SU(d) spin squeezing and many-body entanglement detection with uncertainty relations

G. Vitagliano[1,2]

collaboration with:

P. Hyllus[2], I. Apellaniz[2], I.L. Egusquiza[2], G. Toth[2,3,4]

[1] Institute for Quantum Optics and Quantum Information (IQOQI), Boltzmanngasse 3, 1090 Vienna, Austria
[2] Department of Theoretical Physics, University of the Basque Country UPV/EHU, P.O. Box 644, E-48080 Bilbao, Spain
[3] IKERBASQUE, Basque Foundation for Science, E-48011 Bilbao, Spain
[4] Wigner Research Centre for Physics, Hungarian Academy of Sciences, P.O. Box 49, H-1525 Budapest, Hungary

Entanglement days

Budapest, 26th September 2018

Entanglement criteria from uncertainty relations

- An N-partite separable state is

$$\rho = \sum_i p_i \rho_i^{(1)} \otimes \cdots \otimes \rho_i^{(N)} \quad p_i > 0 \; \sum_i p_i = 1$$

A non-separable state is *entangled*

- Taking $J_x = \sum_{n=1}^{N} j_x^{(n)}$, $J_y = \sum_{n=1}^{N} j_y^{(n)}$ with

$$(\Delta j_x^{(n)})^2 + (\Delta j_y^{(n)})^2 \geq C_j$$

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- We obtain that

$$(\Delta J_x)^2 + (\Delta J_y)^2 < N C_j \quad \Rightarrow \text{entanglement}$$

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i.e., $(\Delta J_x)^2 + (\Delta J_y)^2 \geq N C_j$ a necessary condition for separability
Proof. concavity + $\rho = \bigotimes_n \rho^{(n)} \ \Rightarrow \ (\Delta J_k)^2 = \sum_n (\Delta j_k^{(n)})^2$

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- We obtain that

$$(\Delta J_x)^2 + (\Delta J_y)^2 < NC_j \quad \Rightarrow \text{entanglement}$$

[H. F. Hofmann and S. Takeuchi, PRA, **68** 032103, (2003); Gühne et al. PRL 99 130504 (2007); Gittsovich et al. PRA 78, 052319 (2008);PRA, 82 032306 (2010); PRA 81, 032333 (2010)] see also [L. Dammeier, R. Schwonnek, R. F. Werner, New J. Phys. **17**, 093046 (2015)]

Entanglement of spin squeezed states

From $(\Delta J_x)^2(\Delta J_y)^2 \geq \frac{1}{4}|\langle J_z\rangle|^2$ we define a spin-coherent state as

$$(\Delta J_x)^2 = (\Delta J_y)^2 = \frac{1}{2}|\langle J_z\rangle| = \frac{N}{4}$$

and *spin-squeezed states* as

$$|\langle J_z\rangle| \simeq \frac{N}{2}\ ; \qquad (\Delta J_x)^2 < \frac{N}{4}$$

$$\xi_{\rm s}^2 = \frac{N(\Delta J_x)^2}{\langle J_y\rangle^2 + \langle J_z\rangle^2} < 1 \quad \Rightarrow \text{entanglement}$$

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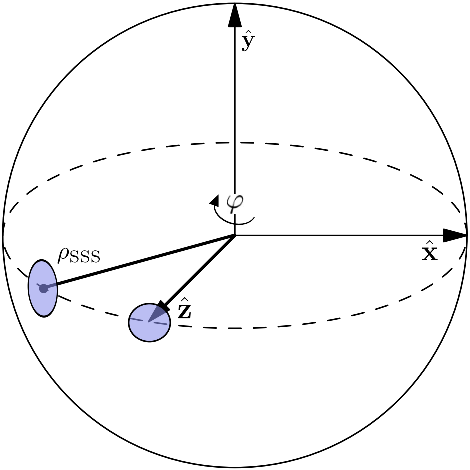
The proof is totally analogous to the previous method



They are also very useful for metrology

[A. Sørensen, L.M. Duan, J.I. Cirac, and P. Zoller, Nature **409**, 63 (2001);M. Kitagawa and M. Ueda, Phys. Rev. A **47**, 5138 (1993); D.J. Wineland, J. J. Bollinger, and W. M. Itano, Phys. Rev. A **50**, 67 (1994).]

