- a Collective Field Theory
- Based on Single-Trace Primary Operators
- That re-produces :the spectrum of conformal dimensions h(Λ_+ ; Λ_-): 1/N=G
- THE DUAL VERSION OF THE THEORY(Bulk AdS HS Gravity)

"Field Theory of Primaries" with JungGi Yoon

Exact Conformal Dimensions

- t'Hooft Variable
  - N , k $\longrightarrow$: $\lambda \equiv \frac{N}{N+k}$ $\quad c = (N-1)\left(1 - \frac{\lambda\left(\lambda + \frac{\lambda}{N}\right)}{1+\frac{\lambda}{N}}\right)$

Conformal dimension

$$h\left(\Lambda_+;\Lambda_-\right) = \frac{\lambda}{2}\left(B_+ - B_-\right) + \frac{1}{2}\sum_{i=1}^{N-1}\left(r_i^+ - r_i^-\right)^2 + \frac{\lambda}{2N}\left(D_+ - D_-\right) - \frac{1}{2N}\left(B_+ - B_-\right)^2$$
$$- \frac{\lambda}{2N^2}\left(B_+^2 - B_-^2\right) + \frac{\frac{\lambda^2}{N^2}}{1+\frac{\lambda}{N}}\left(\frac{1}{2}B_- N + \frac{1}{2}D_- - \frac{B_-^2}{2N}\right)$$

- $r^{\pm}_i$ is the number of boxes in the *i*th row of $\Lambda_{\pm}$.
- $c^{\pm}_i$ is the number of boxes in the *i*th column of $\Lambda_{\pm}$.

$$D_{\pm} = \sum_{i=1}^{N-1}\left(r_i^{\pm}\right)^2 - \sum_{j=1}^{\infty}\left(c_j^{\pm}\right)^2$$

- Exact: 1/N Series

Single-trace primaries

- Simple single-trace primary operators

$$(\,\square\,;\,0\,)=\psi_0 \quad (\,\overline{\square}\,;\,0\,)=\overline{\psi}_0 \quad (\,\square\,;\,\square\,)=\omega_1 \quad (\,\overline{\square}\,;\,\overline{\square}\,)=\overline{\omega}_1$$

- But operators corresponding to higher Young tabloux :some are single some multi-traces
- From the Coulomb gas Representation of Operators(Toda) not clear how to see what would correspond to 'single ' and what would be 'multi' trace operators

Chang and Yin[2011,2012]

- 3-Point Correlation functions :

$$\left\langle \mathcal{O}_{(\Lambda_+^1;\Lambda_-^1)} \mathcal{O}_{(\Lambda_+^2;\Lambda_-^2)} \mathcal{O}_{(\Lambda_+^3;\Lambda_-^3)} \right\rangle = \frac{C_3\left((\Lambda_+^1;\Lambda_-^1),(\Lambda_+^2;\Lambda_-^2),(\Lambda_+^3;\Lambda_-^3)\right)}{|x_{12}|^{\Delta_1+\Delta_2-\Delta_3}\,|x_{23}|^{\Delta_2+\Delta_3-\Delta_1}\,|x_{31}|^{\Delta_3+\Delta_1-\Delta_2}}$$

- One has :

$$\langle \mathcal{O}_{\text{single}} \mathcal{O}_{\text{single}} \mathcal{O}_{\text{single}} \rangle \sim \frac{1}{\sqrt{N}}$$

vs

$$\langle \mathcal{O}_{\text{single}} \mathcal{O}_{\text{single}} \mathcal{O}_{\text{multi}} \rangle \sim 1$$

  - exmpl. $C_3\left((\bar{\square};\bar{\square}),(\bar{\square};\bar{\square}),(\square\square;\square\square)\right) = 1 + \mathcal{O}\left(\frac{1}{N^2}\right)$

single-trace multi-trace

Result

- Through 3-point function [Chang & Yin, 2011]

Identification
$\left(\begin{smallmatrix}\square\\\square\end{smallmatrix}\,;\ 0\right) = \frac{1}{\sqrt{2}}\psi_0^2$
$\left(\left(\square,\overline{\square}\right)\ ;\ 0\right) = \psi_0\overline{\psi}_0$
$\left(\begin{smallmatrix}\square\\\square\end{smallmatrix}\,;\,\begin{smallmatrix}\square\\\square\end{smallmatrix}\right) = \frac{1}{2}\omega_1^2 - \frac{1}{\sqrt{2}}\omega_2$
$\left(\square\square\,;\,\square\square\right) = \frac{1}{2}\omega_1^2 + \frac{1}{\sqrt{2}}\omega_2$
$\left(\begin{smallmatrix}\square\\\square\end{smallmatrix}\,;\,\square\right) = \frac{1}{\sqrt{2}}\left(\psi_1 + \psi_0\omega_1\right)$
$\left(\square\square\,;\,\square\right) = \frac{1}{\sqrt{2}}\left(-\psi_1 + \psi_0\omega_1\right)$

