Efficient classical simulation of lossy boson sampling via separable states

Michał Oszmaniec and Daniel Brod

Entanglement Days, Budapest, 26 September 2018

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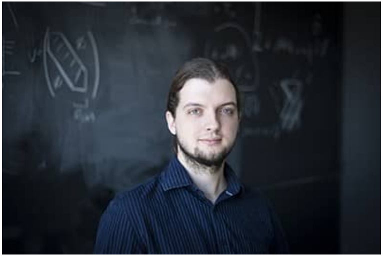
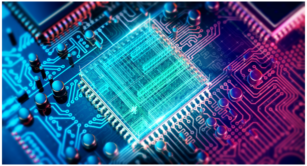


Daniel Brod (UFF Niteroi)

Motivation: Quantum Computational Supremacy



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- Error correction and very clean physical qubits are needed. This results in **gigantic overheads** (> 1000) and a poses great technological challenges.
- An intermediate step: quantum machines of **restricted purpose** that (hopefully) can demonstrate **quantum computational supremacy**[2].
- Possible advantage: **smaller requirements, no error correction needed**.

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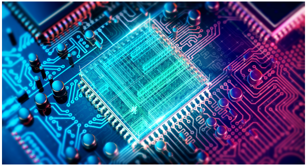

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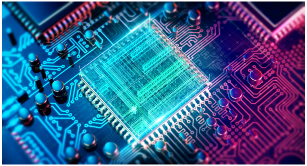

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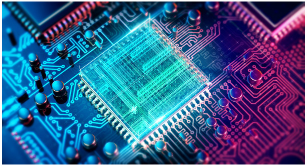

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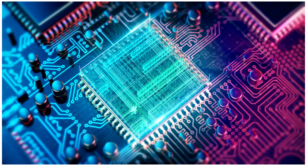

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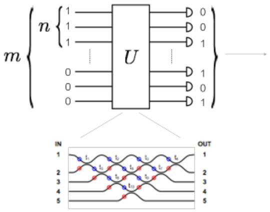

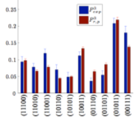

$a_i^\dagger \to \sum_j U_{ji} a_j^\dagger$

- **Task**: sample from the distribution p_U^{BS} for typical $U \in \mathrm{SU}(m)$.
- It is **unlikely** that there exist a classical machine producing a sample from the distribution $\tilde{p}_U$ satisfying

$$\mathrm{TV}(\tilde{p}_U, p_U^{\mathrm{BS}}) \leq \epsilon \text{ in time } T = \mathrm{poly}(n, \frac{1}{\epsilon}) ,$$

where TV - **total variation distance** (∼ distinguishing probability).

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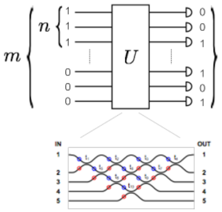

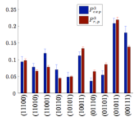

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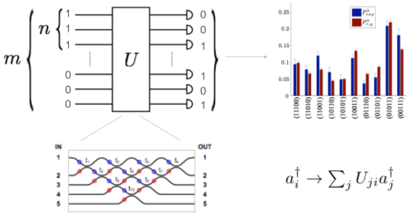

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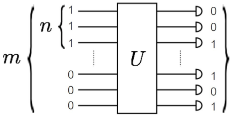

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- A lot of interest due to development of **integrated photonics**.
- State of the art: classical simulation for **up to 50 photons**[4] and *seven* photons[5] in experiments.
- Is Boson-Sampling **scalable**?

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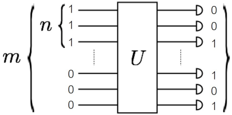

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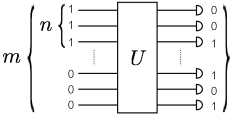

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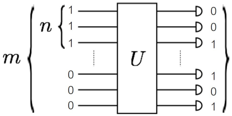

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THIS WORK: EFFICIENT CLASSICAL SIMULATION OF BOSON SAMPLING UNDER PHOTON LOSSES (VIA SEPARABLE STATES)