Generalized spin squeezing

$$\langle J_x^2\rangle + \langle J_y^2\rangle + \langle J_z^2\rangle \leq \frac{N(N+2)}{4}$$
$$(\Delta J_x)^2 + (\Delta J_y)^2 + (\Delta J_z)^2 \geq \frac{N}{2}$$
$$(N-1)\left[(\Delta J_x)^2 + (\Delta J_y)^2\right] - \langle J_z^2\rangle \geq \frac{N(N-2)}{4}$$
$$(N-1)\left[(\Delta J_x)^2\right] - \langle J_y^2\rangle - \langle J_z^2\rangle \geq -\frac{N}{2}$$

Violation of one of them implies entanglement.

[G. Tóth, C. Knapp, O. Gühne and H.J. Briegel, PRL **99**, 250405 (2007); PRA **79** 042334 (2009)]

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Detects Macroscopic singlet states.

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Detects Planar squeezed states.

[G. Tóth, C. Knapp, O. Gühne and H.J. Briegel, PRL **99**, 250405 (2007); PRA **79** 042334 (2009)]

Generalized spin squeezing

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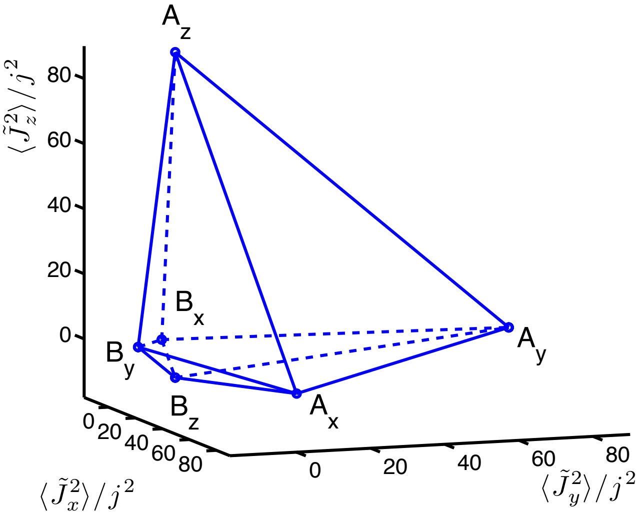
Detects Dicke states and spin squeezed states.

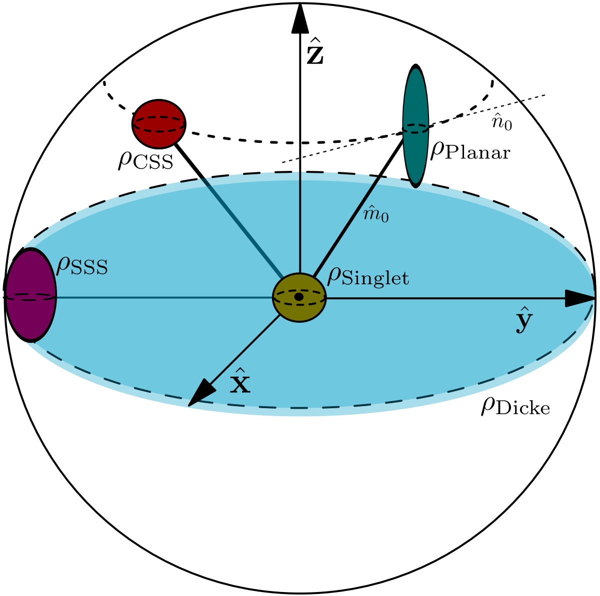
[G. Tóth, C. Knapp, O. Gühne and H.J. Briegel, PRL **99**, 250405 (2007); PRA **79** 042334 (2009)]

It is a complete set of criteria with $(\Delta J_k)^2$ and $\langle J_k^2 \rangle$



the polytope is filled by separable states in the limit $N \gg j$

Generalized Spin Squeezing: summary



Spin Squeezing for $j > \frac{1}{2}$ particles

$$\xi_s^2 = \frac{N(\Delta J_x)^2}{\langle J_y\rangle^2 + \langle J_z\rangle^2} \geq 1$$

is **not** a separability condition for $j > \frac{1}{2}$.