Note : Our notation is different from Yin's one.
e.g. we use ψ_n instead of ϕ_{n-1} of Yin's result

Method

(Three point function method:difficult to generalize)

- Introduce a Fock Space :1-1 relation to single-traces
- Define a Collective Hamiltonian

  (Spectrum of Eigenvalues of H reproduces the exact formula for the conformal dimensions $h(\Lambda_+;\Lambda_-)$)

  Primaries $O(\Lambda_+ ; \Lambda_-)$ appear as Eigenfunctions of *H:we will obtain expressions for Higher primaries in terms of the single traces*.

Fock Space

- Minimal Fock Space
  - $\psi_0, \psi_1, \psi_2, \psi_3, \ldots$: related to states with h ~ O(1).
  - $\omega_1, \omega_2, \omega_3, \omega_4, \ldots$: related to light states.
    - The index in each field is "winding number".
- Two global operators:Superselection Sectors
  - The total number of ψ's : $M = \sum_{n=0}^{\infty} \psi_n \frac{\partial}{\partial \psi_n}$
    - $M = |\Lambda_+| - |\Lambda_-|$
    - leading contribution
  - The winding number : $K = \sum_{n=0}^{\infty} n\psi_n \frac{\partial}{\partial \psi_n} + \sum_{n=1}^{\infty} n\omega_n \frac{\partial}{\partial \omega_n}$
    - $K=|\Lambda_-|$
    - In the semiclassical limit, the winding number gives the leading contribution to conformal dimension.

The Collective Hamiltonian

- Global (includes quadratic terms)

$$H_0 = \frac{\lambda}{2} M - \frac{M^2}{2N} - \frac{\lambda}{2N^2} (M + 2K) M + \frac{\frac{\lambda^2}{N^2}}{1 + \frac{\lambda}{N}} \left[\frac{N}{2} K - \frac{1}{2N} K^2 \right] \qquad H_1 = \frac{1}{2} M$$

- Cubic interactions
  - Two coupling constant : $\frac{\lambda}{N} \gg \frac{\frac{\lambda^2}{N^2}}{1 + \frac{\lambda}{N}}$
    - Interaction between ω's

$$H_2 = \frac{\frac{\lambda^2}{N^2}}{1 + \frac{\lambda}{N}} \left[\frac{1}{2} \sum_{n=2}^{\infty} \sum_{m=1}^{n-1} \sqrt{nm(n-m)} \omega_m \omega_{n-m} \frac{\partial}{\partial \omega_n} + \frac{1}{2} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \sqrt{nm(n+m)} \omega_{n+m} \frac{\partial^2}{\partial \omega_n \partial \omega_m} \right]$$

    - Interaction between ψ and ω

$$H_3 = \frac{\lambda}{N} \left[\sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \sqrt{m} \psi_{n+m} \frac{\partial^2}{\partial \psi_n \partial \omega_m} + \sum_{n=1}^{\infty} \sum_{m=1}^{n} \sqrt{m} \psi_{n-m} \omega_m \frac{\partial}{\partial \psi_n} \right]$$

$$H_4 = \frac{\frac{\lambda^2}{N^2}}{1 + \frac{\lambda}{N}} \left[\sum_{n=0}^{\infty} \sum_{m=1}^{\infty} n \sqrt{m} \psi_{n+m} \frac{\partial^2}{\partial \psi_n \partial \omega_m} + \sum_{n=1}^{\infty} \sum_{m=1}^{n} (n-m) \sqrt{m} \psi_{n-m} \omega_m \frac{\partial}{\partial \psi_n} \right]$$

Full Collective Hamiltonian

- Quartic Interactions
  - only between ψ's

$$H_5 = -\frac{\lambda}{2N}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\sum_{u=0}^{n+m}\psi_{n+m-u}\psi_u\frac{\partial^2}{\partial\psi_n\partial\psi_m}$$

$$H_6 = -\frac{\frac{\lambda^2}{N^2}}{2\left(1+\frac{\lambda}{N}\right)}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\sum_{u=0}^{n+m}F\left(n,m,u\right)\psi_{n+m-u}\psi_u\frac{\partial^2}{\partial\psi_n\partial\psi_m}$$

$$F\left(n,m,u\right)=\begin{cases} u & 0 \leqq u \leqq \min\left(n,m\right) \\ \min\left(n,m\right) & \min\left(n,m\right) \leqq u \leqq \max\left(n,m\right) \\ n+m-u & \max\left(n,m\right) \leqq u \leqq n+m \end{cases}$$

