The limits of large quantum systems

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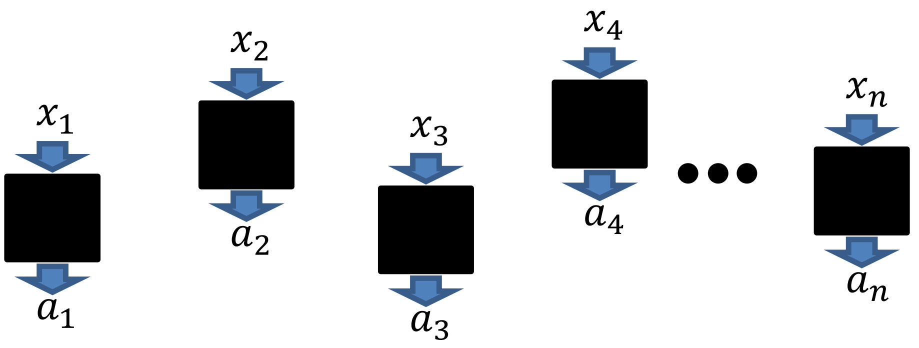
Max-Planck Institute for Gravitational Physics, Potsdam
ICFO – The Institute of Photonic Sciences, Barcelona

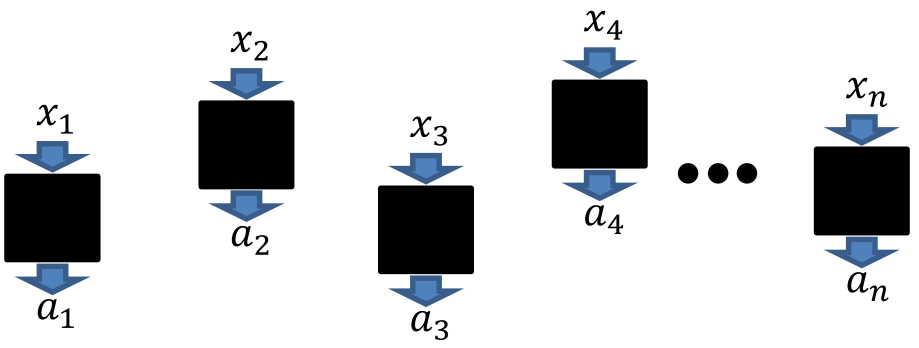
3 tasks in quantum information

Nonlocality detection



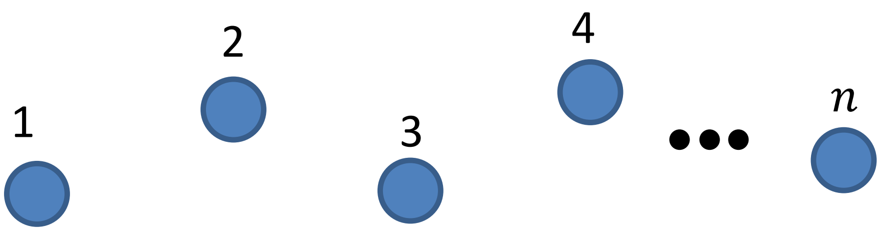
$$P(a_1, \ldots, a_n | x_1, \ldots, x_n) = \sum_{\lambda} p_\lambda P_1(a_1|x_1) \ldots P_n(a_n|x_n)$$

Supra-quantum nonlocality detection

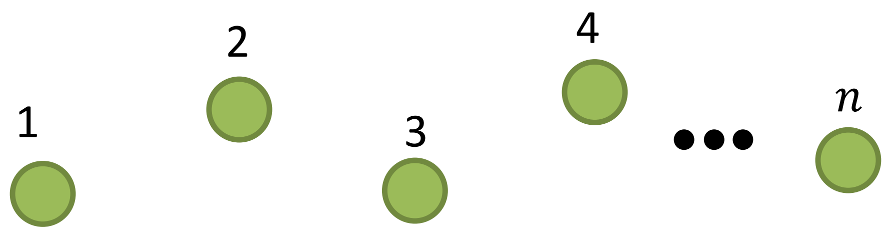


$$P(a_1, \ldots, a_n | x_1, \ldots, x_n) = \left\langle \psi \middle| E^1_{a_1, x_1} \otimes \ldots \otimes E^n_{a_n, x_n} \middle| \psi \right\rangle$$

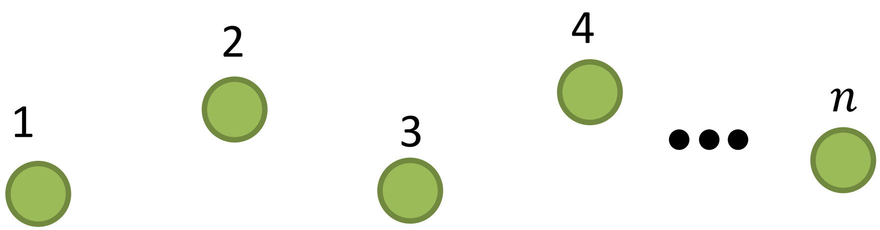
Entanglement detection



$$\rho = \sum_i p_i |\psi_i^1\rangle\langle\psi_i^1| \otimes |\psi_i^2\rangle\langle\psi_i^2| \otimes \ldots \otimes |\psi_i^n\rangle\langle\psi_i^n|$$



Goal: detect a global property of a systems network



Goal: detect a global property of a systems network

General tools

General tools

Nonlocality detection

Supraquantum nonlocality detection

Entanglement detection

General tools

Nonlocality detection

(linear programming)

Supraquantum nonlocality detection

Entanglement detection

General tools

Nonlocality detection

(linear programming)

Supraquantum nonlocality detection

(SDP hierarchies)

Entanglement detection

MN, S. Pironio and A. Acín, Phys. Rev. Lett. 98, 010401 (2007).
A. C. Doherty, Y.-C. Liang, B. Toner and S. Wehner, Proceedings of IEEE Conference on Computational Complexity 2008, pages 199—210.
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General tools

Nonlocality detection

(linear programming)

Supraquantum nonlocality detection

(SDP hierarchies)

Entanglement detection

(SDP hierarchies)

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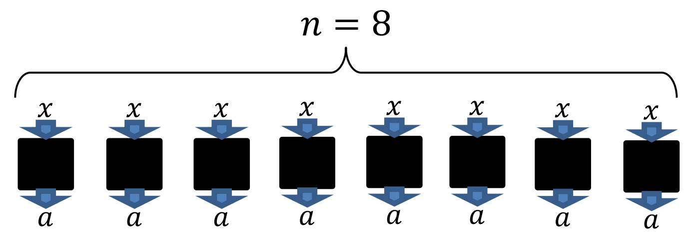
General tools

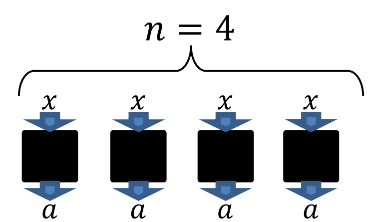
Nonlocality detection

(linear programming)



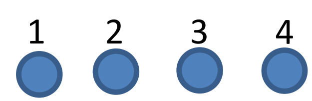
Supraquantum nonlocality detection

(SDP hierarchies)



Entanglement detection

(SDP hierarchies)



(beyond PPT)

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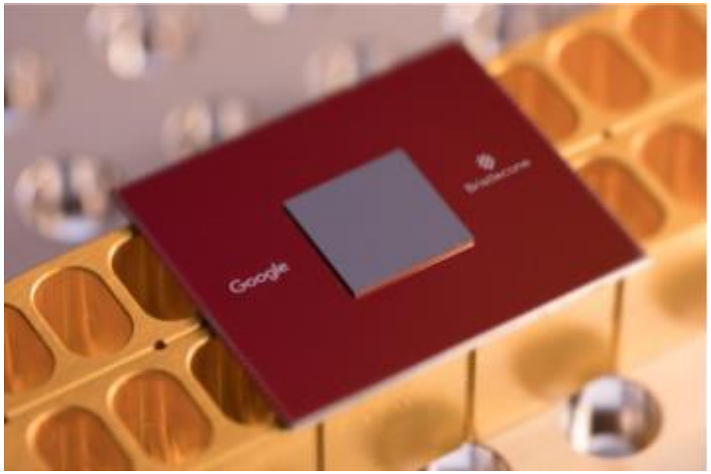
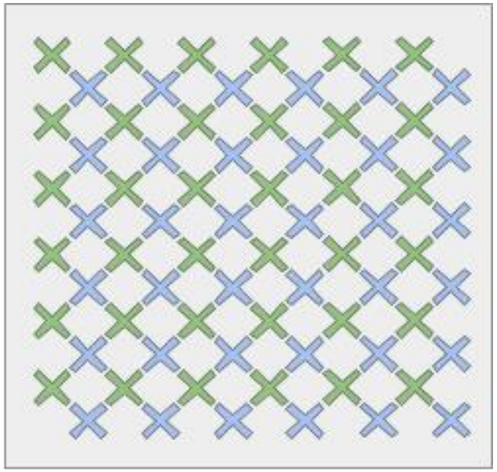
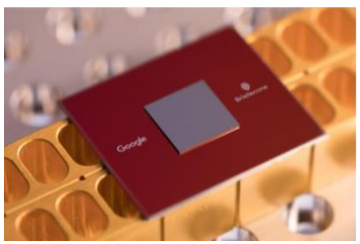
Small quantum systems were interesting yesterday, not now!!

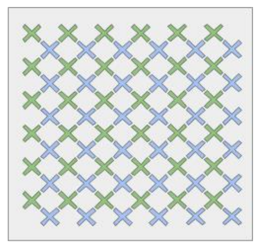




Google’s Bristlecone, 72 qubits

Question I



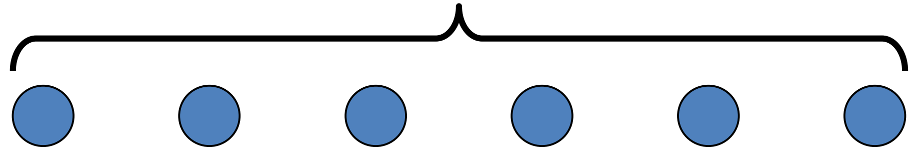


What is this?

What is this?

The quantum many-body problem

k particles



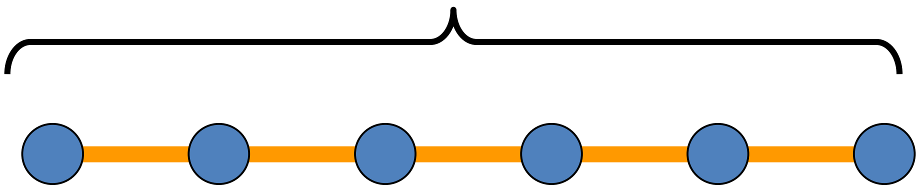
ground state $|\varphi\rangle \in \mathbb{C}^{d^k}$

Impossible to even *store* for d=2, k~50

What is this?

The quantum many-body problem

k particles



Many natural states admit an approximate compact description

$$|\psi\rangle = \sum_{i_1,\dots,i_k} tr(\sigma A_{i_1} \dots A_{i_k}) |i_1\rangle \otimes \dots \otimes |i_k\rangle$$

D. Pérez-García, F. Verstraete, M.M. Wolf and J.I. Cirac, Quantum Inf. Comput. 7, 401 (2007).

What is this?

The quantum many-body problem

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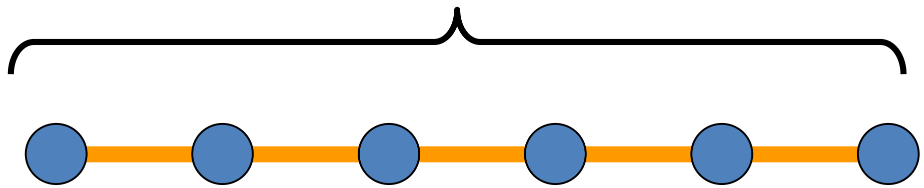
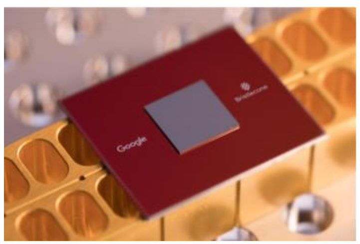
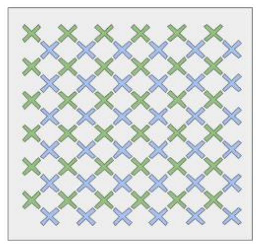


The parameters determining the state can be estimated with a polynomial number of experiments.

M. Cramer, M. B. Plenio, S. T. Flammia, D. Gross, S. D. Bartlett, R. Somma, O. Landon-Cardinal, Y.-K. Liu, and D. Poulin, Nat. Commun. **1**, 149 (2010).
T. Baumgratz, D. Gross, M. Cramer, and M. B. Plenio, Phys. Rev. Lett. **111**, 020401 (2013).

