Entanglement and Codes

C. Eltschka, O. Gühne, M. Grassl, F. Huber, J. Siewert

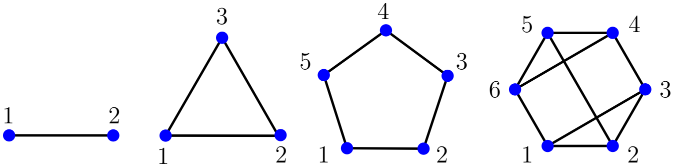
ICFO Barcelona

Part I: Absolutely maximally entangled states

Part II: Entanglement and codes

Part III: Highly entangled subspaces / QMDS codes

Part I: Absolutely maximally entangled states



How entangled can two couples get?

A. Higuchi, A. Sudbery *

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A. Higuchi and A. Sudbery, Phys. Lett. A **273, 213 (2000)**

Absolutely maximally entangled states

Definition (AME states)

A pure state $|\phi_{n,D}\rangle$ is called *absolutely maximally entangled* (AME), if it shows maximal entanglement over all bipartitions.

($\equiv$ all its reductions to $\lfloor \frac{n}{2} \rfloor$ parties are maximally mixed)

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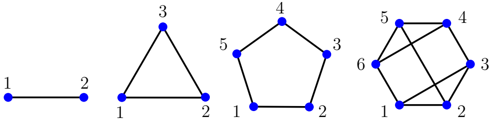
Example

For all prime dimensions (graph states):



$\longrightarrow$ arbitrary dimensions: prime-decomposition $D = p_1 p_2 \dots p_r$:

$$|\phi_{n,D}\rangle = |\phi_{n,p_1}\rangle \otimes |\phi_{n,p_2}\rangle \otimes \dots \otimes |\phi_{n,p_r}\rangle$$

Bounds on AME state existence

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A. Scott, Phys. Lett. A **69**, 052330 (2004)

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- Open problem: provide tight bounds for the existence of AME states (and quantum codes).

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"Bloch-style" proof by contradiction:

Assume a 4-qubit AME state $\varrho_{ABCD} = |\phi\rangle\langle\phi|$ exists.

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b) Decompose

$$\varrho_{ABC} = \frac{1}{2^3}\Big(\mathbb{1} + \underbrace{\sum_{\alpha,\beta,\gamma\in\{x,y,z\}} c_{\alpha\beta\gamma}\,\sigma_\alpha \otimes \sigma_\beta \otimes \sigma_\gamma}_{P_3 \ (\neq 0)}\Big)$$

Note that there are no terms of e.g. the form

$$\sum_{\alpha,\beta\in\{x,y,z\}} c_{\alpha\beta}\,\sigma_\alpha \otimes \sigma_\beta \otimes \sigma_0 \quad !$$

$\nexists$ Four-qubit AME, continued (I)

c) From projector relation $\varrho_{ABC}^2 = \frac{1}{2}\varrho_{ABC}$,

$$(P_3)^2 = \frac{1}{2}\{P_3, P_3\} \stackrel{!}{=} 3\,\mathbb{1} + 2P_3$$

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d) Above, in $\{P_3, P_3\}$ two different Paulis either

$$\sigma_j \sigma_k = i\,\epsilon_{jkl}\sigma_l, \quad j \neq k$$
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e) To contribute to P_3 on the RHS, three pairs of Paulis need to produce three other Paulis. Factor of $\boldsymbol{i}^3$ appears, and term vanishes in anticommutator. Thus $P_3 = 0$. Contradiction!

$\Longrightarrow \nexists$ four-qubit AME state. $\square$

$\nexists$ Seven-qubit AME

Proof by contradiction:

Assume a 7-qubit AME state $\varrho = |\phi\rangle\langle\phi|$ exists.