Eigenstates:Characters

- General states

$$(\Lambda_+;\Lambda_-) = \sum c(\{a\},\{b\}) \prod_{n=1}^{|\Lambda_+|-|\Lambda_-|} \psi_{a_n} \prod_{m=1}^{\infty} \omega_{b_m} \quad \text{with} \quad \sum_{n=1}^{|\Lambda_+|-|\Lambda_-|} a_n + \sum_{m=1}^{\infty} b_m = |\Lambda_-|$$

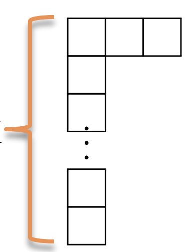
  - The number of ψ's $=|\Lambda_+|-|\Lambda_-|$
  - Total winding number $=|\Lambda_-|$

- Pure ω States:Schur Polynomials

$$(\Lambda;\Lambda) = P_n(\Lambda;\{\sqrt{j}\omega_j\}) = \frac{1}{n!}\sum_{g\in S_n}\left[ch_\Lambda(g)\prod_{j=1}^{\infty}\left(\sqrt{j}\omega_j\right)^{\lambda(g)_j}\right] \quad \text{where } n = |\Lambda|$$

  - Conversely, expressing ω_j in terms of (Λ;Λ):Single traces

$$\omega_n = \frac{1}{\sqrt{n}}\sum_{k=1}^{n}(-1)^k\left(\Lambda_k^{(n)};\Lambda_k^{(n)}\right)$$

    - where $\Lambda_k^{(n)}$ is a Young tableau with n boxes such that k 
    - e.g.

$$\omega_4 = \frac{1}{\sqrt{4}}\left[\left(\yng(4);\yng(4)\right) - \left(\yng(3,1);\yng(3,1)\right) + \left(\yng(2,1,1);\yng(2,1,1)\right) - \left(\yng(1,1,1,1);\yng(1,1,1,1)\right)\right]$$

Generalized Schur Polynomials

Eigenfunction	Conformal Dimension
$(\,\tiny\yng(1,1)\,;\,\square\,) = \frac{1}{\sqrt{2}}(-\psi_1 + \psi_0\omega_1)$	$E_{2,1} - \frac{\lambda}{N}$
$(\,\tiny\yng(2)\,;\,\square\,) = \frac{1}{\sqrt{2}}(\psi_1 + \psi_0\omega_1)$	$E_{2,1} + \frac{\lambda}{N}$

where $E_{2,1} = \frac{1}{2}(1+\lambda) - \frac{1}{2N} - \frac{3\lambda}{2N^2} + \epsilon\left(\frac{1}{2}N - \frac{1}{2N}\right)$

Higher Examples :

Eigenfunction	Conformal Dimension
$\left(\yng(1,1,1)\,;\,\yng(1,1)\right) = \frac{1}{\sqrt{3}}\psi_2 - \frac{1}{\sqrt{3}}\psi_1\omega_1 + \frac{1}{2\sqrt{3}}\psi_0\omega_1^2 - \frac{1}{\sqrt{6}}\psi_0\omega_2$	$E_{3,2} - 2\frac{\lambda}{N} - \epsilon$
$\left(\yng(2,1)\,;\,\yng(2)\right) = -\frac{1}{\sqrt{6}}\psi_2 - \frac{1}{\sqrt{6}}\psi_1\omega_1 + \frac{1}{\sqrt{6}}\psi_0\omega_1^2 + \frac{1}{\sqrt{3}}\psi_0\omega_2$	$E_{3,2} - \frac{\lambda}{N} + \epsilon$
$\left(\yng(2,1)\,;\,\yng(1,1)\right) = -\frac{1}{\sqrt{6}}\psi_2 + \frac{1}{\sqrt{6}}\psi_1\omega_1 + \frac{1}{\sqrt{6}}\psi_0\omega_1^2 - \frac{1}{\sqrt{3}}\psi_0\omega_2$	$E_{3,2} + \frac{\lambda}{N} - \epsilon$
$\left(\yng(3)\,;\,\yng(2)\right) = \frac{1}{\sqrt{3}}\psi_2 + \frac{1}{\sqrt{3}}\psi_1\omega_1 + \frac{1}{2\sqrt{3}}\psi_0\omega_1^2 + \frac{1}{\sqrt{6}}\psi_0\omega_2$	$E_{3,2} + 2\frac{\lambda}{N} + \epsilon$

where $E_{3,2} = \frac{1}{2}(1+\lambda) - \frac{1}{2N} - \frac{5\lambda}{2N^2} + \epsilon\left(N - \frac{2}{N}\right)$ and $\epsilon = \dfrac{\frac{\lambda^2}{N^2}}{1+\frac{\lambda}{N}}$