Question I





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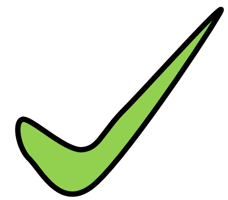
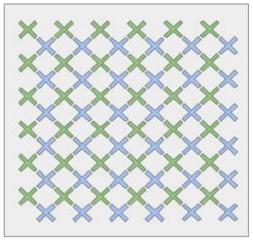


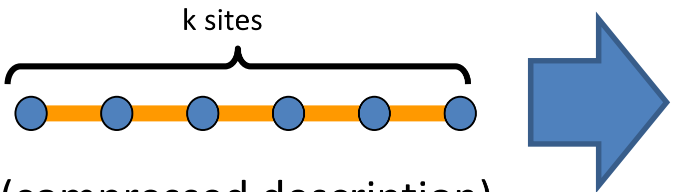
Question II



Is it quantum?

Is it quantum?

The many-body quantum information problem



(compressed description)

Entanglement?

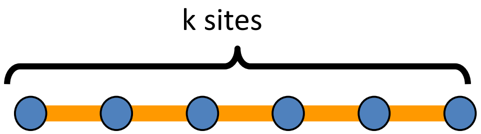
Nonlocality?

Supra-quantum nonlocality?

Is it quantum?

The many-body quantum information problem

General tools



(compressed description)

Entanglement?	Exp[k] operations
Nonlocality?	Exp[k] operations
Supra-quantum nonlocality?	Exp[k] operations

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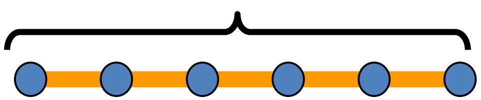
Is it quantum?

The many-body quantum information problem

General tools

k sites



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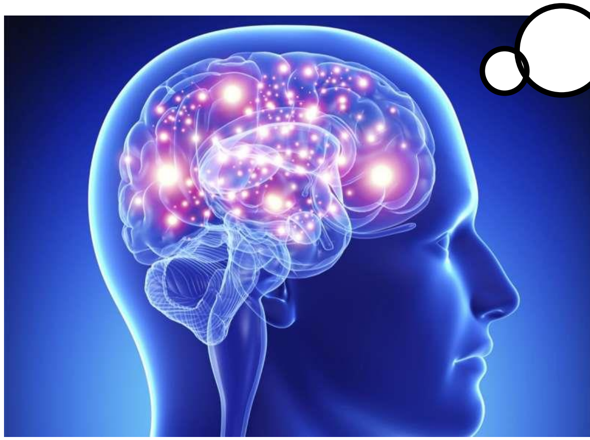
Human reasoning

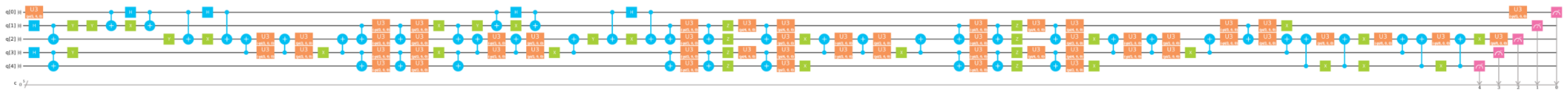


Short term memory ⇨ 7 ± 2 items

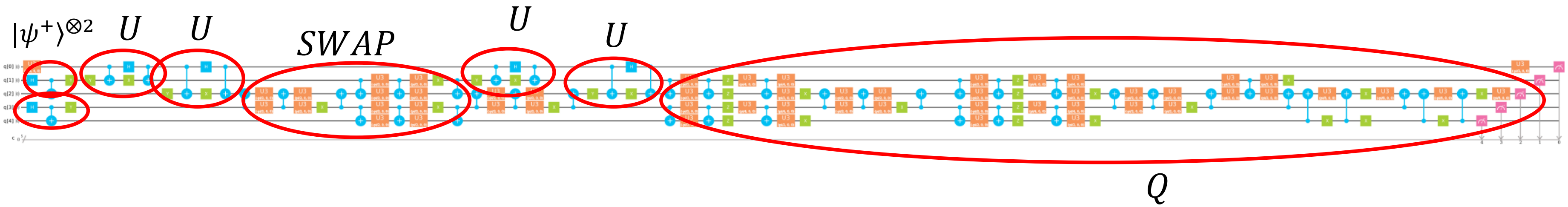
G. A. Miller, Psychological Review. 63 (2): 81–97 (1956).

Human reasoning

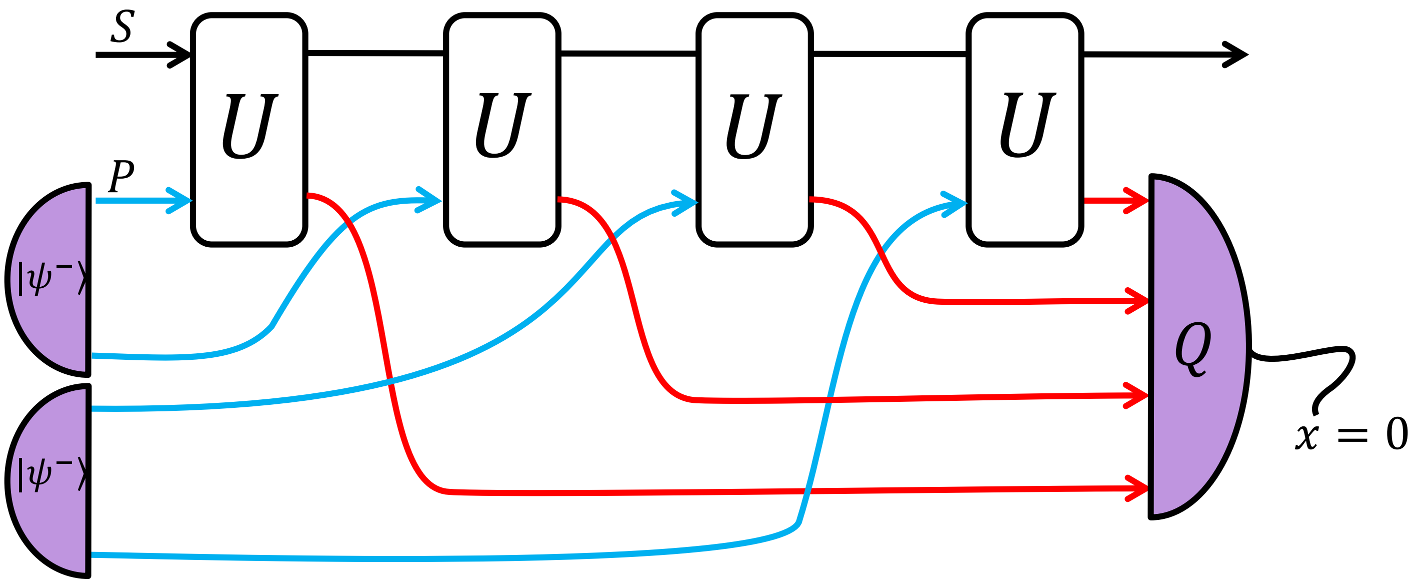
How do humans manage to devise a circuit?