(a) We use the Bloch decomposition and sort the correlations:

$$\varrho \sim \sum_{\alpha_1 \dots \alpha_n} r_{\alpha_1,\dots,\alpha_n} \sigma_{\alpha_1} \otimes \dots \otimes \sigma_{\alpha_N} \sim \left(\mathbb{1}^{\otimes n} + \sum_{j=4}^{7} P_j \right).$$

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(b) General (qubit) parity rule for $\{P_j, P_k\}$:

$$\begin{array}{rcl} \{\text{even}, \text{even}\} & \longrightarrow & \text{even} \\ \{\text{odd}, \text{odd}\} & \longrightarrow & \text{even} \\ \{\text{odd}, \text{even}\} & \longrightarrow & \text{odd}\,. \end{array}$$

$\nexists$ Seven-qubit AME, continued (I)

(c) The four- and five-qubit reductions fulfill "projector relations"

$$\varrho_{(4)}^2 = \frac{1}{8}\varrho_{(4)} \qquad\qquad \varrho_{(5)}^2 = \frac{1}{4}\varrho_{(5)} \,.$$

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$$\varrho_{(4)} \otimes \mathbb{1}^{\otimes 3} \, |\phi\rangle = \frac{1}{8} \, |\phi\rangle \qquad \varrho_{(5)} \otimes \mathbb{1}^{\otimes 2} \, |\phi\rangle = \frac{1}{4} \, |\phi\rangle \, .$$

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(d) Expand $\varrho_{(4)}$ and $\varrho_{(5)}$ in the Bloch basis

$$\varrho_{(4)} = \frac{1}{2^4}(\mathbb{1} + P_4)\,, \qquad\qquad \varrho_{(5)} = \frac{1}{2^5}(\mathbb{1} + \sum_{j=1}^{5} P_4^{[j]} \otimes \mathbb{1}^{(j)} + P_5)\,.$$

$\nexists$ Seven-qubit AME, continued (II)

(e) Resulting eigenvalue equations:

$$P_4^{[j]} \otimes \mathbb{1}^{\otimes 3} \, |\phi\rangle = 1 \, |\phi\rangle \, , \qquad P_5 \otimes \mathbb{1}^{\otimes 2} \, |\phi\rangle = 2 \, |\phi\rangle \ .$$

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$$\{\sum_{j=1}^{5} P_4^{[j]} \otimes \mathbb{1}^{(j)}, P_5\} = 6P_5 \qquad \text{(odd part)}$$

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(g) Multiplying with $|\phi\rangle$ from the right:

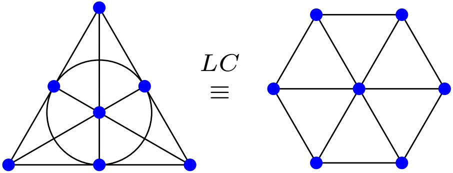
$$(5 \cdot 1 \cdot 2 + 2 \cdot 5 \cdot 1) \, |\phi\rangle \ \neq 6 \cdot 2 \, |\phi\rangle \ .$$

$\Longrightarrow$ $\nexists$ seven-qubit AME.
(similar contradiction found for all $n \neq 2, 3, 5, 6$.) $\square$

A best approximation...

Result

A seven qubit AME does not exist. At most 32 out of 35 three-body RDMs can be maximally mixed.



FH, O. Gühne, J. Siewert, Phys. Rev. Lett. **118**, 200502 (2017)

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- Consider correlation constraints from generalized state inversion / shadow inequality (talk by Jens)

$$\mathrm{Tr}(\mathcal{I}_T[\varrho]\varrho) = \sum_{S \subseteq \{1 \dots n\}} (-1)^{|S \cap T|} \, \mathrm{tr}[\varrho_S^2] \, \geq \, 0 \, .$$

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Example ($\nexists$ four-qubit AME)