Note that terms proportional to ψ_0 are Schur polynomial of $\sqrt{n}\,\omega_n$

Examples : Continued

Eigenfunction	Conformal Dimension
$([1,1,1,1,1];[1,1]) = \frac{3}{2\sqrt{15}}\psi_2\psi_0^2 + \frac{3}{2\sqrt{15}}\psi_1^2\psi_0 - \frac{3}{2\sqrt{15}}\psi_1\psi_0^2\omega_1 + \frac{1}{4\sqrt{15}}\psi_0^3\omega_1^2 - \frac{1}{2\sqrt{30}}\psi_0^3\omega_2$	$E_{5,2} - 9\frac{\lambda}{N} - \epsilon$
$([2,1,1,1];[2]) = -\frac{3}{2\sqrt{15}}\psi_2\psi_0^2 - \frac{3}{2\sqrt{15}}\psi_1\psi_0^2\omega_1 + \frac{1}{2\sqrt{15}}\psi_0^3\omega_1^2 + \frac{1}{\sqrt{30}}\psi_0^3\omega_2$	$E_{5,2} - 6\frac{\lambda}{N} + \epsilon$
$([2,1,1,1];[1,1]) = \frac{1}{2\sqrt{15}}\psi_2\psi_0^2 - \frac{2}{\sqrt{15}}\psi_1^2\psi_0 - \frac{1}{2\sqrt{15}}\psi_1\psi_0^2\omega_1 + \frac{1}{2\sqrt{15}}\psi_0^3\omega_1^2 - \frac{1}{\sqrt{30}}\psi_0^3\omega_2$	$E_{5,2} - 4\frac{\lambda}{N} - \epsilon$
$([2,2,1];[1,1]) = -\frac{1}{2\sqrt{3}}\psi_2\psi_0^2 + \frac{1}{2\sqrt{3}}\psi_1^2\psi_0 + \frac{1}{2\sqrt{3}}\psi_1\psi_0^2\omega_1 + \frac{1}{4\sqrt{3}}\psi_0^3\omega_1^2 - \frac{1}{2\sqrt{6}}\psi_0^3\omega_2$	$E_{5,2} - \frac{\lambda}{N} - \epsilon$
$([3,1,1];[2]) = \frac{1}{\sqrt{10}}\psi_2\psi_0^2 + \frac{1}{\sqrt{10}}\psi_1\psi_0^2\omega_1 + \frac{1}{2\sqrt{10}}\psi_0^3\omega_1^2 + \frac{1}{2\sqrt{5}}\psi_0^3\omega_2$	$E_{5,2} - \frac{\lambda}{N} + \epsilon$

where $E_{5,2} = \frac{3}{2}(1+\lambda) - \frac{9}{2N} - \frac{21\lambda}{2N^2} + \epsilon\left(N - \frac{2}{N}\right)$ and $\epsilon = \frac{\frac{\lambda^2}{N^2}}{1+\frac{\lambda}{N}}$

Note: This Generalized ‘Schur” Polynomial relations are new:Bantay Orbifold CFT S_n

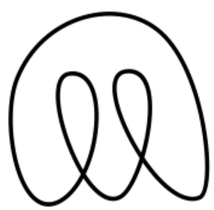
Matrix-vector /Inegrable:Spin-Calogero

$$H_{\mathrm{MV}} = \sum_{i,j} \frac{1}{2} m_M \frac{\partial^2}{\partial M_{ij} \partial M_{ji}} + \sum_{a=1}^{n_f} \frac{1}{2} m_a \frac{\partial}{\partial \bar{x}_i^a} \frac{\partial}{\partial x_i^a}$$

- $M_{ij}(t)$: SU(N) matrix field   i, j=1, 2, … , N (color)
  x_i^a : complex vector field   a=1, 2, … , n_f (flavor)

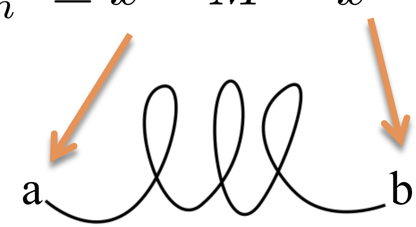
- Under M $\longrightarrow$ VMV^{-1}, $x^a \longrightarrow Vx^a$ ($V \in SU(N)$)
  Invariant variables:

$$\phi_n \equiv tr\,(M^n)$$



Closed Loop

$$\psi_n^{ab} \equiv \bar{x}^a \cdot M^n \cdot x^b$$

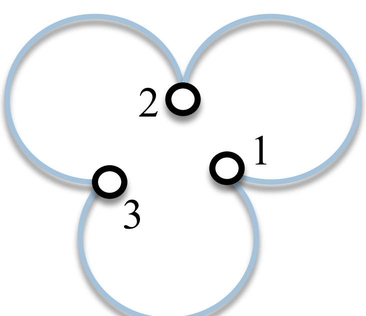


Open Loop

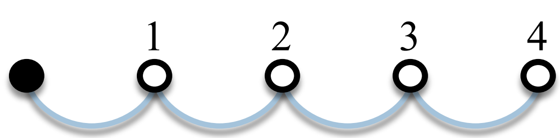
Geometrical Interpretation : H_3

$$H_3 = \frac{\lambda}{N}\left[\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} \underbrace{m\psi_{n+m}\frac{\partial^2}{\partial\psi_n\partial\phi_m}}_{\text{joining}} + \sum_{n=1}^{\infty}\sum_{m=1}^{n} \underbrace{\psi_{n-m}\phi_m\frac{\partial}{\partial\psi_n}}_{\text{splitting}}\right]$$

- Splitting ψ_n into ψ_{n-m} and ϕ_m
  - (coefficient)=(the number of the following possible ways)
  - Cut one end of point in ψ_n. (e.g. black point of loop 2)
  - Cut the other point (e.g. point 1~4 of loop 2) to get ϕ_m.
- Joining ψ_n and ϕ_m into ψ_{n+m}
  - (coefficient)=(the number of the following possible ways)
  - Cut one end of ψ_n(e.g. black point of loop 2).
  - Cut one point of ϕ_m (e.g. point 1~3 of loop 1) and glue them.



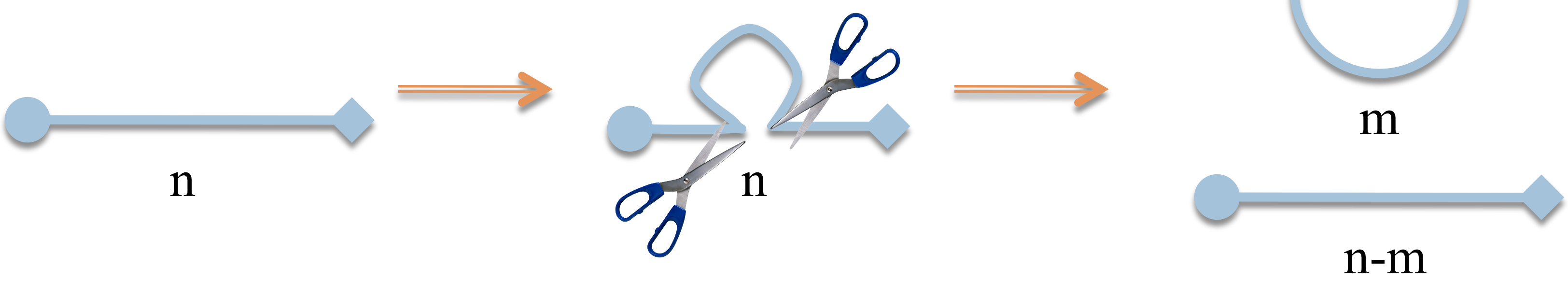
Loop 1



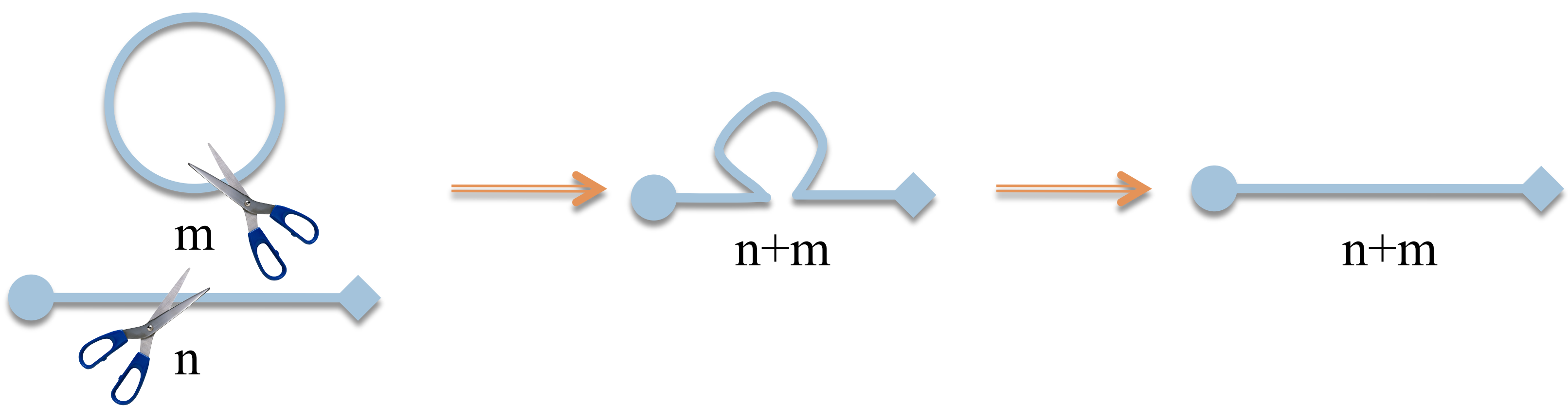
Loop 2

H_4

- Splitting $\sum_{n=1}^{\infty}\sum_{m=1}^{n}(n-m)\,\psi_{n-m}\phi_m\frac{\partial}{\partial\psi_n}$