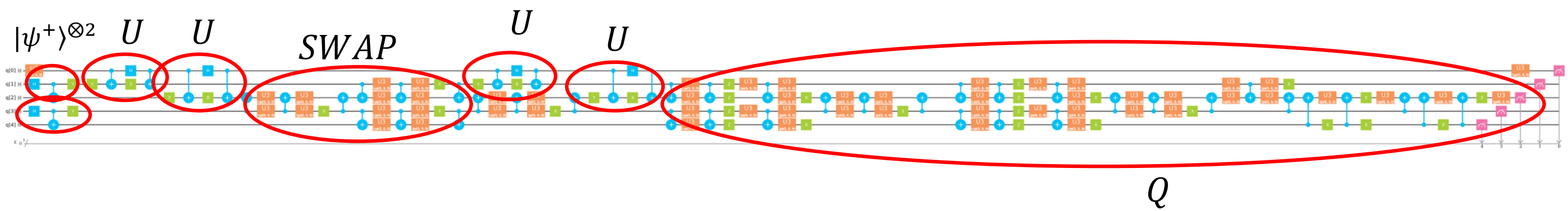


Human reasoning

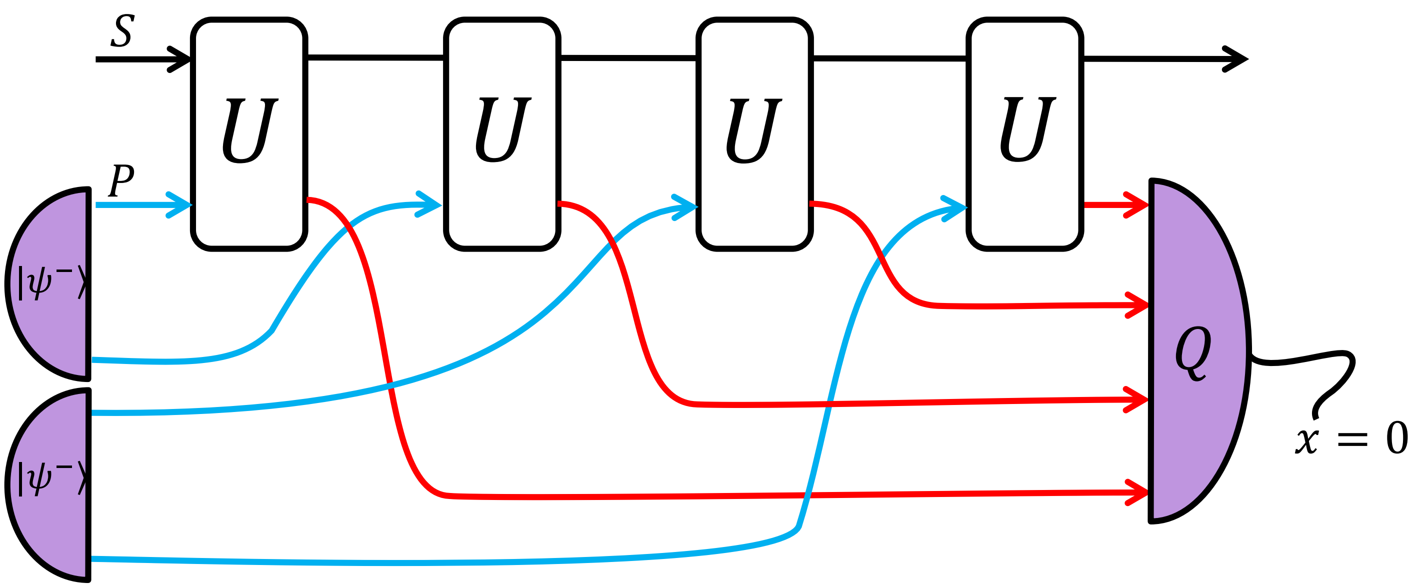


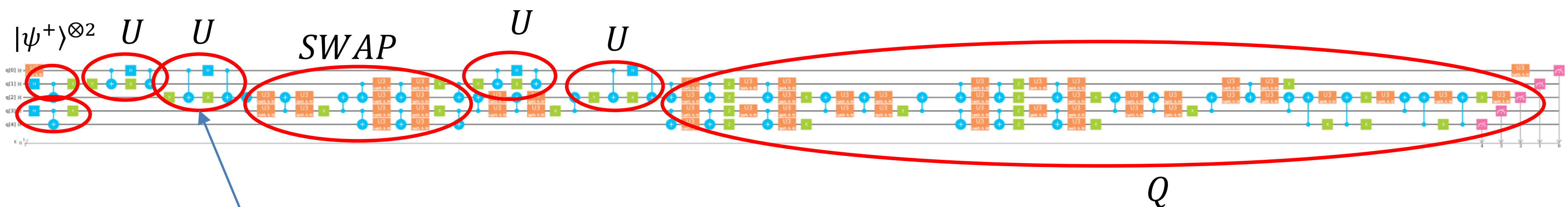
Human reasoning





Human reasoning

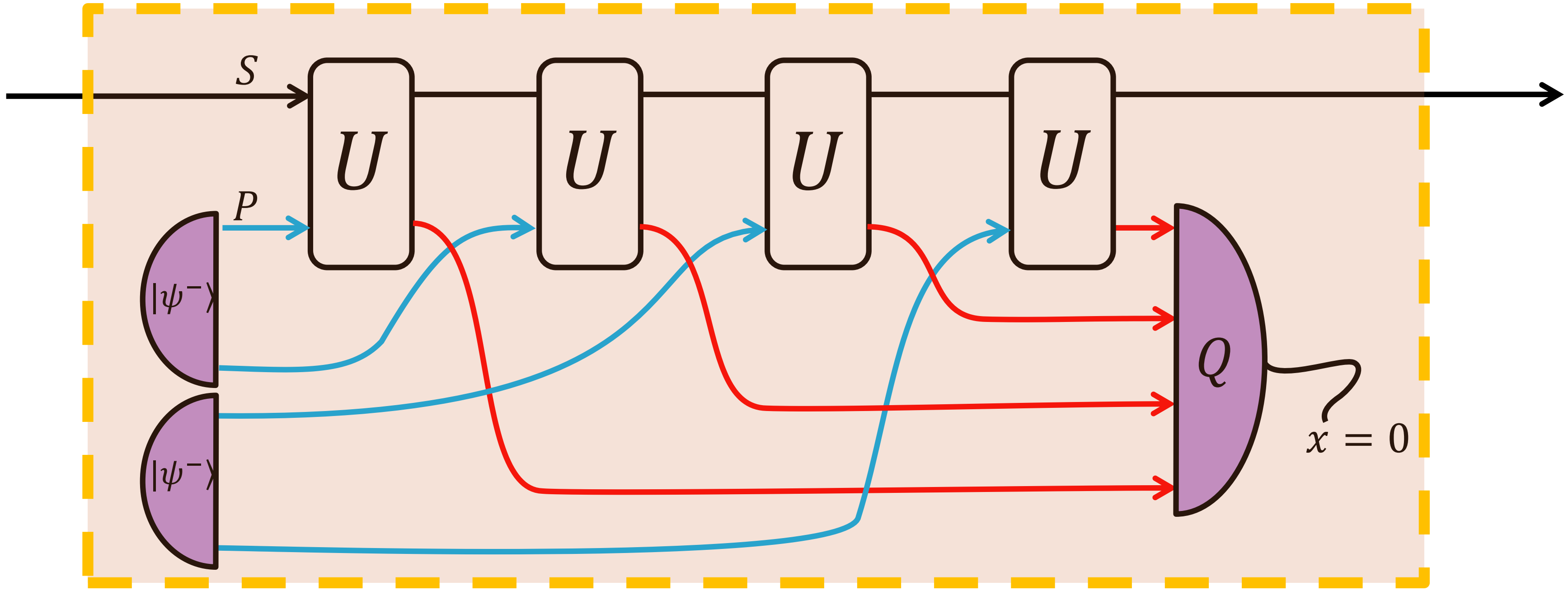




chunk

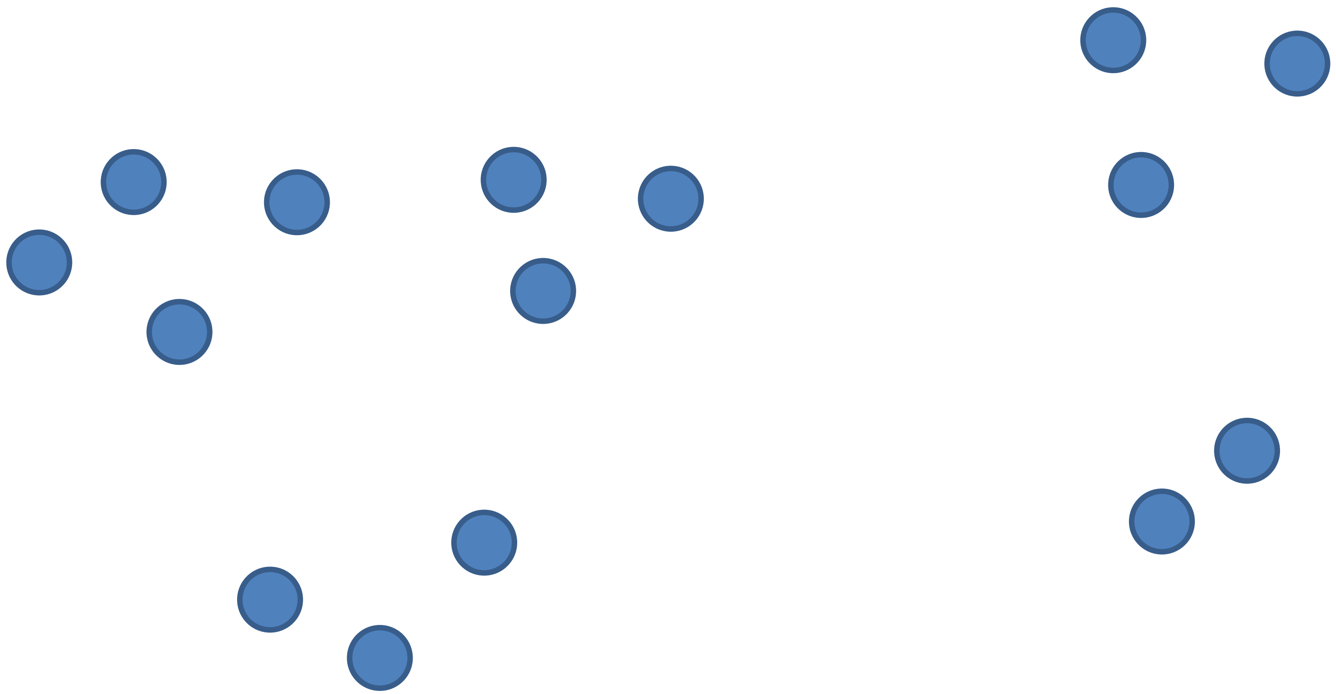
Collection of elementary units that have been inter-associated and stored in memory repeatedly and act as a coherent, integrated group when retrieved.

Human reasoning



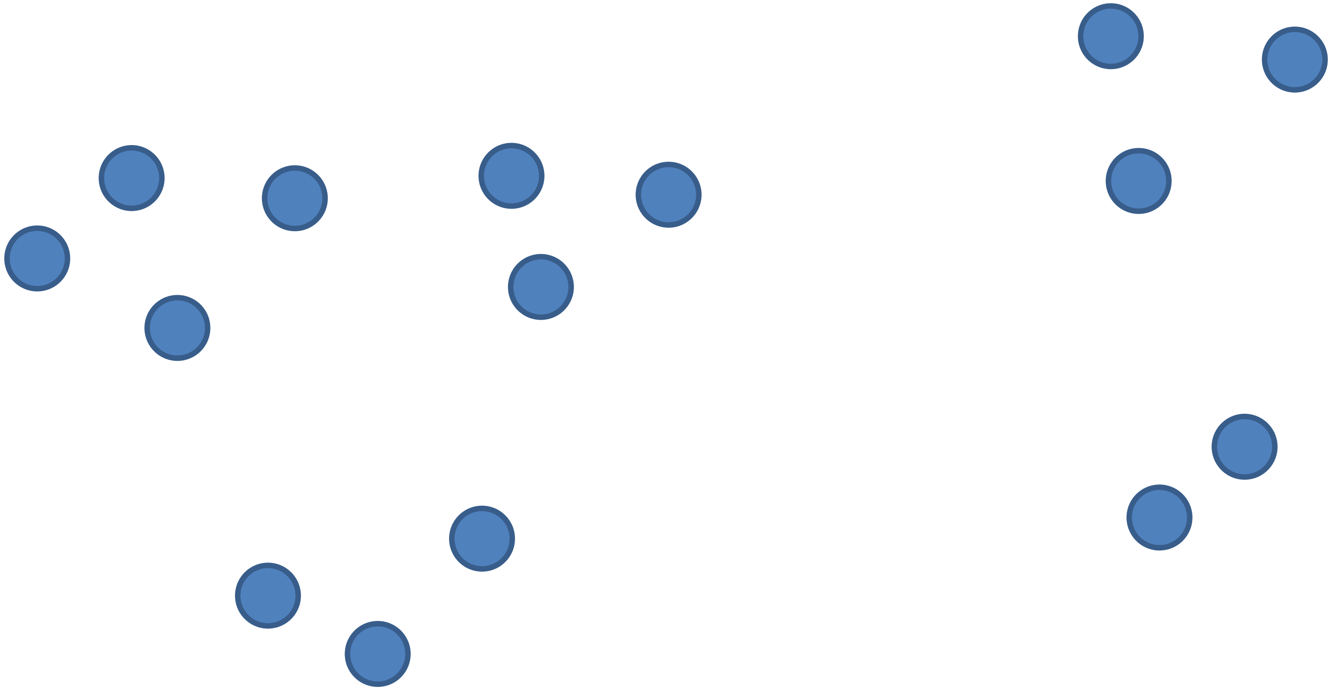
Weird quantum circuit I

Many-body quantum information problem



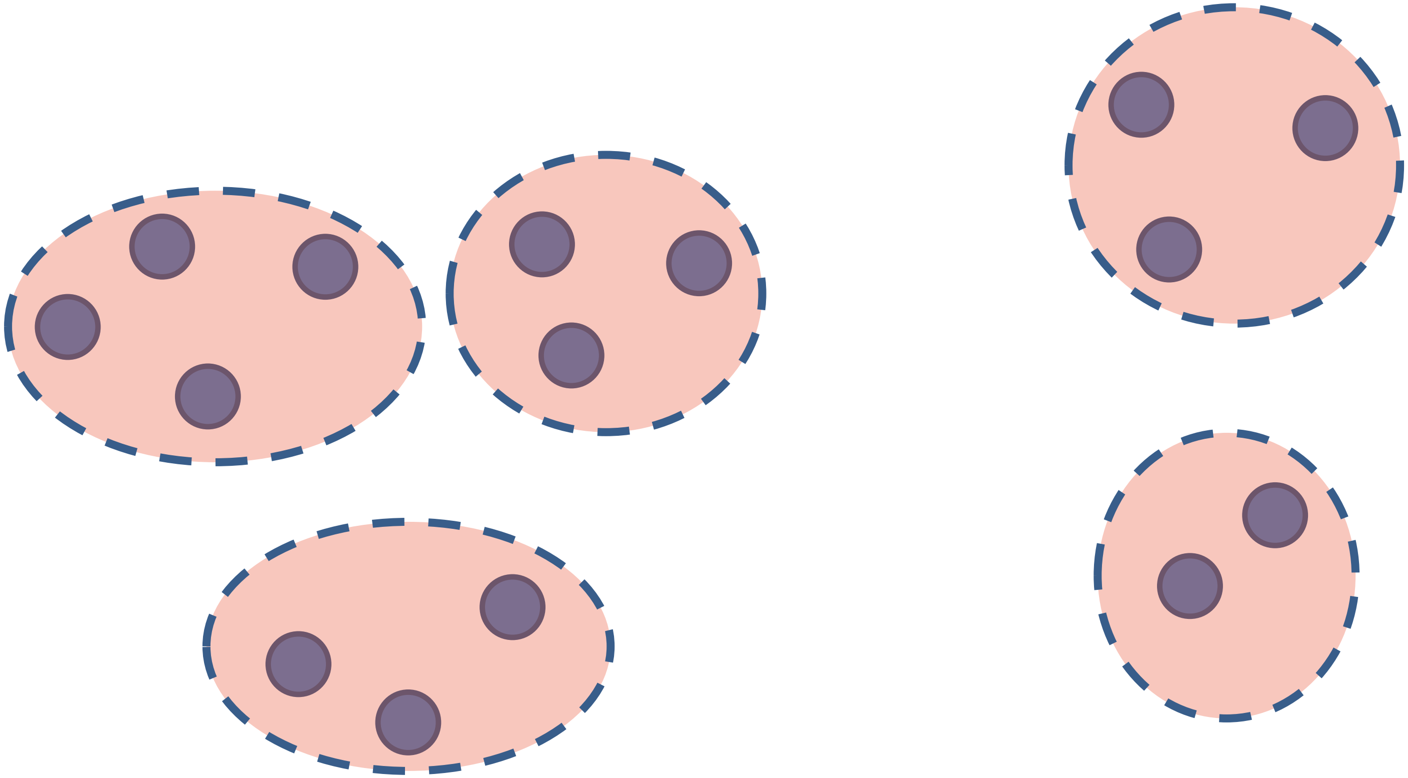
Many-body quantum information problem

Global property?

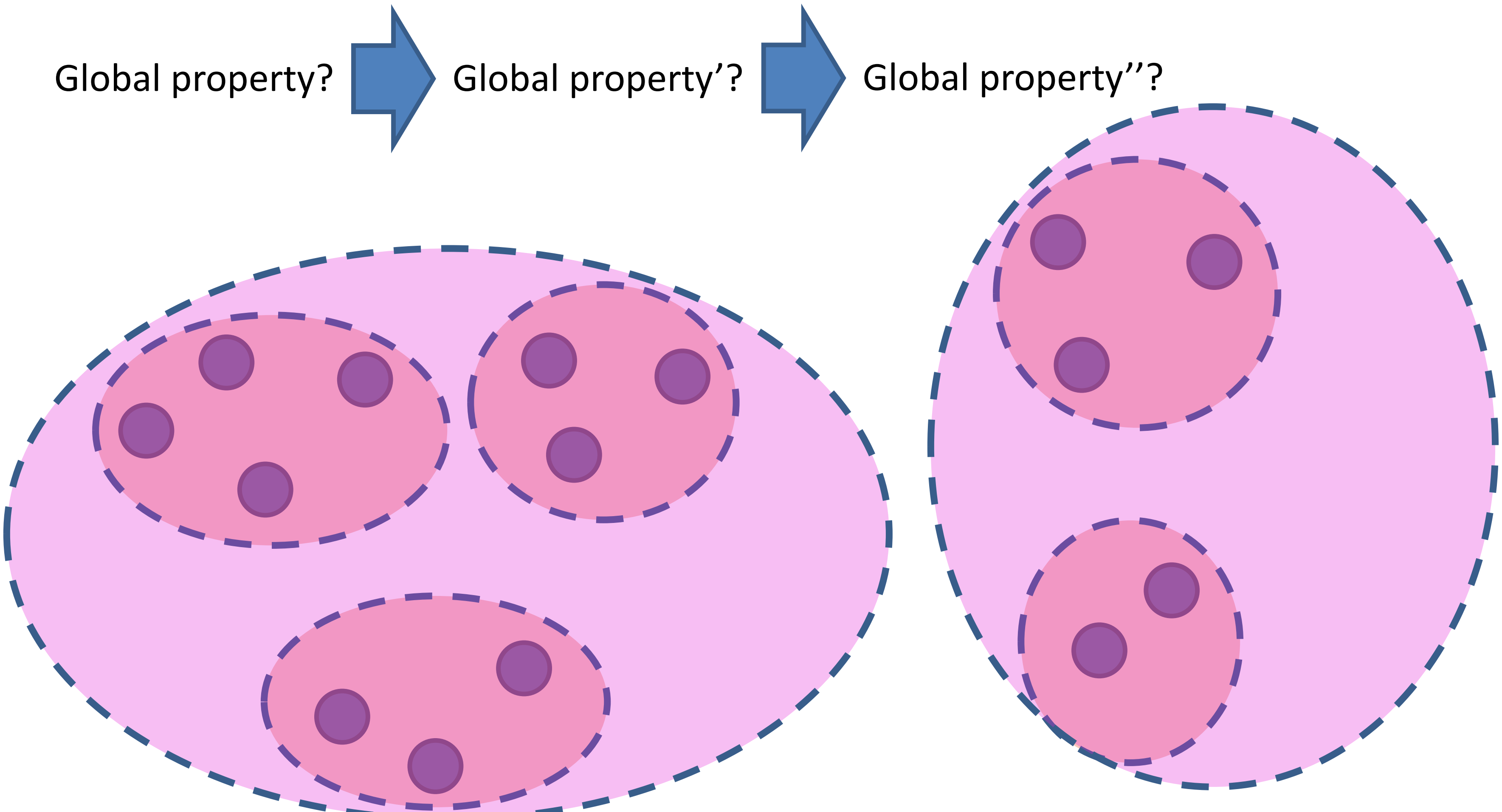


Many-body quantum information problem

Global property? → Global property’?