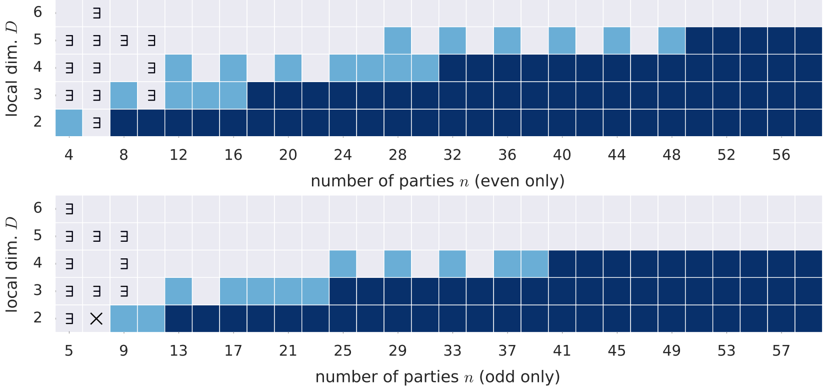
$$\mathrm{Tr}(\mathcal{I}_{1234}[\varrho]\varrho) = 1 - \sum_i \mathrm{tr}(\rho_i^2) + \sum_{i<j} \mathrm{tr}(\varrho_{ij}^2) - \sum_{i<j<k} \mathrm{tr}(\rho_{ijk}^2) + \mathrm{tr}(\varrho^2)$$

$$= 1 - 4\frac{1}{2} + 6\frac{1}{4} - 4\frac{1}{2} + 1 \;=\; -\frac{1}{2} \not\geq 0$$

Bounds on higher-dimensional AME states

Further bounds

A further 27 higher-dimensional AME states $\nexists$ (light blue).



$\exists$: state exists

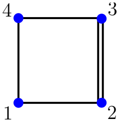
dark blue: excluded by Scott's bound

light blue: excluded by the shadow inequality

Mixed-dimensional AME states

Consider maximally entangled systems of mixed dimensions (e.g. qubit-qutrit), with maximal entanglement across every bipartition:

2x2x2x2: $\nexists$ four-qubit AME (proof at the beginning)
2x2x2x3: $\nexists$ shadow inequality
2x2x3x3: $\nexists$ shadow inequality
2x3x3x3: $\exists$ see new state below
3x3x3x3: $\exists$ four-qutrit AME (c.f. Karol's talk)



$$
\begin{aligned}
|\phi_{2333}\rangle = & -\alpha\,|0011\rangle - \beta\,|0012\rangle + \beta\,|0021\rangle + \alpha\,|0022\rangle \\
& -\beta\,|0101\rangle + \alpha\,|0102\rangle + \beta\,|0110\rangle + \alpha\,|0120\rangle \\
& -\alpha\,|0201\rangle + \beta\,|0202\rangle - \alpha\,|0210\rangle - \beta\,|0220\rangle \\
& -\beta\,|1011\rangle + \alpha\,|1012\rangle - \alpha\,|1021\rangle + \beta\,|1022\rangle \\
& +\alpha\,|1101\rangle + \beta\,|1102\rangle - \alpha\,|1110\rangle + \beta\,|1120\rangle \\
& -\beta\,|1201\rangle - \alpha\,|1202\rangle - \beta\,|1210\rangle + \alpha\,|1220\rangle \qquad . \\
& 12(\alpha^2+\beta^2) = 1\,, 54\alpha\beta = 1\,.
\end{aligned}
$$

FH, C. Eltschka, J. Siewert, O. Gühne, J. Phys. A: Math. Theor. **51**, 175301 (2018)

Part II: Entanglement and Quantum Codes

dimension of code space

$$((n, K, d))_D \longrightarrow \text{local dimension}$$

number of parties distance

Quantum codes

A quantum code is a *subspace* of a multipartite system:
Denote by $\mathcal{Q}$ a subspace of $(\mathbb{C}^d)^{\otimes n}$ spanned by an ONB $\{|v_i\rangle\}$. Let $\Pi = \sum_i^K |v_i\rangle\langle v_i|$ be the projector onto it, with $\mathrm{rank}(\Pi) = K$.