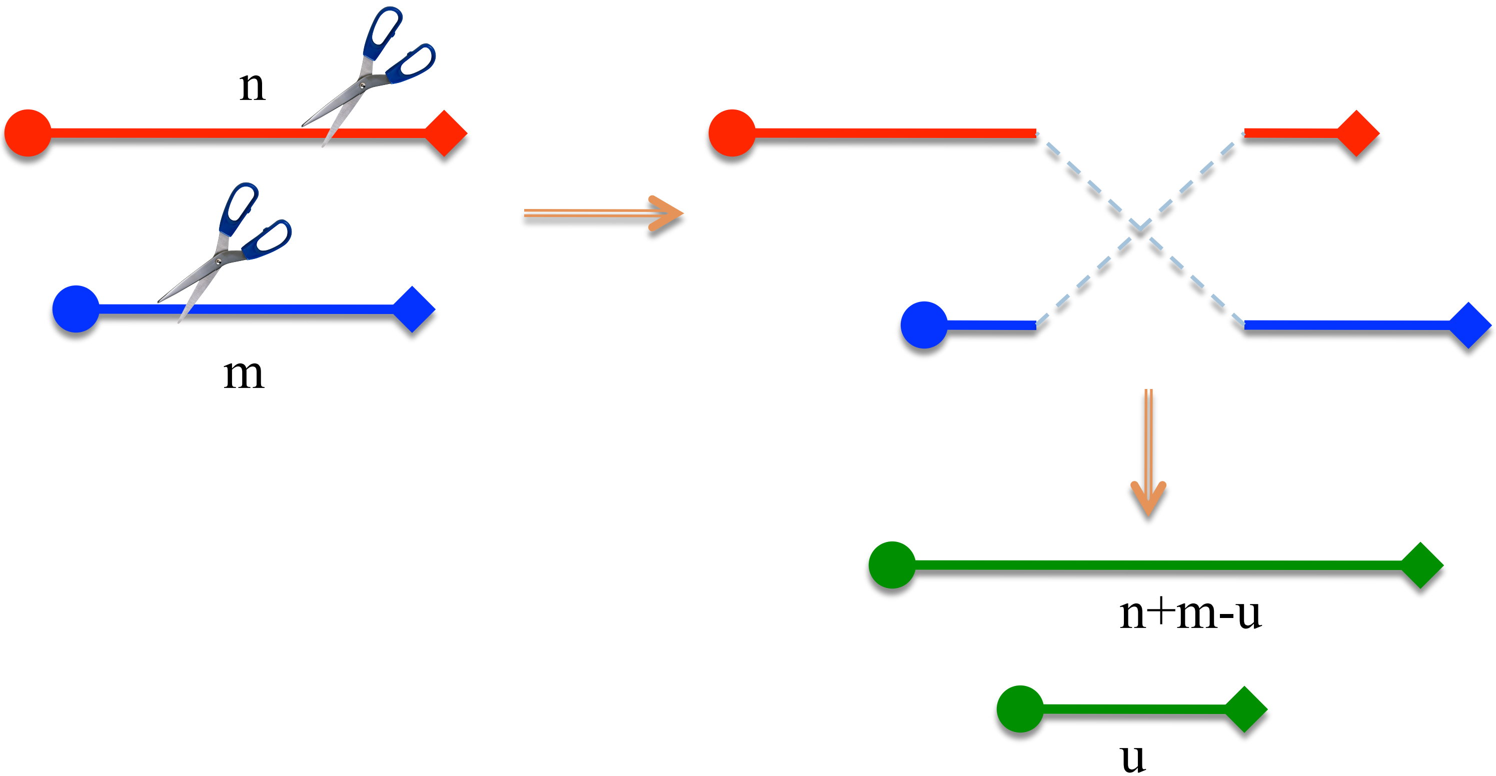
- Joining $\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}nm\psi_{n+m}\frac{\partial^2}{\partial\psi_n\partial\phi_m}$