Many-body quantum information problem



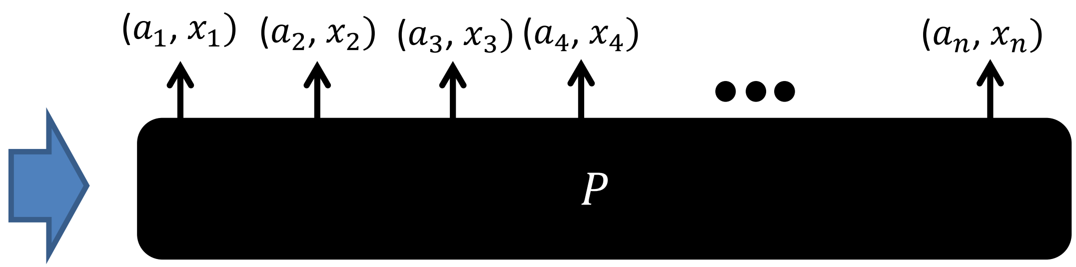
The chunking method: connector theory

Nonlocality detection

Nonlocality detection

Tensor notation

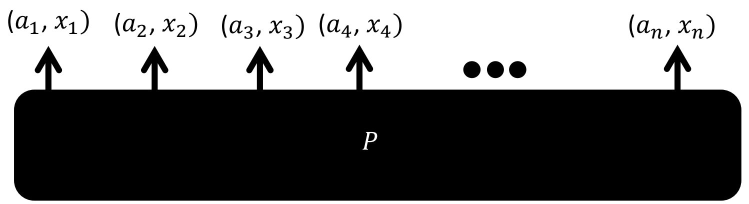
$$P(a_1,\dots,a_n|x_1,\dots,x_n)$$



Nonlocality detection

Nonlocality problem

Input:



Output: $P(a_1, \dots, a_n | x_1, \dots, x_n) = \sum_\lambda p_\lambda P_1(a_1|x_1) \dots P_n(a_n|x_n)$ "No"

$P(a_1, \dots, a_n | x_1, \dots, x_n) \neq \sum_\lambda p_\lambda P_1(a_1|x_1) \dots P_n(a_n|x_n)$ "Yes"

Nonlocality detection

GPT: LOC-world

(A, X)

$A, X \in \mathbb{N}$

Basic (single-site) system types

Nonlocality detection

GPT: LOC-world

In quantum theory:

(A, X)

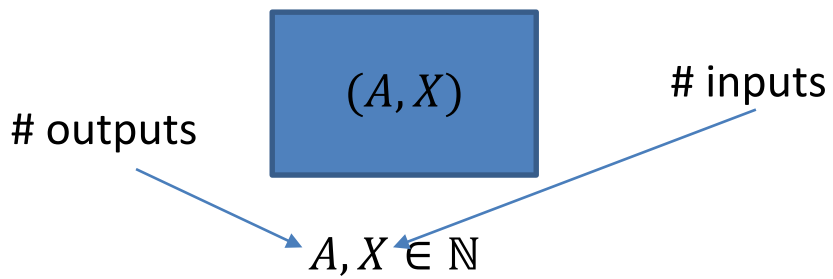
$\mathbb{C}^d$

$A, X \in \mathbb{N}$

Basic (single-site) system types

Nonlocality detection

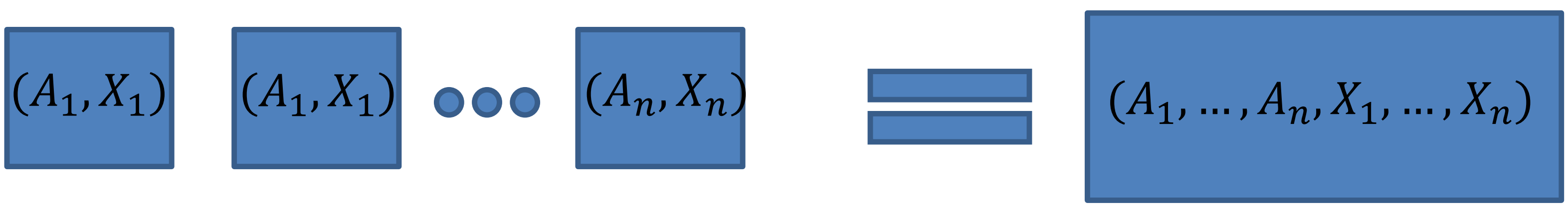
GPT: LOC-world



Basic (single-site) system types

Nonlocality detection

GPT: LOC-world



Composition rule for several basic systems

Nonlocality detection

GPT: LOC-world

$(A_1, \dots, A_n, X_1, \dots, X_n)$

State space $\Rightarrow$ $\mathbb{R}^{A_1 \dots A_n X_1 \dots X_n}$

Set of states $\Rightarrow$

$$P(a_1, \dots, a_n | x_1, \dots, x_n) = \sum_\lambda p_\lambda P_1(a_1|x_1) \dots P_n(a_n|x_n)$$

(set of local/classical behaviors)

In quantum theory:

$\rho \geq 0$

Nonlocality detection

GPT: LOC-world

Norm of a state

$$\boxed{N(P) = \sum_{a_1,\ldots,a_n} P(a_1, \ldots, a_n | x_1, \ldots, x_n)}$$

$$\left(\begin{array}{c}\text{In quantum theory:} \\[1em] N(\rho) = tr(\rho)\end{array}\right)$$

Nonlocality detection

GPT: LOC-world

Operations

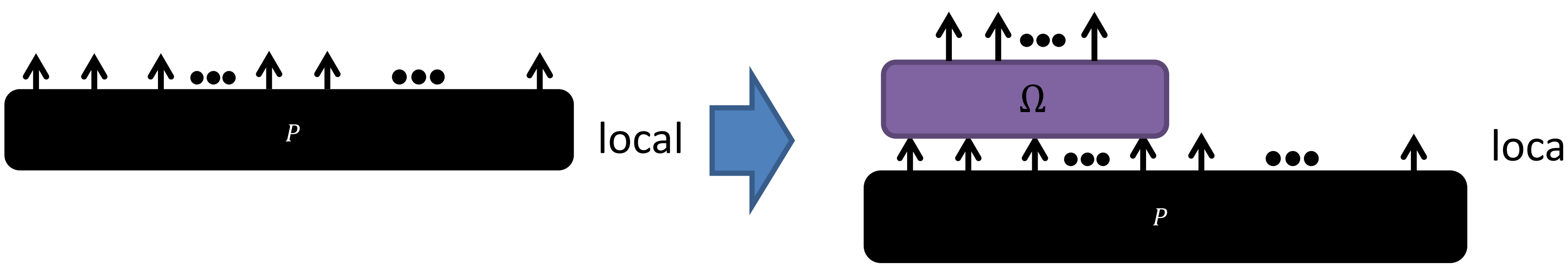
$$(A_1, \dots, A_n, X_1, \dots, X_n) \xrightarrow{\Omega} (B_1, \dots, B_{n'}, Y_1, \dots, Y_{n'})$$

Nonlocality detection

GPT: LOC-world

Operations

$$(A_1, \dots, A_n, X_1, \dots, X_n) \xrightarrow{\Omega} (B_1, \dots, B_{n'}, Y_1, \dots, Y_{n'})$$

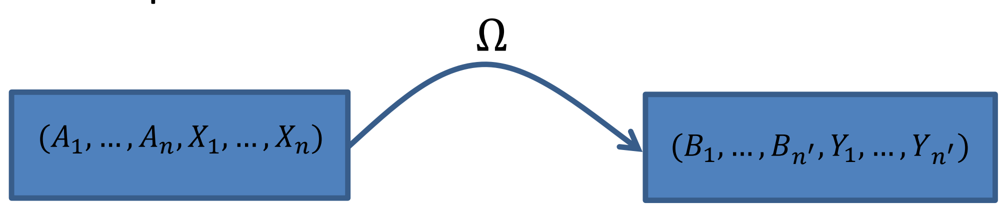


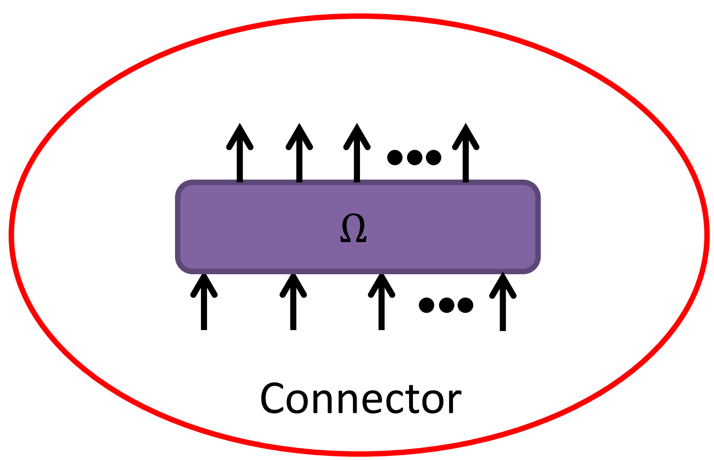
$$N(P) \geq N(\Omega(P))$$

Nonlocality detection

GPT: LOC-world

Operations





Tensor with directed edges representing a transformation in LOC-world

Nonlocality detection

Characterization of connectors in LOC-world

$$(A_1, \dots, A_n, X_1, \dots, X_n) \xrightarrow{\Omega} (B_1, \dots, B_{n'}, Y_1, \dots, Y_{n'})$$

List of deterministic boxes $\{P_k\}_k$

List of deterministic boxes $\{Q_l\}_l$

Ω, valid transformation in LOC-world $\Longleftrightarrow$

$$\Omega(P_k) = \sum_l \mu_l^k Q_l \,,$$

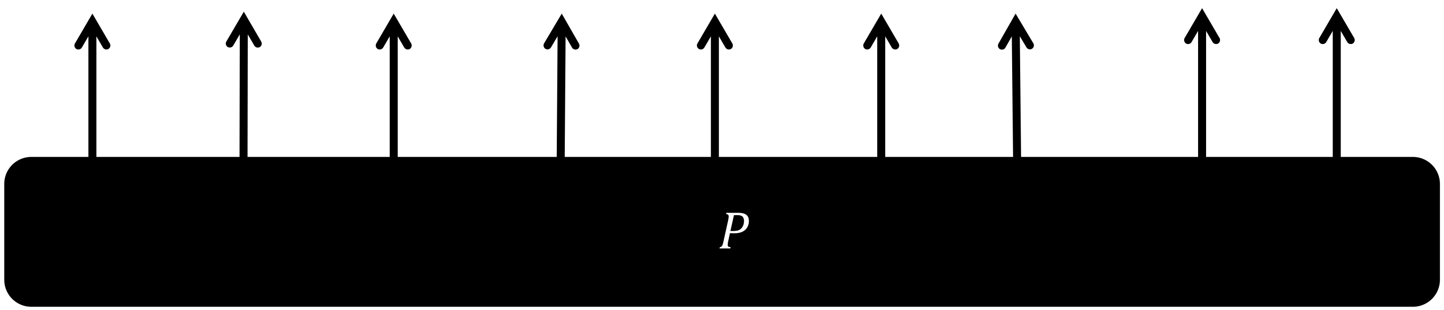
$$\mu_l^k \geq 0, \sum_l \mu_l^k \leq 1, \text{for all } k$$

Linear program!!