- However, defining modified second moments

$$\langle \tilde{J}_k^2\rangle = \langle J_k^2\rangle - \sum_n \langle (j_k^{(n)})^2\rangle$$

$$(\tilde{\Delta} J_k)^2 = \langle \tilde{J}_k^2\rangle - \langle J_k\rangle^2$$

- Normalized as

$$\langle J_l\rangle \to \frac{1}{2j}\langle J_l\rangle \qquad \langle \tilde{J}_l^2\rangle \to \frac{1}{4j^2}\langle \tilde{J}_l^2\rangle$$

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$$\xi_s^2 = \frac{N(\Delta J_x)^2}{\langle J_y \rangle^2 + \langle J_z \rangle^2} = N \frac{(\tilde{\Delta} J_x)^2 + \frac{N}{4}}{\langle J_y \rangle^2 + \langle J_z \rangle^2} \geq 1$$

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$$\xi_s^2 = \frac{N(\Delta J_x)^2}{\langle J_y \rangle^2 + \langle J_z \rangle^2} = N\frac{(\tilde{\Delta} J_x)^2 + Nj^2}{\langle J_y \rangle^2 + \langle J_z \rangle^2} \geq 1$$

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  it becomes a valid condition for spin-j systems

Spin Squeezing for $j > \frac{1}{2}$ particles

The complete set of SSIs

$$\langle J_x^2 \rangle + \langle J_y^2 \rangle + \langle J_z^2 \rangle \le \frac{N(N+2)}{4}$$

$$(\Delta J_x)^2 + (\Delta J_y)^2 + (\Delta J_z)^2 \ge \frac{N}{2}$$

$$(N-1)\left[(\Delta J_x)^2 + (\Delta J_y)^2\right] - \langle J_z^2 \rangle \ge \frac{N(N-2)}{4}$$

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Spin Squeezing for $j > \frac{1}{2}$ particles

The complete set of SSIs becomes, for $j > \frac{1}{2}$

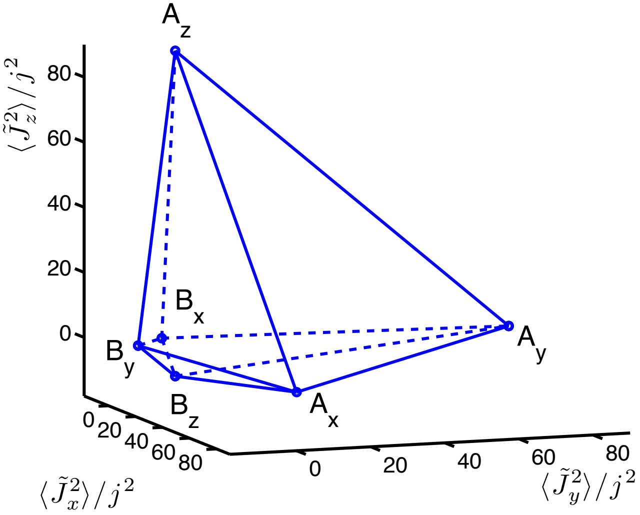
$$\langle J_x^2 \rangle + \langle J_y^2 \rangle + \langle J_z^2 \rangle \leq Nj(Nj+1)$$

$$(\Delta J_x)^2 + (\Delta J_y)^2 + (\Delta J_z)^2 \geq Nj$$

$$(N-1)\left[(\Delta J_k)^2 + (\Delta J_l)^2\right] \geq \langle J_m^2 \rangle - N\sum_n \langle (j_m^{(n)})^2 \rangle + N(N-1)j$$

$$\langle J_k^2 \rangle + \langle J_l^2 \rangle - Nj(Nj+1) \leq N(\Delta J_m)^2 - N\sum_n \langle (j_m^{(n)})^2 \rangle$$