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Theorem (Knill-Laflamme error-conditions)

The subspace $\mathcal{Q}$ is a QECC of distance at least d, if and only if for all operators with $|\operatorname{supp}(E)| < d$,

$$\langle v_i| E |v_j\rangle = \delta_{ij} C_E \qquad (= \delta_{ij} \operatorname{tr}[E] \ \text{“pure”})$$

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- If distance is d, then errors on $\lfloor \frac{(d-1)}{2} \rfloor$ particles can be corrected.
- $\mathcal{Q}$ is denoted as a $(\!(n, K, d)\!)_D$ code.

E. Knill, R. Laflamme, and L. Viola, Phys. Rev. Let. **84**, 2525 (2000).

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a) For all $|\phi\rangle \in \mathcal{Q}$, and all subsets $|S| < d$

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b) Let $\varrho = \Pi/K$. For all subsets $|S| < d$,

$$K\,\mathrm{tr}[\varrho_{S^c}^2] = \mathrm{tr}[\varrho_S^2] \quad (= 1/D^{|S|})$$

$\longrightarrow$ "constraints on purities of complementary reductions"

E. Rains, IEEE Trans. Inf. Theory **44**, 4 (1998)

Part III: QMDS codes & highly entangled subspaces

$$\begin{bmatrix} 1 & 0 & 0 & 1 & \omega & \omega \\ 0 & 1 & 0 & \omega & 1 & \omega \\ 0 & 0 & 1 & \omega & \omega & 1 \\ \omega & 0 & 0 & \omega & \bar{\omega} & \bar{\omega} \\ 0 & \omega & 0 & \bar{\omega} & \omega & \bar{\omega} \\ 0 & 0 & \omega & \bar{\omega} & \bar{\omega} & \omega \end{bmatrix}$$

$$\begin{bmatrix} X & I & I & X & Y & Y \\ I & X & I & Y & X & Y \\ I & I & X & Y & Y & X \\ Y & I & I & Y & Z & Z \\ I & Y & I & Z & Y & Z \\ I & I & Y & Z & Z & Y \end{bmatrix}$$

Highly entangled subspaces

Definition

A pure state $|\phi\rangle$, whose reductions onto r parties are all maximally mixed, is termed *r-uniform*. A *r-uniform subspace* (rUS) is a subspace of $(\mathbb{C}^D)^{\otimes n}$, in which every vector is at least r-uniform.

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Observation (pure QECC $\equiv$ r-uniform subspace)

The following objects are equivalent:

a) *a pure $((n, K, d))_D$ quantum error correcting code.*

b) *a $(d-1)$-uniform subspace in $(\mathbb{C}^D)^{\otimes n}$ of dimension K.*

Bounds on codes

Theorem (Quantum Singleton bound)

Let $\mathcal{Q}$ be a $((n, K, d))_D$ quantum error correcting code. Then

$$n + 2 \geq \log_D K + 2d$$

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- If equality above, the code is called *quantum maximum distance separable* (QMDS)
- Fact: QMDS codes are pure (have maximally mixed marginals).

$\longrightarrow$ QMDS codes are the largest possible r-uniform subspaces.

New codes from old

New codes can be constructed from old ones:

Theorem

Let $((n, K, d))_D$ *be a pure QECC with* $n, d \geq 2$. *Then there exists a pure code* $((n-1, DK, d-1))_D$.

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$\longrightarrow$ corresponds to taking a partial trace over one particle.

QMDS families

...apply to QMDS codes:

Example

$$\begin{array}{ll} ((6, 2^0, 4))_2 \ \exists \\ ((5, 2^1, 3))_2 \ \exists \\ ((4, 2^2, 2))_2 \ \exists \\ \downarrow \ ((3, 2^3, 1))_2 \ \exists \end{array} \qquad \begin{array}{ll} ((12, 3^0, 7))_3 & \nexists \\ ((11, 3^1, 6))_3 & \nexists \\ ((10, 3^2, 5))_3 & \nexists \\ ((9, 3^3, 4))_3 & \nexists \\ ((8, 3^4, 3))_3 & \exists \\ ((7, 3^5, 2))_3 & \exists \\ \downarrow \ ((6, 3^6, 1))_3 & \exists \end{array}$$

- Family of codes / highly entangled subspaces determined by $n + k$.
- For a given family, if the parent-AME does not exist, what is the uppermost member?