Nonlocality detection

Connecting connectors

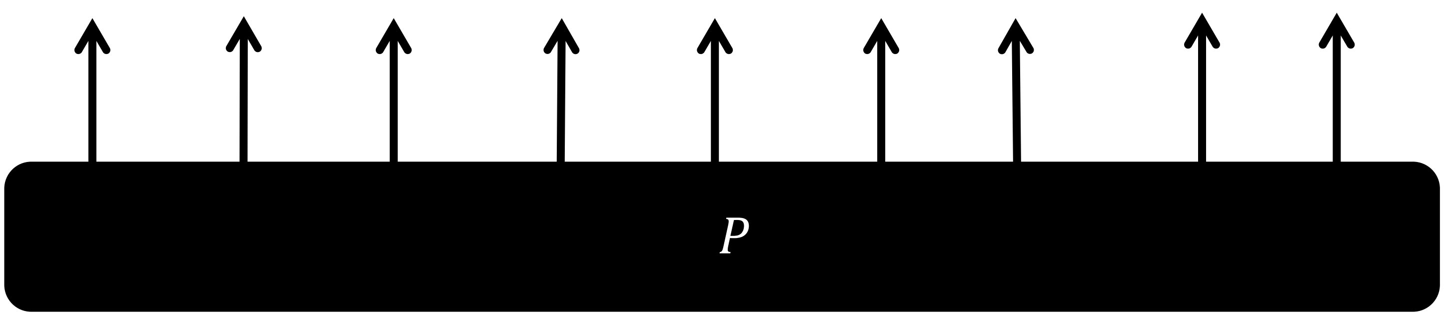
P, local?