It is a complete set of criteria with $(\tilde{\Delta} J_k)^2$ and $\langle \tilde{J}_k^2 \rangle$



the polytope is filled by separable states in the limit $N \gg j$

Extremal points of the polytope

States corresponding to the extremal points of the polytope are of two types

$$\rho_{Y_k} = p\rho_{+,k}^{\otimes N} + (1-p)\rho_{-,k}^{\otimes N}$$
$$\rho_{X_k} = \rho_{+,k}^{\otimes N_+} \otimes \rho_{-,k}^{\otimes N-N_+},$$

where $\rho_{\pm,k}$ are single-particle states such that $\langle j_k \rangle = \pm j$, $0 \leq p \leq 1$ and $N_+ = Np$ must be integer

Such states can be found corresponding to (neglecting differences of order j/N) all vertices of the polytope

(they can be seen as a generalization of spin-coherent states)

A compact form for the complete set

- Let us define the following correlation matrices

$$
\begin{aligned}
C_{kl} &:= \frac{1}{2}\langle J_k J_l + J_l J_k \rangle \\
\Gamma_{kl} &:= C_{kl} - \langle J_k \rangle \langle J_l \rangle \\
Q_{kl} &:= \frac{1}{N}\sum_n \left(\frac{1}{2}\langle j_k^{(n)} j_l^{(n)} + j_l^{(n)} j_k^{(n)} \rangle \right) \\
\mathfrak{X} &:= \Gamma + \tfrac{1}{N-1} C - \tfrac{N^2}{N-1} Q
\end{aligned}
$$

- The complete set becomes

$$
\mathrm{Tr}\,(\Gamma) - \sum_{k=1}^{I} \lambda_k^{\mathrm{pos}}(\mathfrak{X}) - Nj \geq 0 \qquad (1)
$$

- Eq. (1) follows *just from* $(\Delta j_x^{(n)})^2 + (\Delta j_y^{(n)})^2 + (\Delta j_z^{(n)})^2 \geq j$
(*Proof. idea:* $\lambda_k^{\mathrm{pos}}(\mathfrak{X}) = 0$ for product states + concavity)

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Further generalization: $SU(d)$-squeezing criteria

- For $d > 2$-dimensional particles we can operators different from spin components
- In particular a full local orthogonal basis $\{g_k\}_{k=0}^{d^2-1}$ is such that

$$\sum_k (\Delta g_k)^2 \geq d - 1$$

- Then, by considering $G_k = \sum_{n=1}^{N} g_k^{(n)}$ we find that

$$\mathrm{Tr}\,(\Gamma) - \sum_{k=1}^{I} \lambda_k^{\mathrm{pos}}(\mathfrak{X}) - N(d-1) \geq 0 \qquad (2)$$

is another set of entanglement criteria
(with similar definitions of Γ, C and $\mathfrak{X}$)

Relation with two-body marginals

- Consider the average two-body marginal density matrix

$$\rho_{\mathrm{av2}} := \tfrac{1}{N(N-1)} \sum_{i \neq j} \rho_{ij}$$

- and the quantities

$$\bar{\mathfrak{X}}_{kl} := \langle g_k \otimes g_l \rangle_{\mathrm{av2}} - \langle g_k \rangle \langle g_l \rangle_{\mathrm{av2}}$$

$$\langle F \rangle_{\mathrm{av2}} = \sum_{k=0}^{d^2-1} \langle g_k \otimes g_k \rangle_{\mathrm{av2}} \quad \text{(i.e., } F \text{ is the flip operator)}$$