Bound on the existence of QMDS codes

Maximal length of QMDS codes

A $((n, K, d))_D$ QMDS code of distance $d \geq 3$
[$\equiv$ $(d-1)$-uniform subspace in $(\mathbb{C}^D)^{\otimes n}$ of dimension K]
must satisfy

$$n \leq D^2 + d - 2\,, \qquad \text{or equivalently}$$

$$n + k \leq 2(D^2 - 1)\,.$$

FH and M. Grassl, in preparation.

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- Extends Scott's AME bound and stabilizer QMDS bounds to *all* QMDS codes.
- Further bounds from the shadow inequality / generalized inversion $\mathrm{tr}(\mathcal{I}_T[\varrho]\varrho) \geq 0$.

Examples

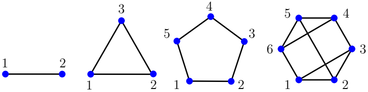
Example

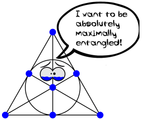
All QMDS-families of local dimension $D = 3$:

$\mathbf{n+k}$	**bound**	**achieved**	
4	$[\![4,0,3]\!]_3$	$[\![4,0,3]\!]_3$	(optimal)
6	$[\![6,0,4]\!]_3$	$[\![6,0,4]\!]_3$	(optimal)
8	$[\![6,2,3]\!]_3$	$[\![6,2,3]\!]_3$	(optimal)
10	$[\![10,0,6]\!]_3$	$[\![10,0,6]\!]_3$	(optimal)
12	$[\![8,4,3]\!]_3$	$[\![8,4,3]\!]_3$	(optimal)
14	$[\![11,3,5]\!]_3$	$[\![10,4,4]\!]_3$	
16	$[\![11,5,4]\!]_3$	$[\![10,6,3]\!]_3$	

Summary of Results

- Arbitrarily strong quantum correlations are not allowed. Qubit AME states only exist for $n = 2, 3, 5, 6$.





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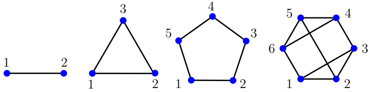
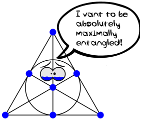
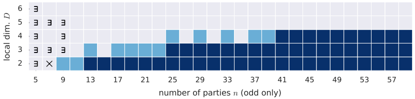




FH, O. Gühne, J. Siewert, Phys. Rev. Lett. **118**, 200502 (2017)

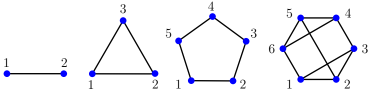
- Constraints on higher-dimensional / mixed-dimensional AME states.

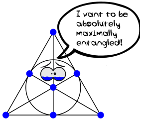


FH, C. Eltschka, J. Siewert, O. Gühne, JPA: Math. Theor. **51**, 175301 (2018)

Summary of Results

- Arbitrarily strong quantum correlations are not allowed. Qubit AME states only exist for $n = 2, 3, 5, 6$.





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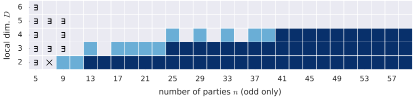


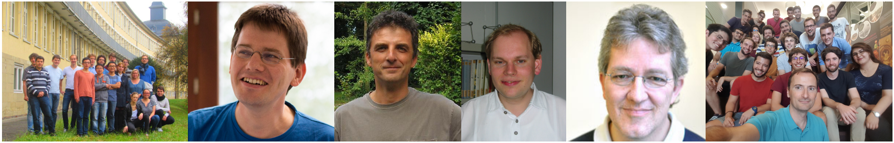
FH, C. Eltschka, J. Siewert, O. Gühne, JPA: Math. Theor. **51**, 175301 (2018)

- Bounds for the existence QMDS codes / highly entangled subspaces

FH and M. Grassl, in preparation.

Thank you for your attention . . .

. . . and thanks to my collaborators!



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