Nonlocality detection

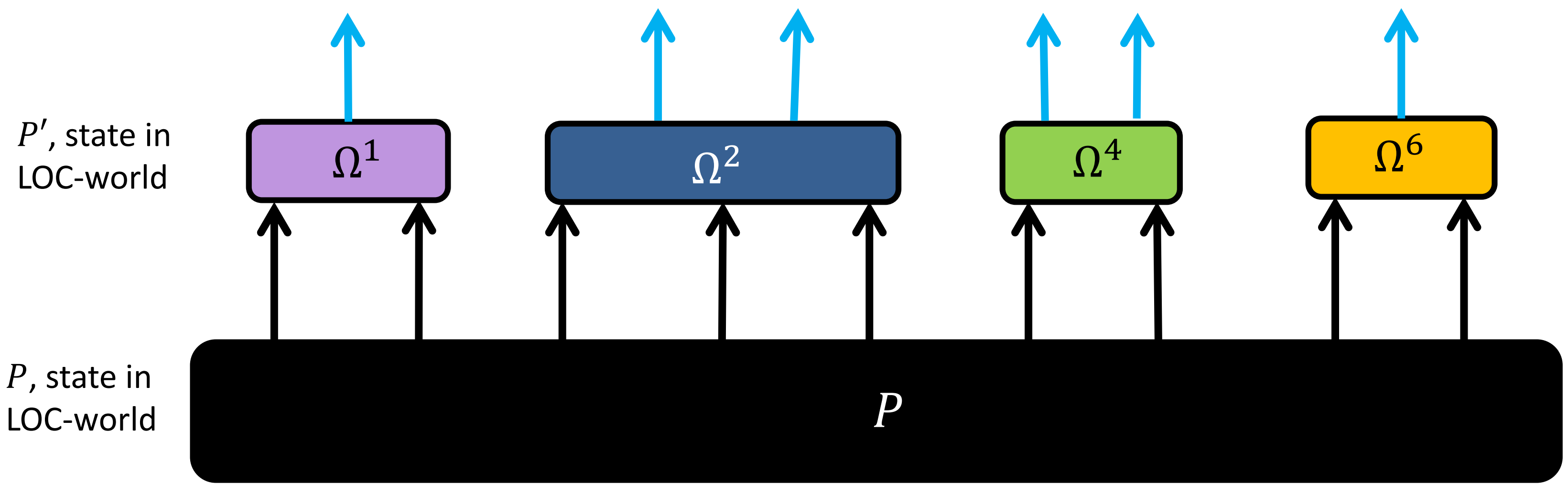
Connecting connectors

P, state in LOC-world



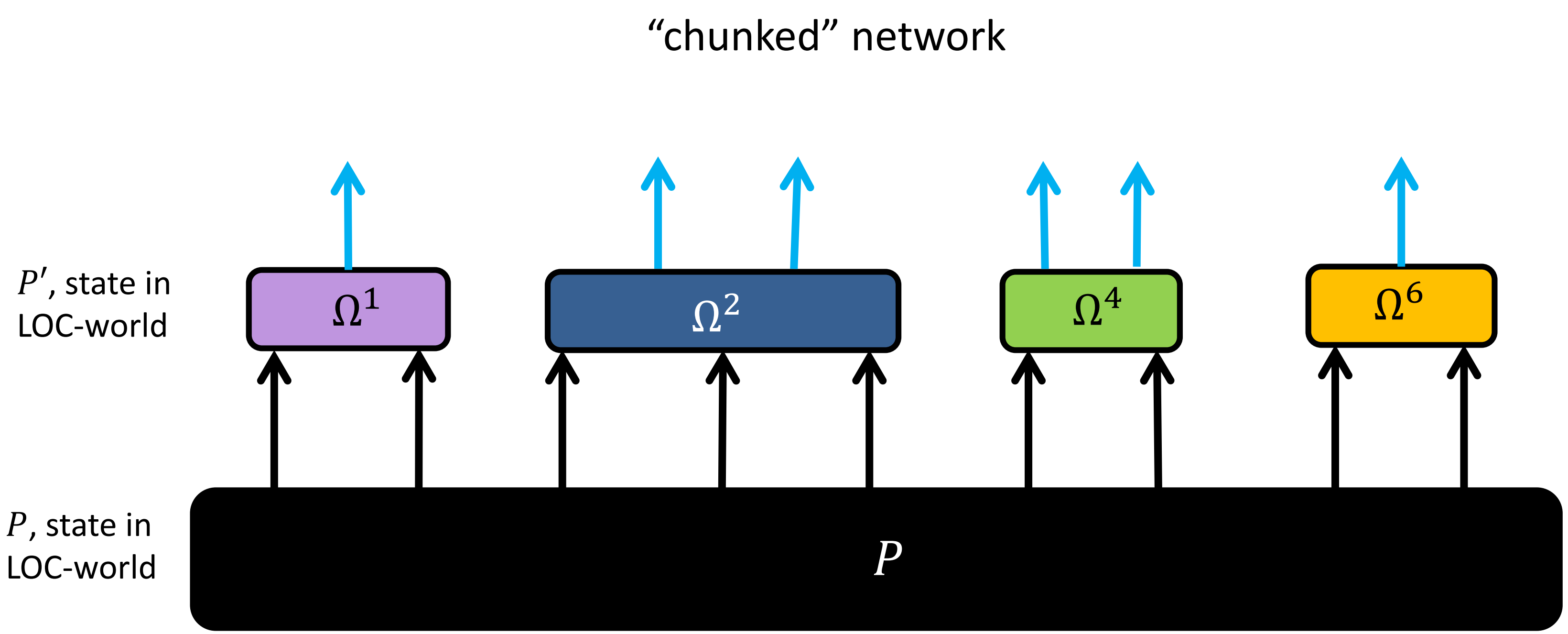
Nonlocality detection

Connecting connectors



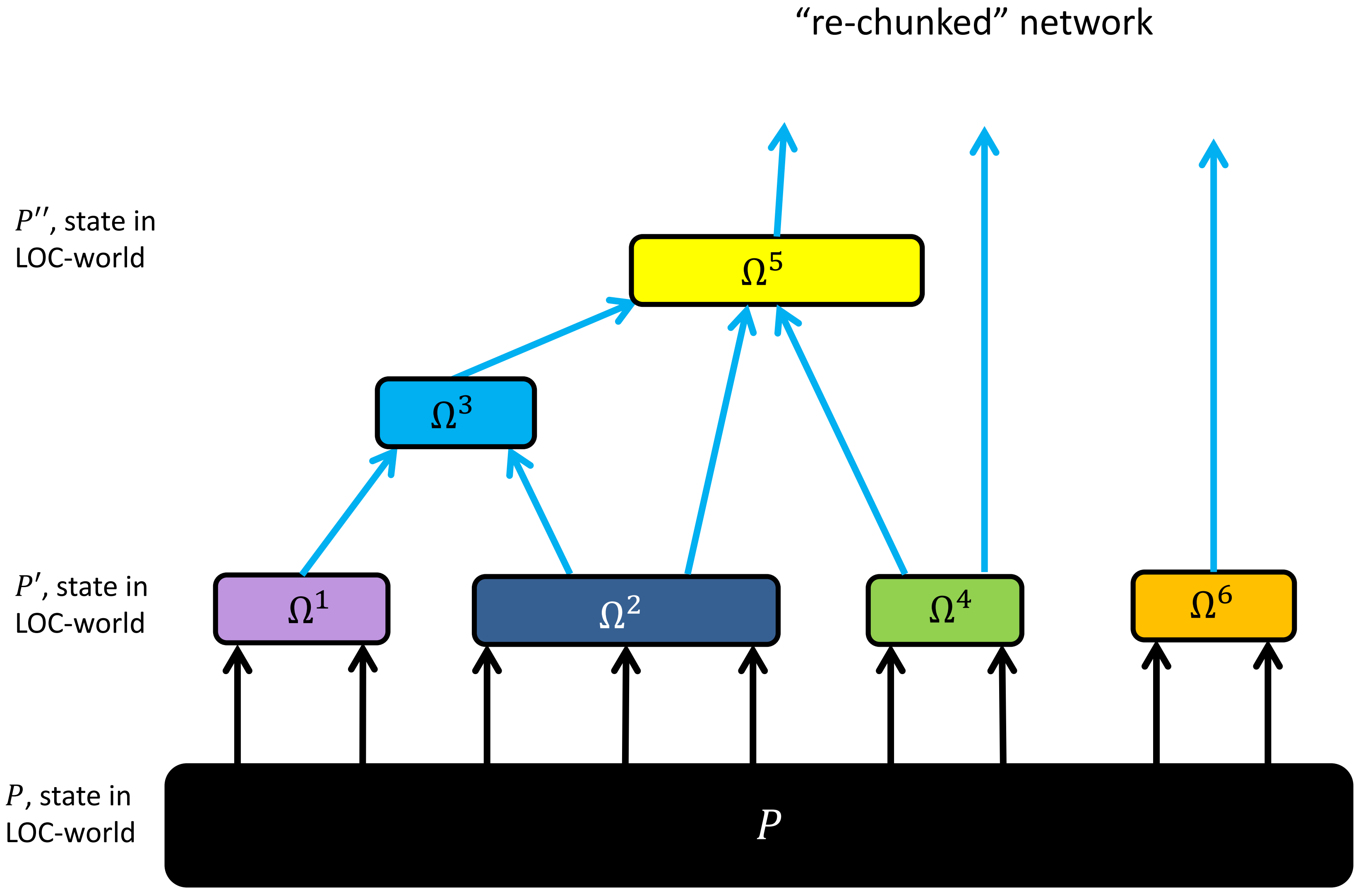
Nonlocality detection

Connecting connectors



Nonlocality detection

Connecting connectors



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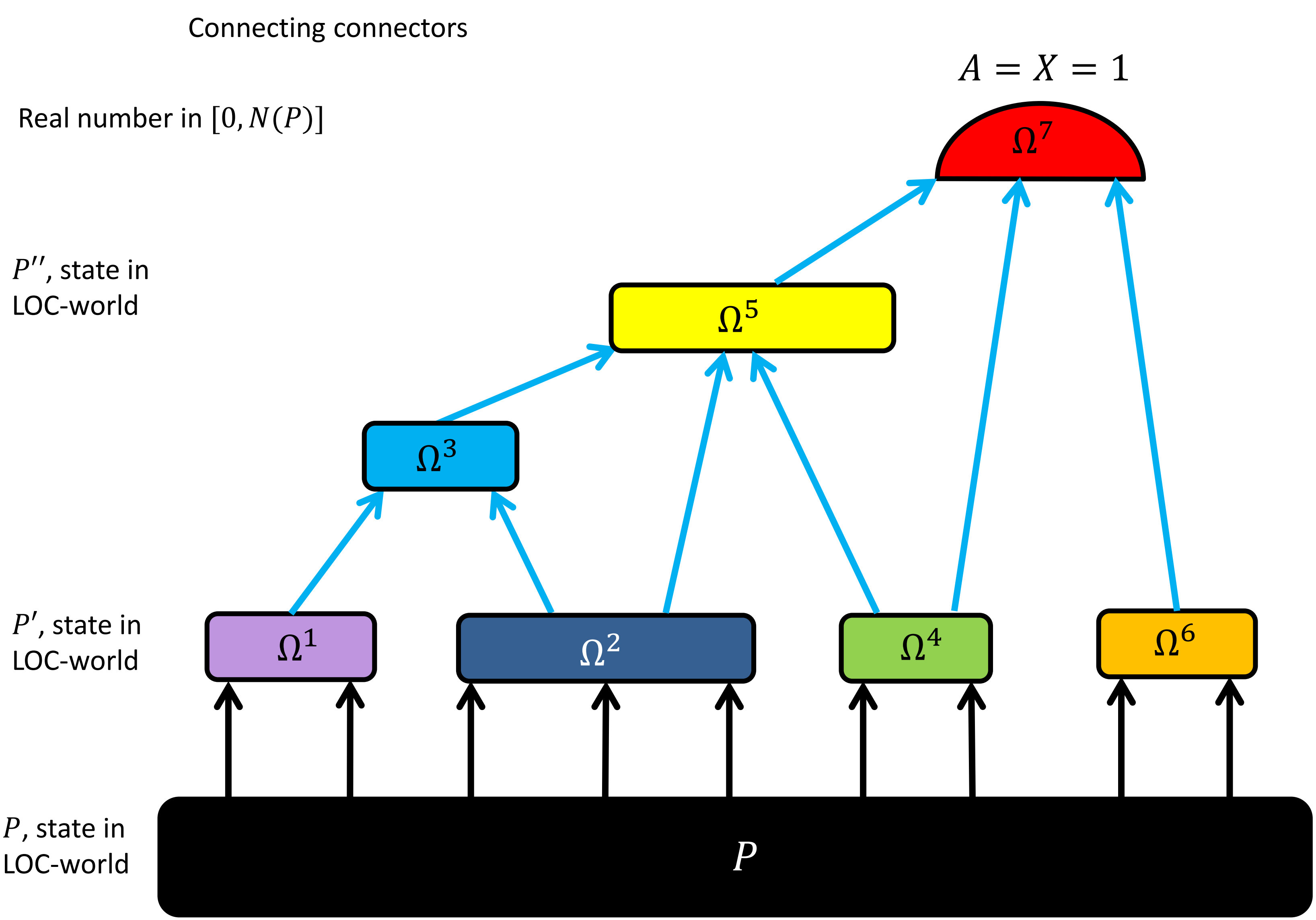
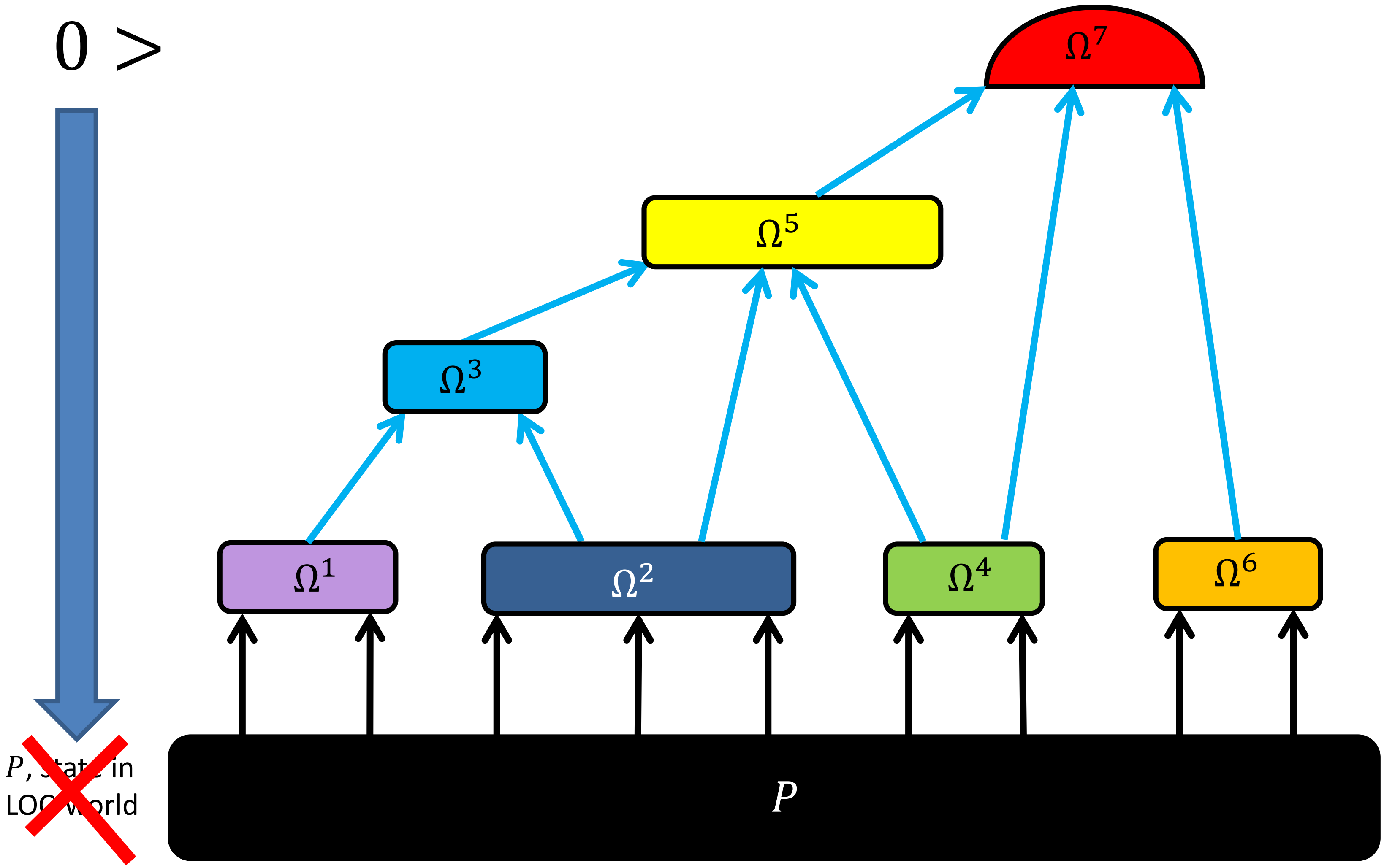


Nonlocality detection

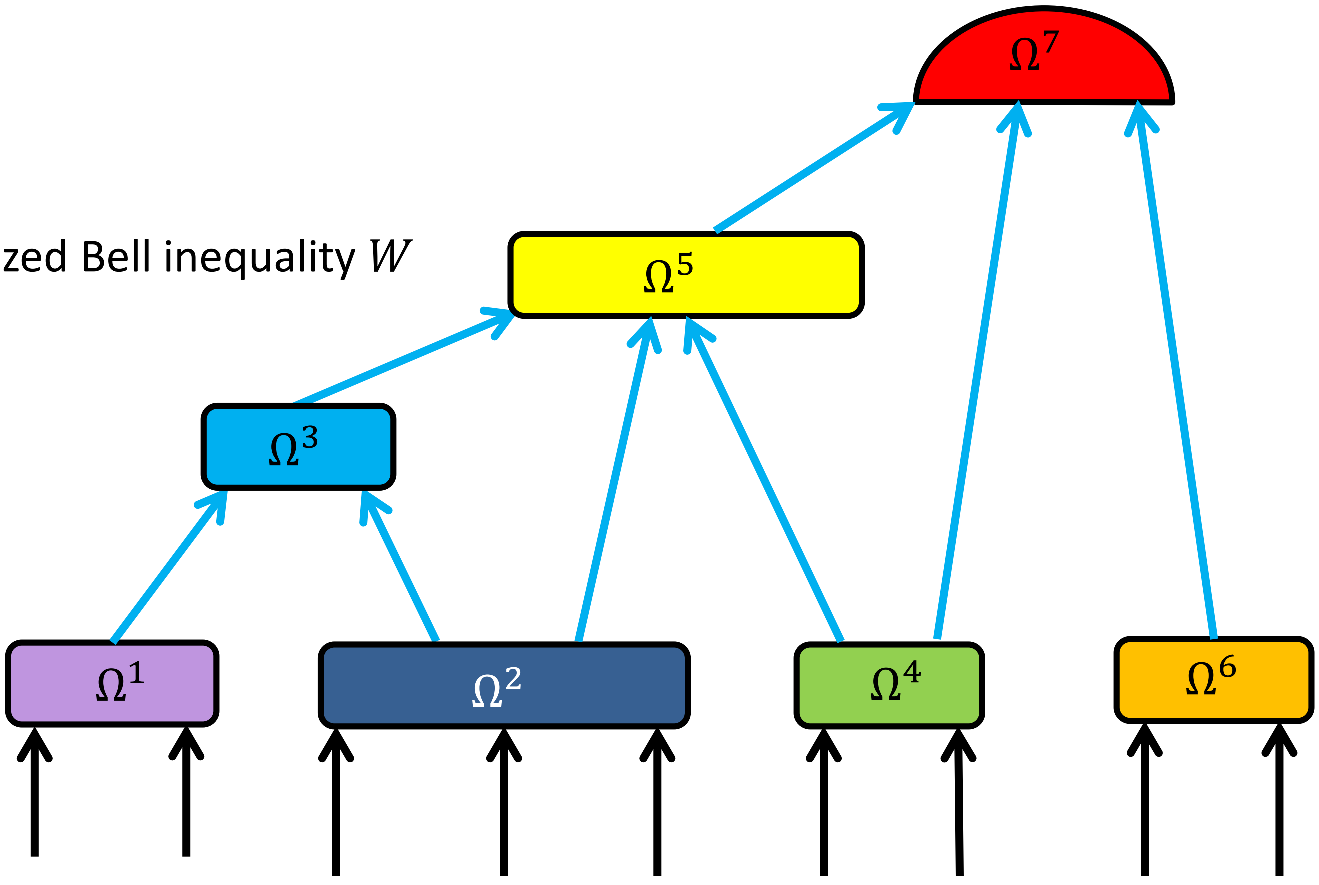
Connecting connectors



Nonlocality detection

Connecting connectors

Normalized Bell inequality W

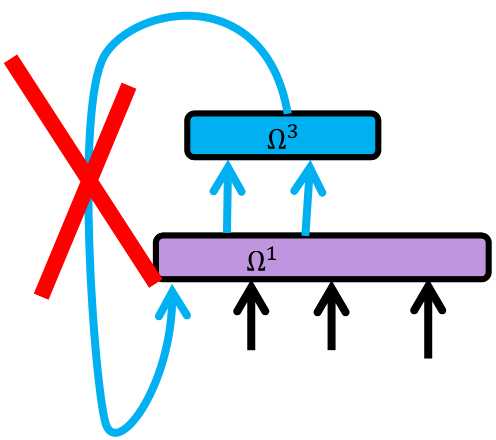


$0 \leq W(P) \leq 1$, for all P, classical

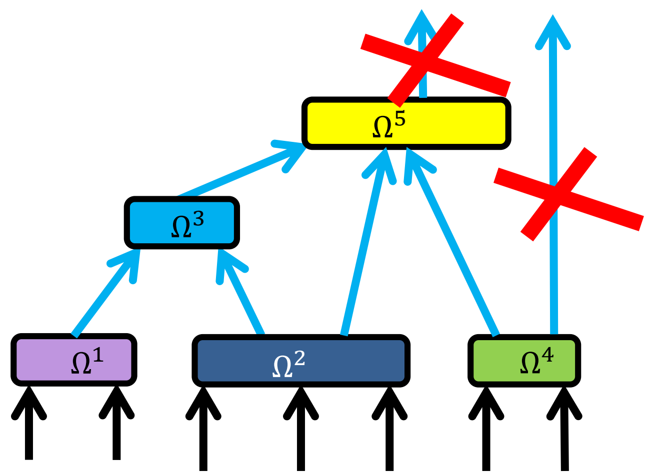
Entanglement detection

Connecting connectors

Any contraction of connectors with…



No cycles



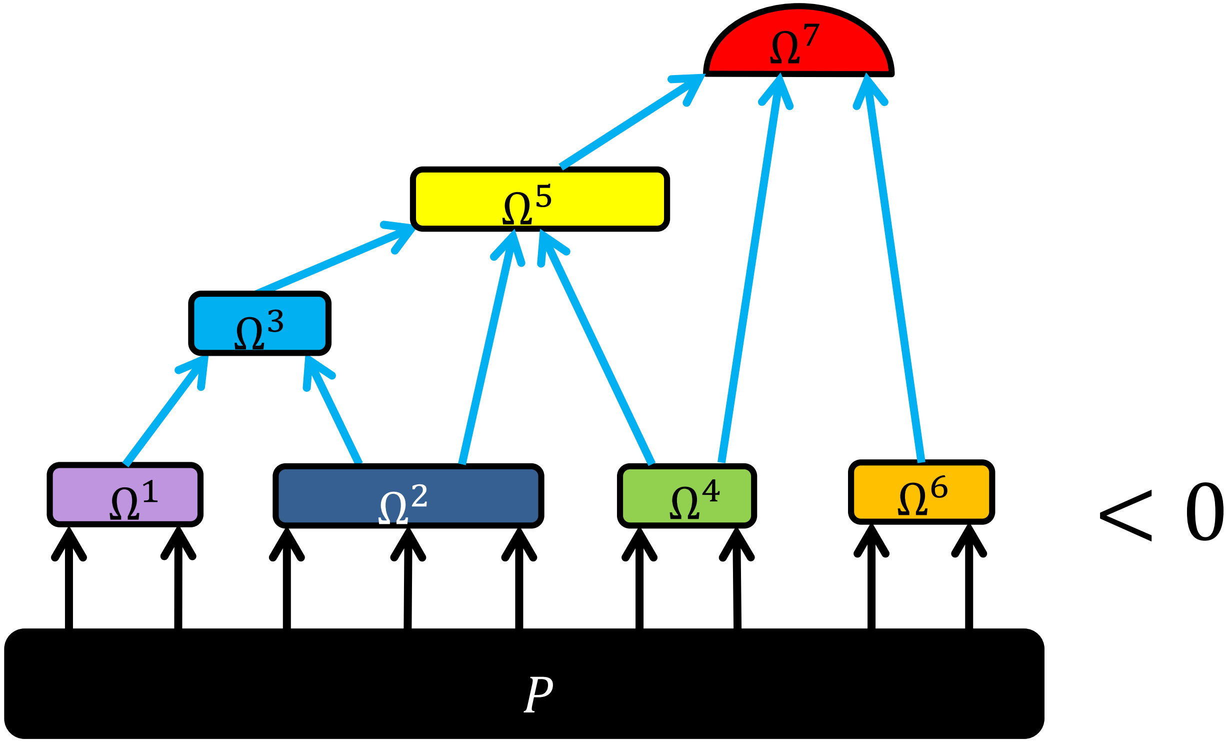
No output arrows

…defines a normalized Bell inequality

Nonlocality detection

Applications

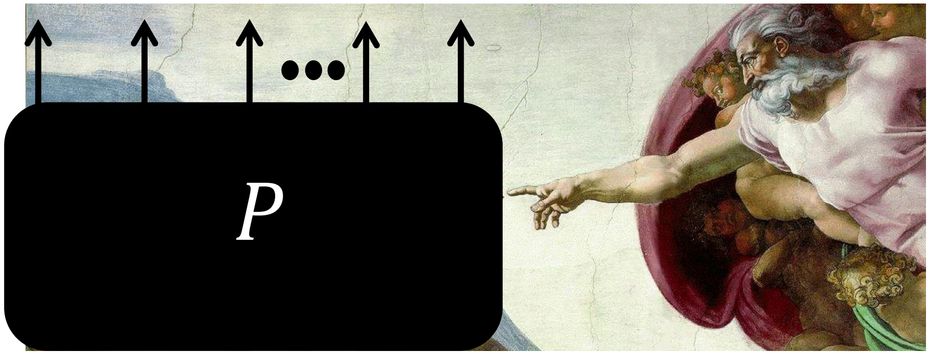
Finding the right connectors can be tricky!