- we can express the criteria as

$$\bar{\xi}_{\mathrm{su(d)}}(\rho) := \sum_{k=1}^{I} \lambda_k^{\mathrm{neg}}(\bar{\mathfrak{X}}_\rho) + \tfrac{1}{N}(1 - \langle F \rangle_{\mathrm{av2}}) \geq 0, \tag{3}$$

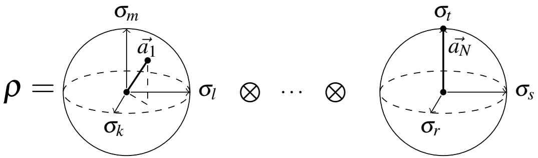
(thereby defining a $su(d)$-squeezing entanglement parameter)

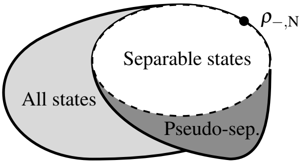
“Pseudo”-completeness of the $SU(d)$ inequalities

We define an N-partite *pseudo-separable* state as

$$\rho = \sum_i p_i \rho_i^{(1)} \otimes \cdots \otimes \rho_i^{(N)} \quad p_i > 0 \; \sum_i p_i = 1$$

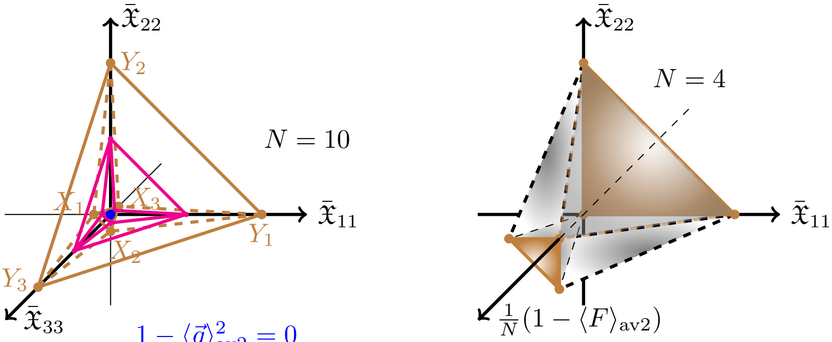
where $\rho_i^{(n)}$ satisfy $\sum_k (\Delta g_k)^2 \geq d-1$ but need not be positive





"Pseudo"-completeness of the $SU(d)$ inequalities

The $SU(d)$ inequalities define a polytope completely filled by pseudo-separable states (in the limit $N \gg d$)



$1 - \langle \vec{g} \rangle^2_{\mathrm{av2}} = 0$

$1 - \langle \vec{g} \rangle^2_{\mathrm{av2}} = 1/2$

$1 - \langle \vec{g} \rangle^2_{\mathrm{av2}} = 1$

$\bar{\xi}_{\mathrm{su(d)}}$ can be seen as a signed distance from the polytope

Extremal points of the $SU(d)$ polytope

States corresponding to the extremal points of the polytope are of two types

$$\rho_{Y_k} = p\rho_{+,k}^{\otimes N} + (1-p)\rho_{-,k}^{\otimes N}$$
$$\rho_{X_k} = \rho_{+,k}^{\otimes N_+} \otimes \rho_{-,k}^{\otimes N-N_+},$$

where

$$\rho_{\pm,k} := \frac{\mathbb{1}}{d} \pm \sqrt{\frac{d-1}{d}}\sigma_k$$

are (not-necessarily positive) single-particle matrices such that $\langle\sigma_k\rangle = \pm\sqrt{\frac{d-1}{d}}$, $0 \leq p \leq 1$ and $N_+ = Np$ must be integer

Such states can be found corresponding to (in the limit $N \gg 1$) all vertices of the polytope

States detected: $SU(d)$ singlet

As a first example consider the $SU(d)$ singlet, that is obtained from products of totally anti-symmetric d-particle states (and is $U^{\otimes N}$ invariant).