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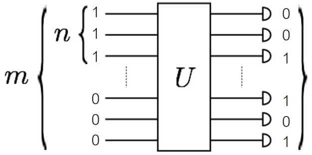
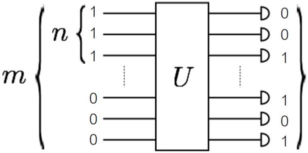
Outline of the talk



- Motivation and introduction to Boson Sampling
- Technical tools and the idea of classical simulation
- Classical simulation of lossy Boson Sampling for:
  - (a) Fixed-loss model
  - (b) Uniform beamsplitter loss model
  - (c) Lossy linear optical network

First vs. Second Quantisation



- Hilbert space $\mathcal{H} = \mathrm{Fock}_b(\mathbb{C}^m) = \bigoplus_{l=0}^{\infty} \mathrm{Sym}^l\left(\mathbb{C}^m\right)$.
- Typically input state ρ has a fixed number of particles n.
- Linear-optical transformation $U \in \mathrm{SU}(m)$ defines mappings

$$a_i^\dagger \mapsto \sum_j U_{ji} a_j^\dagger \quad , \quad \rho \mapsto U^{\otimes n} \rho (U^\dagger)^{\otimes n} \; .$$

- Inclusion $\mathrm{Sym}^n\left(\mathbb{C}^m\right) \subset \left(\mathbb{C}^m\right)^{\otimes n}$ gives rise to **particle entanglement**.
- Particle entanglement is different than **mode entanglement** originating form the decomposition $\mathrm{Fock}_b(\mathbb{C}^m) \approx \bigotimes_{i=1}^{m} \mathrm{Fock}_b(\mathbb{C})$.

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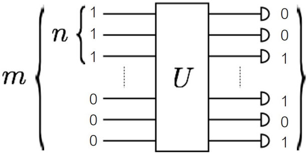

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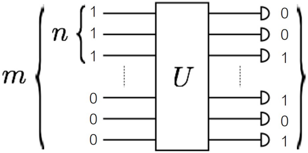

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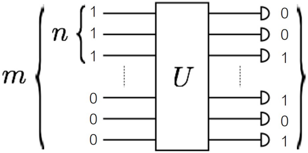

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Particle separable bosonic states

An n particle bosonic state ρ is called **particle separable** ($\rho \in \mathrm{Sep}$) iff

$$\sigma = \sum_{\alpha} p_\alpha |\phi_\alpha\rangle\langle\phi_\alpha|^{\otimes n} \ , \text{ where } \{p_\alpha\} \text{ - prob. dist.}$$

Important features:

- **Easy update** of states $|\phi\rangle^{\otimes n}$ under linear optics (acting like $U^{\otimes n}$)
- The particle-number statistics of the state $(U|\phi\rangle)^{\otimes n}$ is **efficiently classically simulable** for any U and $|\phi\rangle$.
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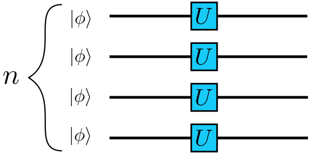

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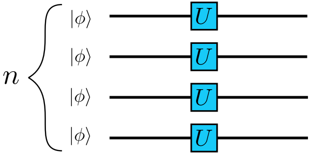

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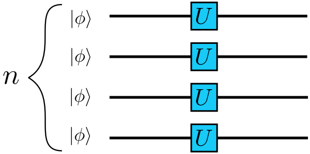

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Models of particle losses

- **Model 1**: A fixed number of particles are lost in mode symmetric-manner

$$\rho \mapsto \rho_l = \mathrm{tr}_{n-l}(\rho)\ .$$

- **Model 2**: Every particle is lost with probability $(1-\eta)$. Equivalently: column of beamsplitters with trasmitivity η.