Nonlocality detection

Applications

Heusristic I: quantum correlations are not God-given

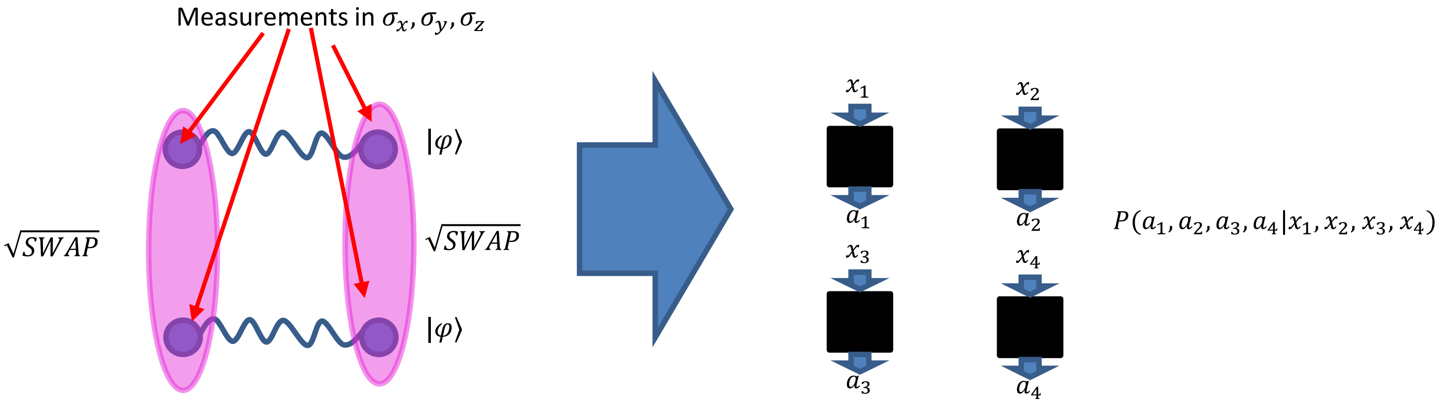


Highly infrequent situation

Nonlocality detection

Applications

Preparation of the box

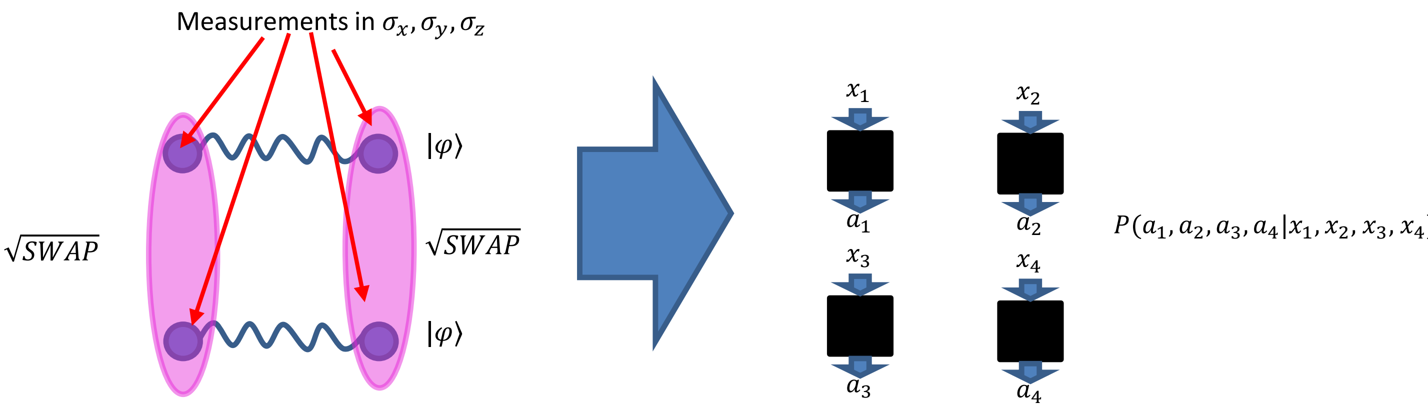


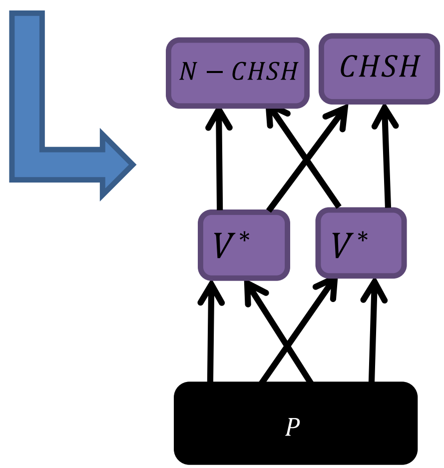
$$\boxed{|\varphi\rangle = (e^{i\pi\sigma_y/4} \otimes \mathbb{I})|\psi^+\rangle}$$

Nonlocality detection

Applications

Preparation of the box





“right” connectors

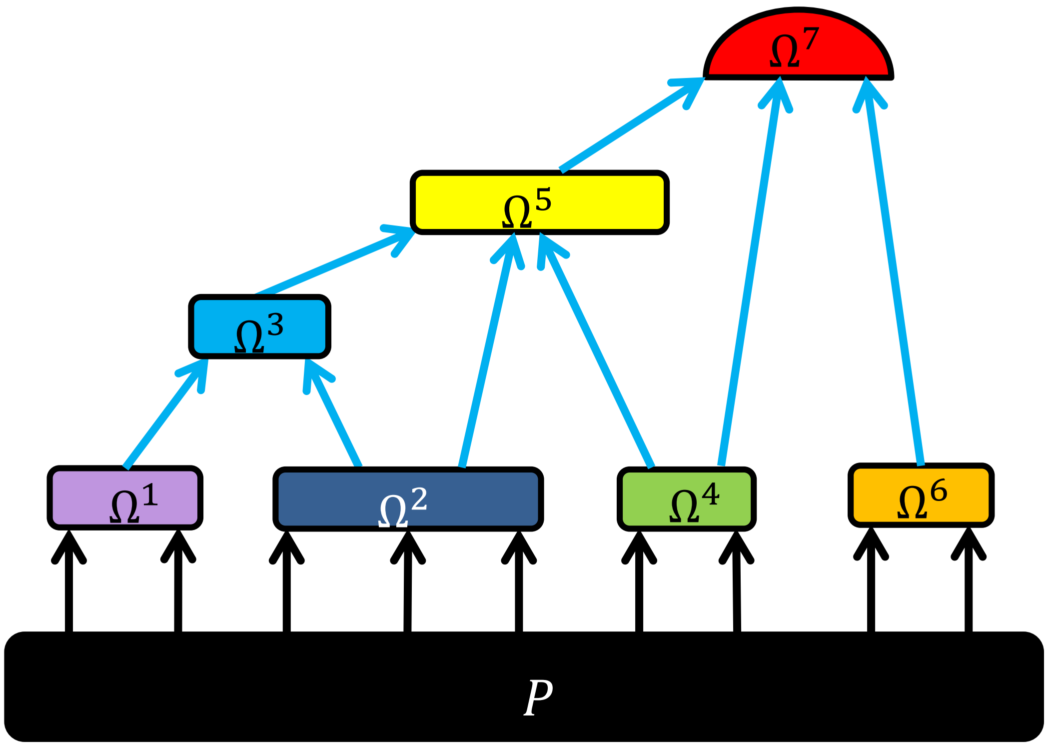
≈ -1.395

$$\boxed{|\varphi\rangle = (e^{i\pi\sigma_y/4} \otimes \mathbb{I})|\psi^+\rangle}$$