Such a state, mixed with white noise that has a two-body marginal

$$\mathrm{Tr}_{N-2}(\rho_{\mathrm{Nsing}}) = \frac{\mathbb{1}\otimes\mathbb{1}}{d^2} - \sum_k \frac{1}{d}\frac{1-p}{N-1}\sigma_k \otimes \sigma_k, \tag{4}$$

where p is the white noise probability

- It is detected with $\bar{\xi}_{\mathrm{su(d)}} < 0$ (e.g., for $p = 0$ with $\bar{\xi}_{\mathrm{su(d)}} = -\frac{d-1}{N}$)
- with noise tolerance

  $$p < 1 - \frac{1}{d+1}$$

  independently of N
- Note also that (4) is separable for $p > (d^2 - N)/(d^2 - 1)$

States detected: $SU(d)$ singlet

The $SU(d)$ singlet is also obtained as the ground state of

$$H = \sum_{n \neq m=1}^{N} \sum_{k=1}^{d^2-1} g_k^{(n)} \otimes g_k^{(m)}$$

Thermal states $\rho_T = \exp(-H/T)/\mathrm{Tr}\,(\exp(-H/T))$ are also detected the are PPT with respecto to all bipartitions

Table: Maximal temperature $T_{\mathrm{su(d)}}$ until which the $su(d)$ singlet is detected with $\xi_{\mathrm{su(d)}}$ vs maximal temperature T_{NPT} for which it is NPT for at least one bipartition.

N	2	3	4	5	6
$T_{\mathrm{su(d)}}$	2.54	1.53	0.86	0.45	0.285
T_{NPT}	5.09	1.92	0.87	0.42	0.21

States detected: $SU(d)$ vs Spin squeezing

We can also compare the spin squeezing parameter with the $SU(d)$ one

- We can consider thermal states of the two Hamiltonians

$$H_{\mathrm{su(d)}} = \sum_{n\neq m=1}^{N} \sum_{k=1}^{d^2-1} g_k^{(n)} \otimes g_k^{(m)}$$

$$H_{\mathrm{spin}} = \sum_{n\neq m=1}^{N} \sum_{k=x,y,z} j_k^{(n)} \otimes j_k^{(m)}$$

Table: Maximal temperatures T_{spin} and $T_{\mathrm{su(d)}}$ until which respectively spin squeezing and $SU(d)$ squeezing parameter detects thermal states of H_{spin} and $H_{\mathrm{su(d)}}$ as entangled.

N	2	3	4	5
$T_{\mathrm{spin}}(H_{\mathrm{spin}})$	1.88	0.88	0.49	0.28
$T_{\mathrm{spin}}(H_{\mathrm{su(d)}})$	0.03	0.60	0.26	0.13
$T_{\mathrm{su(d)}}(H_{\mathrm{spin}})$	1.26	0.26	0.08	0.03
$T_{\mathrm{su(d)}}(H_{\mathrm{su(d)}})$	0.93	0.68	0.31	0.16

Conclusions

Summary

1. We have studied generalizations of spin squeezing entanglement criteria
   - We have shown a method to generalize all spin squeezing criteria from spin-$1/2$ to higher spin particles
   - We have constructed complete sets of criteria for spin operators and for $su(d)$ operators
2. We have studied boundary states of the criteria and states that violate them
   - The criteria with spin operators define a polytope completely filled by separable states and detect thermal states of (especially permutationally invariant) spin models (e.g., spin singlet)
   - The criteria with $su(d)$ operators define a polytope completely filled by "pseudo"-separable states and detect thermal states of (especially permutationally invariant) $su(d)$ models (e.g., $su(d)$ singlet)

References

- GV, P. Hyllus, I.L. Egusquiza, and G. Tóth, PRL **107**, 240502 (2011)
- GV, I. Apellaniz, I.L. Egusquiza, and G. Tóth, PRA **89**, 032307 (2014)
- + in preparation

THANK YOU FOR YOUR ATTENTION!