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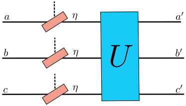

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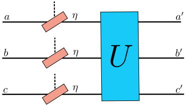

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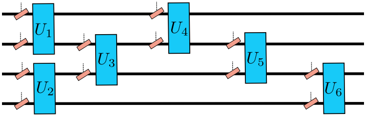

General Simulation Strategy

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$$\rho_{l,n} = \frac{1}{\binom{n}{l}} \sum_{\sum_i x_i = l,\ 0 \le x_i \le 1} |x_1,\ldots,x_n\rangle\langle x_1,\ldots,x_n| \ .$$

- **Main idea**: Approximate $\rho_{l,n}$ by **symmetric separable** states in **trace distance**.

$$\Delta_l = \min_{\sigma \in \mathrm{Sep}} \mathrm{d}_{\mathrm{tr}}(\sigma, \rho_{l,n}) \ .$$

- Finding the optimal σ_* gives the **immediate classical simulation** of Boson Sampling to accuracy Δ_l in TV (a figure of merit for BS),

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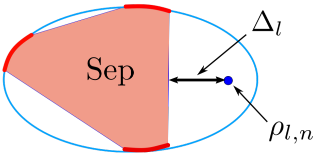

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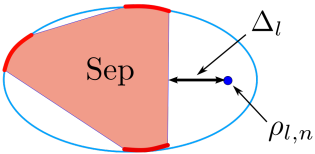

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Classical simulation for $n - l$ particles lost

$$\rho_{l,n} = \frac{1}{\binom{n}{l}} \sum_{\sum_i x_i = l,\ 0 \leq x_i \leq 1} |x_1, \ldots, x_n\rangle\langle x_1, \ldots, x_n|$$

RESULT (Closest seperable state to a lossy Fock state)

Trace distance of $\rho_{l,n}$ to the set of symmetric separable l-particle states is

$$\Delta_l = 1 - \frac{n!}{n^l (n-l)!} \ .$$

Moreover, an optimal separable state σ_* attaining Δ_l can be chosen to be

$$\sigma_* = \frac{1}{(2\pi)^n} \int_0^{2\pi} d\varphi_1 \ldots \int_0^{2\pi} d\varphi_n \left(V_{\varphi_1,\ldots,\varphi_n} |\phi_0\rangle\langle\phi_0| V^\dagger_{\varphi_1,\ldots,\varphi_n} \right)^{\otimes l} \ ,$$

where $|\phi_0\rangle = (1/\sqrt{n}) \sum_{i=1}^n |i\rangle$ and $V_{\varphi_1,\ldots,\varphi_n} = \exp\left(-\mathrm{i} \sum_{i=1}^n \varphi_i |i\rangle\langle i|\right)$.

Consequence: Lossy Boson-Sampling can be **efficiently approximated** to accuracy Δ_l in TV-distance. Moreover,

$$l = \mathrm{o}(\sqrt{n}) \Rightarrow \Delta_l \approx \frac{l^2}{2n} \ , \quad l = \omega(\sqrt{n}) \Rightarrow \Delta_l \to 1 \ .$$

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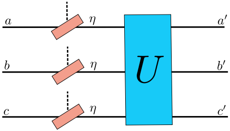
Classical simulation for a beamsplitter loss model



The input state for a uniform beamsplitter loss model with transmitivity η,

$$\rho_\eta = \sum_{l=0}^{n} \eta^l (1-\eta)^{n-l} \binom{n}{l} \rho_{l,n} \ .$$

We take a probabilistic mixture of optimal separable states with different l,

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We get $\mathrm{d_{tr}}\left(\rho_\eta, \sigma_\eta\right) \approx \Delta_{\eta n} = \frac{\eta^2 n}{2}$, so effectively **the same conclusion as before holds for average number $\langle l \rangle = \eta n$ of photons left in the network**.

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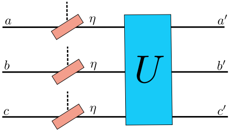

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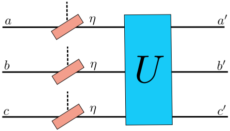

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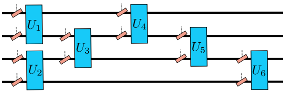
Gdańsk in the Autumn

Classical simulation for the realistic loss model

$\Lambda_{\mathcal{N}}$



RESULT (Extracting uniform losses from a lossy network)

Let s be a **smallest number of times a particle traverses a beamsplitter as it propagates through the network** $\mathcal{N}$. Let $\Lambda_{\mathcal{N}}$ be the channel associated the network $\mathcal{N}$. Then **it is possible to "pull-out" uniform losses of transmitivity** $\eta_{\text{eff}} = \eta^s$ from the channel $\Lambda_{\mathcal{N}}$:

$$\Lambda_{\mathcal{N}} = \tilde{\Lambda}_{\mathcal{N}} \circ \Lambda_{\eta_{\text{eff}}} \ ,$$

where $\Lambda_{\eta_{\text{eff}}}$ - **beamspliter loss model**, $\tilde{\Lambda}_{\mathcal{N}}$ -**still a linear optics channel**.