Example : H_6

- Interaction between two open loops

$$\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\sum_{u=0}^{n+m}\frac{1}{2}F\left(n,m,u\right)\psi_{n+m-u}\psi_{u}\frac{\partial^{2}}{\partial\psi_{n}\partial\psi_{m}}$$



Extra Dimension : AdS x S_1

- The construction implies an extra dimension in Addition to AdS3:Matrix Models $\phi_n(t) = tr(M^n(t))$

Fourier transformation

$$\phi(x,t) = \int \frac{dk}{2\pi} e^{-inx} \phi_n(t)$$

x : extra dimension

- Minimal model Holography might require extension of Vasilievs AdS3 HS theory [conjectured by Yin , Ginzburg Talk 2012]

DERIVATIVES and DESCENDANTS

- Descendants
  - Dynamical generators : W^s_{-k} $\quad k \geqq s$
    - Corresponds to higher spin fields $\longrightarrow Z_{\mathrm{hs}}$
    - e.g. Virasoro algebra (s=2)

$$(L_{-n}\psi) = \frac{1}{(n-2)!}\left(\partial^{n-2}T\psi\right)$$

  - Kinematical(Wedge) generators : W^s_{-k} $\quad k < s$
    - e.g. Virasoro algebra (s=2)

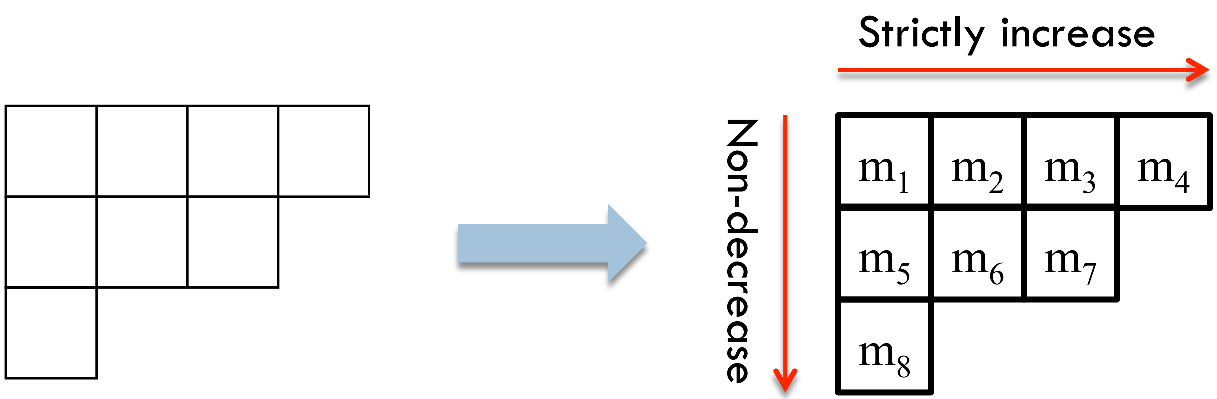
$$(L_{-1}\psi) = \partial\psi$$

    - will dress the single traces
- Full action needs Young operator

Double Young Operators

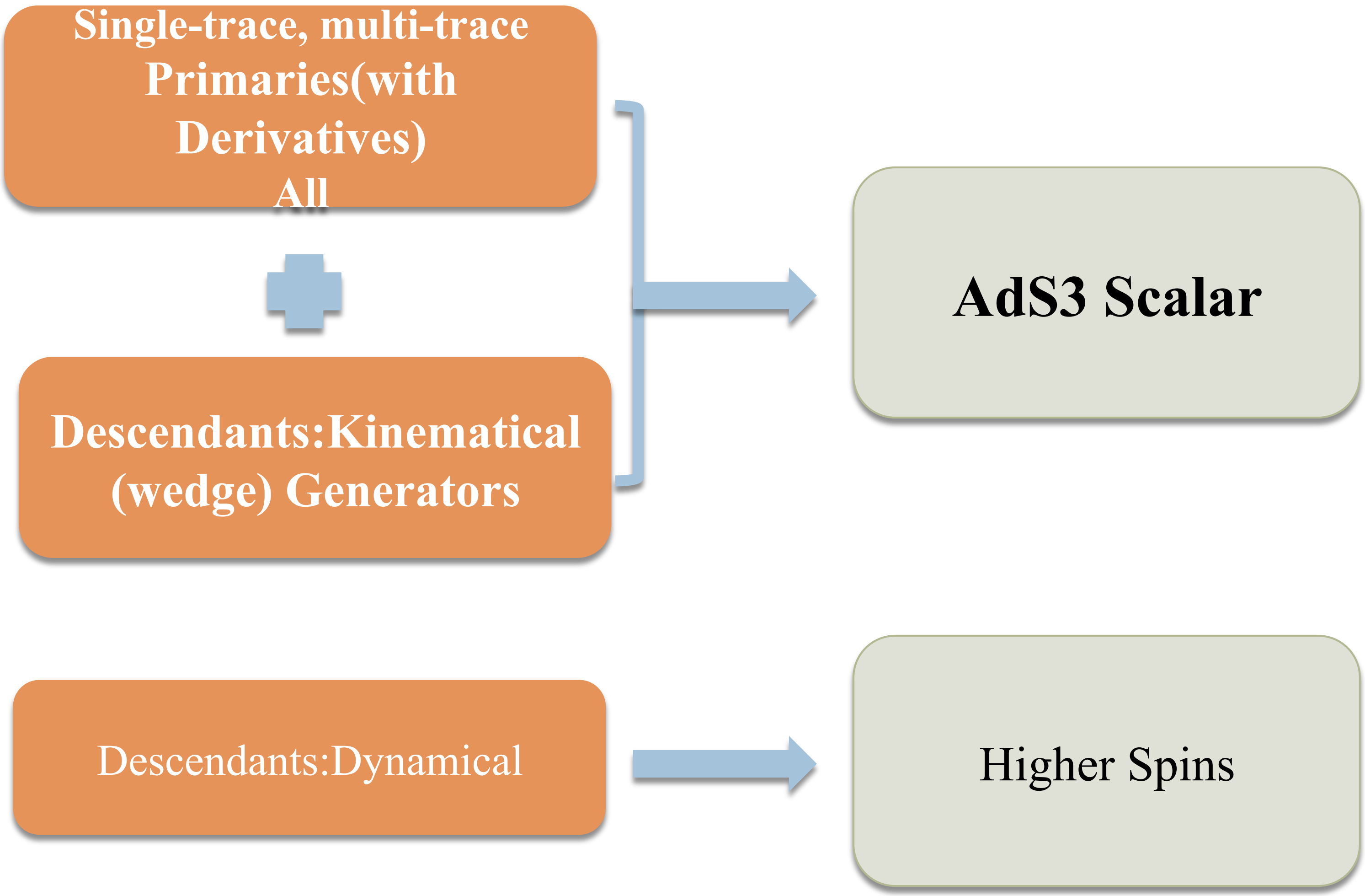
- Can generate derivative primaries and kinematics descendants

- Need two Young tableaux filled with non-negative integers.
  - From $(R;0)$, one can fill Young tableau R with non-negative integers. $\rightarrow$ Two Young tableaux R_1, R_2
  - Non-decreasing in a column, strictly increasing in a row.



R $\qquad$ R_1, R_2

Complete Hilbert Space



Hilbert Space of Scalar Field in AdS_3

- Action $S = \int dx^3 \sqrt{|g|} \frac{1}{2} \left(-g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - m^2 \Phi^2 \right)$
- In global coordinates,

$$\Phi = e^{-i\omega t + il\varphi} \frac{(\tanh r)^{|l|}}{(\cosh r)^{2h}} \, {}_2F_1 \left(h + \frac{1}{2} \left(|l| + \omega \right), h + \frac{1}{2} \left(|l| - \omega \right), |l| + 1; \tanh^2 r \right)$$

where $h = \frac{1}{2} (1 \pm \lambda)$

- Hamiltonian

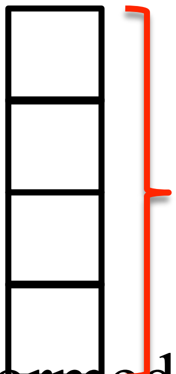
$$H = \sum_{l=-\infty}^{\infty} \sum_{n=0}^{\infty} \frac{\omega_{l,n}}{2} \left(a_{l,n} a^\dagger_{l,n} + a^\dagger_{l,n} a_{l,n} \right)$$

- Frequencies

$$\omega_{l,n} = 2h + |l| + 2n \quad (n = 0, 1, 2, \cdots)$$

Finite N:Exclusion

- CFT construction introduces a cut-off in $N \sim \frac{1}{G}$



$$N \quad : \quad \frac{1}{\sqrt{N!}}\psi_0^N \longrightarrow a^N = 0$$

- q-deformed oscillators [Jevicki & Ramgoolam, 1999]

$$a_q a_q^\dagger - q^{-1} a_q^\dagger a_q = q^{\hat{n}} \qquad\qquad a_q^N = 0$$

- Question:How do we see integer quantization of G in Vasiliev's HS Gravity?

Conclusion and Future

- Using Characters of CFT as variables we develop a Field Theory (for Primaries):It represents an Ultra-local description of Higher Spin Gravity

W_N Group: Primaries/ Descendants (give a full description of AdS space-time theory.

Semi-classical Limit (Defects, Black Holes)

Wigner/Einstein :Gravity Emerges from Systems with Large Symmetry groups through Quantum Effects