- **Efficient classical simultion of lossy Boson sampling device** to accuracy $\Delta \approx \frac{n\eta^{2s}}{2}$ in TV- distance.
- **Typically** $s \gtrsim n$. In fact **even if** $s \approx \log(n)$ we can still have $\Delta \to 0$ (for fixed η)!

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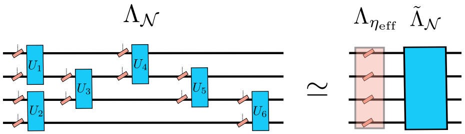

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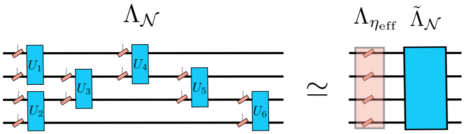

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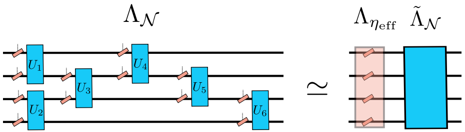

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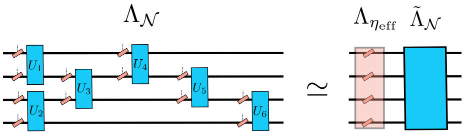

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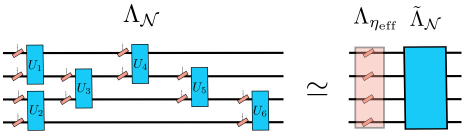

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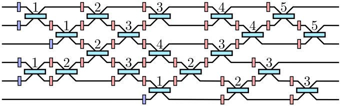
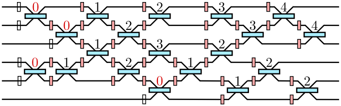
Example: Quantum Tetris

(I)



(II)



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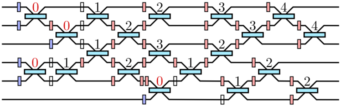

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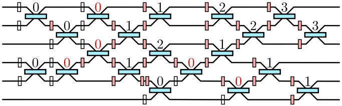

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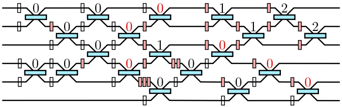
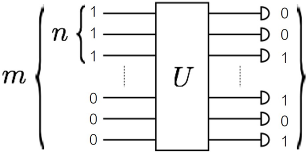


Conclusions



- Linear-optical networks **are heavily affected** by photon losses.
- **Consequence for lossy Boson Sampling devices**: classical simulation of output statistics to precision Δ in TV - distance:
  (a) If s number of photons that are left $l = o(\sqrt{n})$, then $\Delta \approx \frac{l^2}{2n}$.
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- **Limitation on the construction** linear optical networks imposed by losses.

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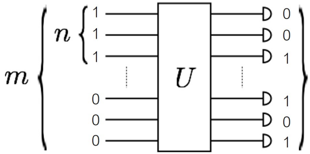

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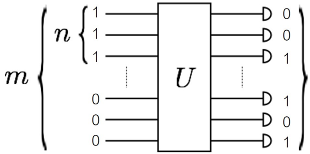

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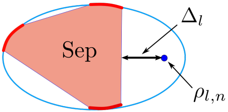
Open problems and future research directions



- Is it possible to obtain a proper ϵ-simulation of lossy boson sampling[6]?
- Using total variation distance instead of trace distance?
- Generalization to non-uniform losses.
- de- Finetti theorem for diagonal symmetric states?
- Similar techniques to other quantum supremacy proposals?

[6] S. Rahimi-Keshari *et al.*, Phys. Rev. X **6**, 021039 (2016)

Thank you for your attention!

Check out `arXiv` or NJP for the full paper[7]

[7] Also: `arXiv:1712.10037` for independent work by R. Garcia-Parton *et al.*