Nonlocality detection

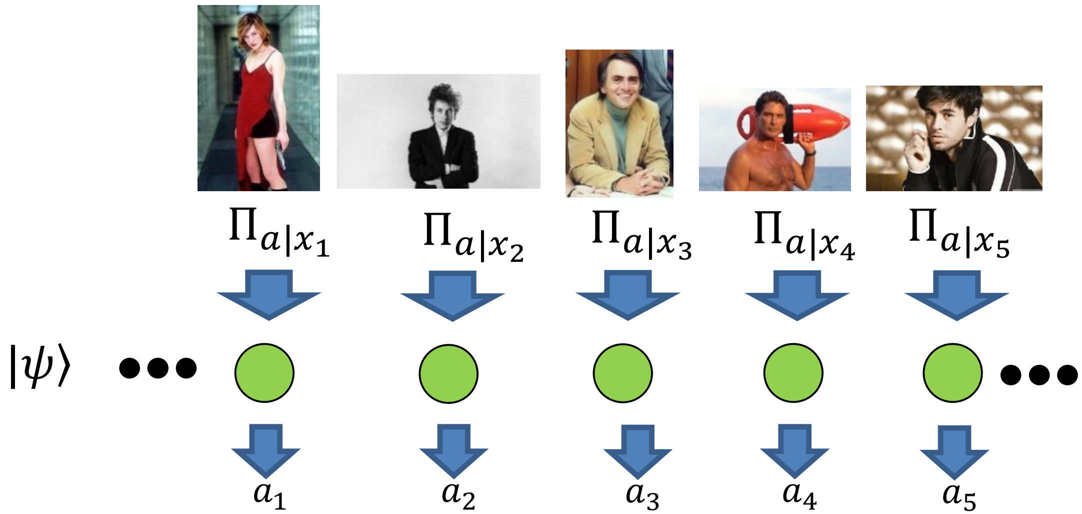
Applications

Heuristic II: use see-saw



Nonlocality detection

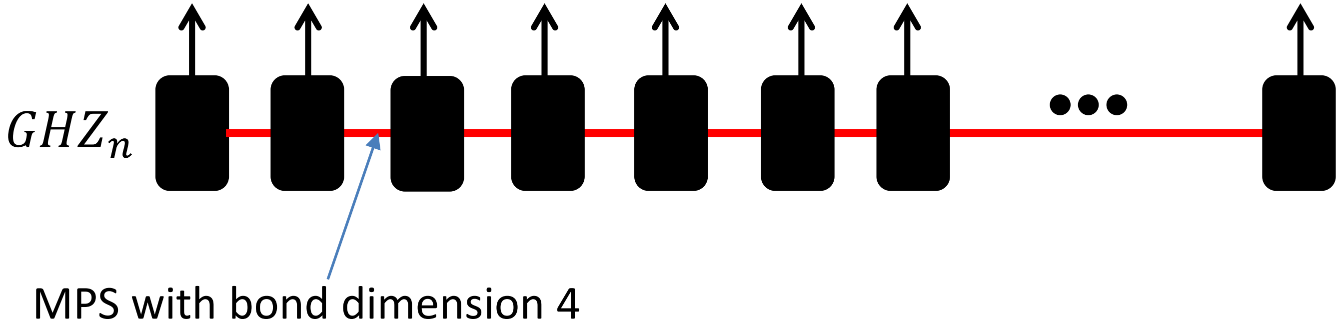
Applications



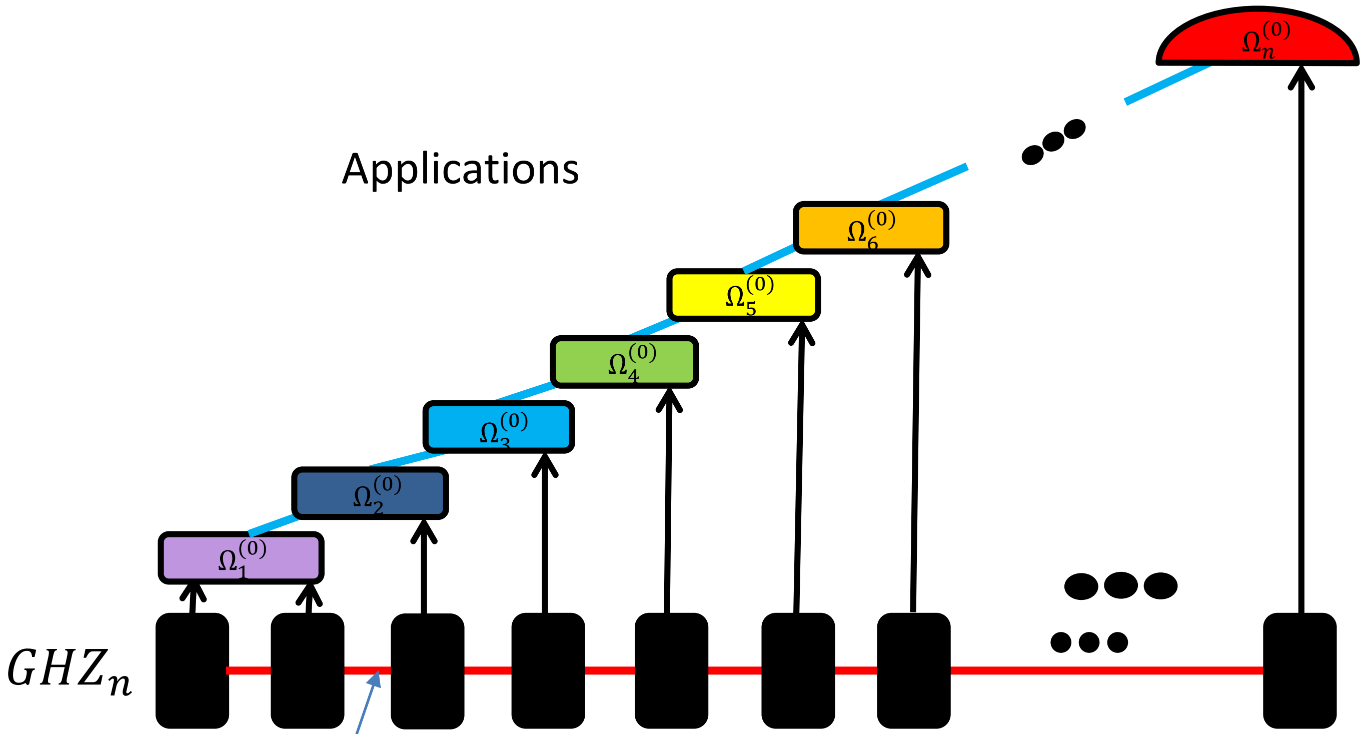
$$\left(|\psi\rangle = |GHZ_n\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle^{\otimes n} + |1\rangle^{\otimes n}\right) \quad \Pi_{a|0} = \frac{\mathbb{I} + (-1)^a \sigma_x}{2} \quad \Pi_{a|1} = \frac{\mathbb{I} + (-1)^a \sigma_y}{2} \right)$$

Nonlocality detection

Applications



Nonlocality detection

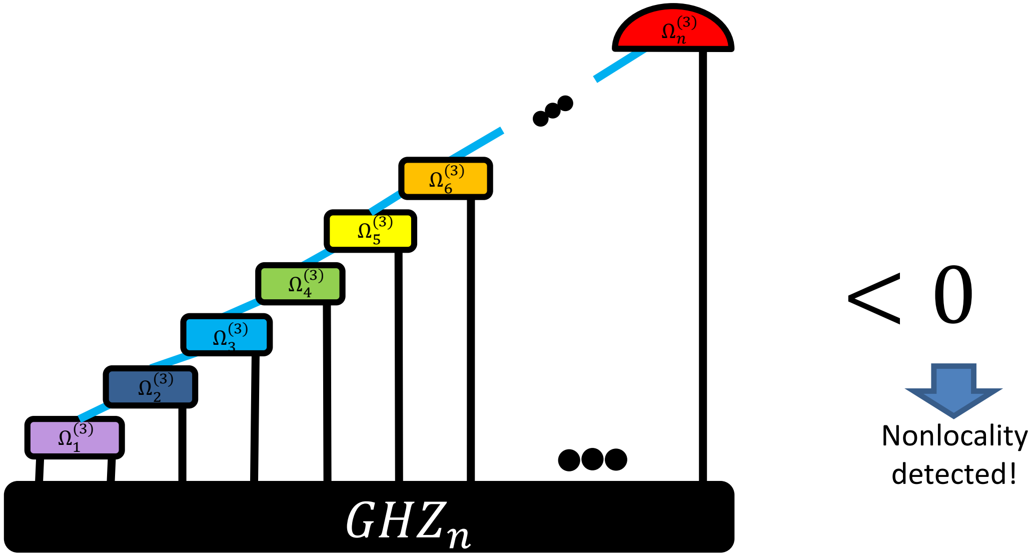


MPS with bond dimension 4

The contraction between the witness and the box can be carried with $O(n)$ operations!!

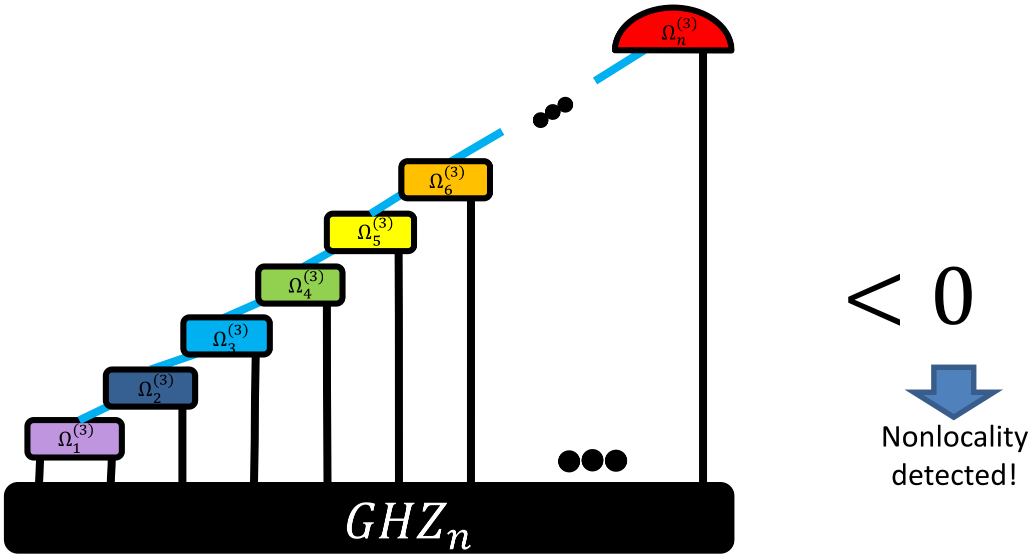
Nonlocality detection

Applications



Nonlocality detection

Applications

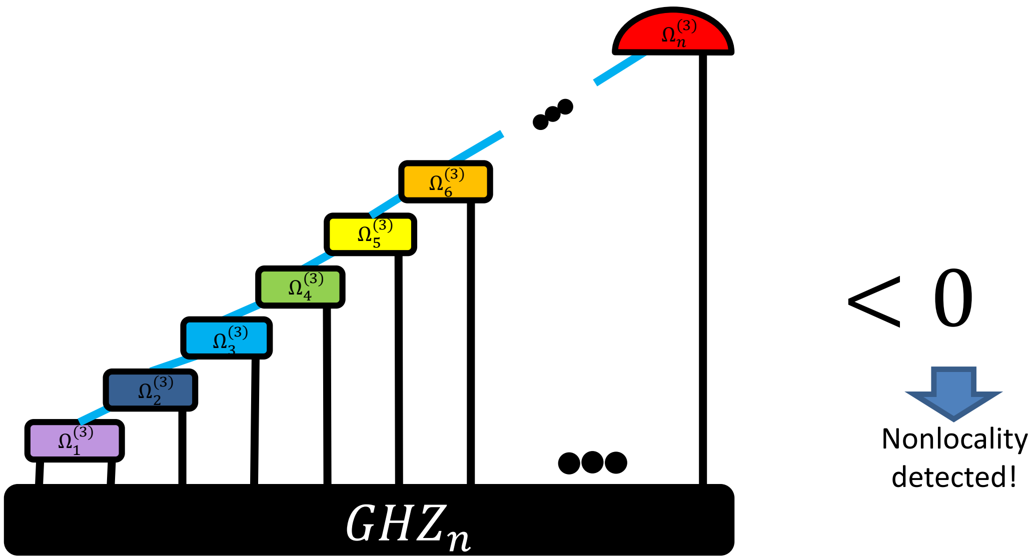


The violation B_n scales exponentially with n.

(note that $0 \leq B_n(P) \leq 1$, for all P, local)

Nonlocality detection

Applications



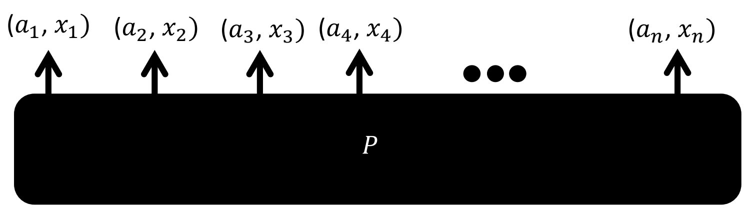
For $n = 200$, the computer detects nonlocality in a few seconds!!

Supra-quantum nonlocality detection

Supra-quantum nonlocality detection

Quantum nonlocality problem

Input:



Output:

$$P(a_1, \dots, a_n | x_1, \dots, x_n) = \langle\psi| \Pi^1_{a_1|x_1} \otimes \cdots \otimes \Pi^n_{a_n|x_n} |\psi\rangle$$ “No”

$$\Pi^k_{a|x} \in B\left(H^k\right), \sum_a \Pi^k_{a|x} = \mathbb{I},$$

$$\Pi^k_{a|x} = \left(\Pi^k_{a|x}\right)^2 = (\Pi^k_{a|x})^\dagger$$

Otherwise “Yes”

Supra-quantum nonlocality detection

GPT: QUANT-world

$(A_1, \dots, A_n, X_1, \dots, X_n)$

State space

$\mathbb{R}^{A_1 \dots A_n X_1 \dots X_n}$

(same as in LOC-world)

Set of states

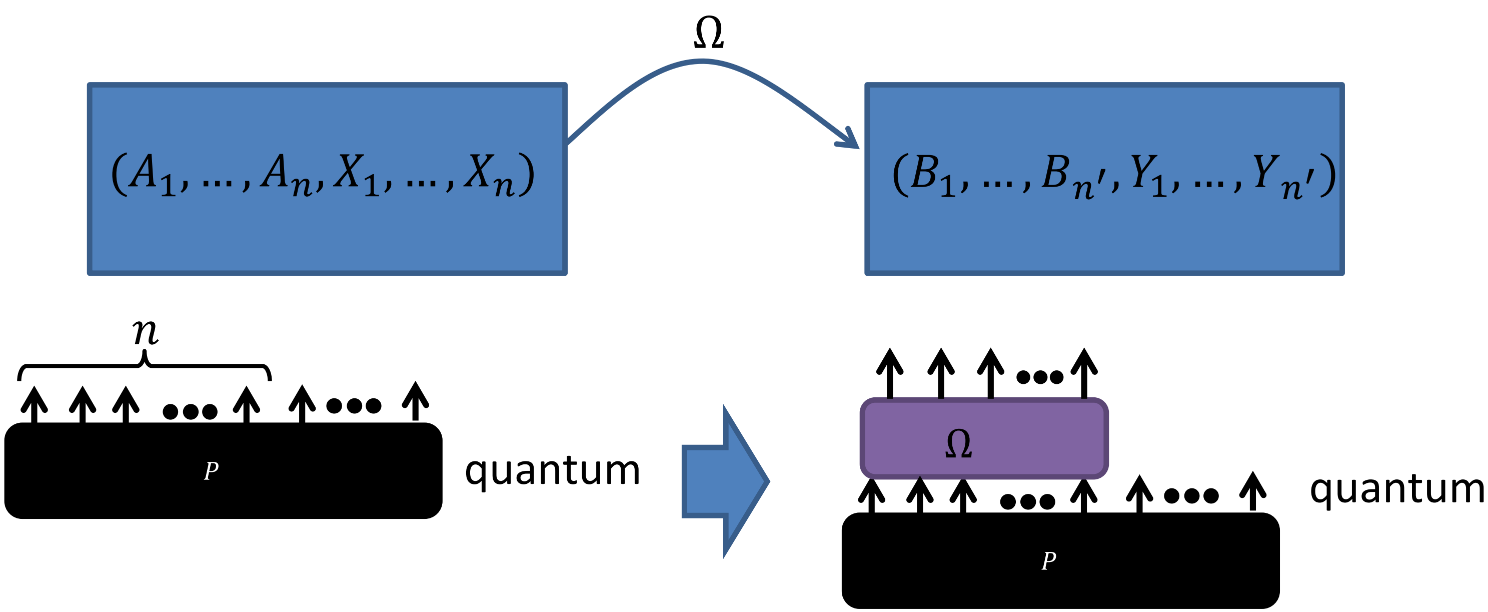
$P(a_1, \dots, a_n | x_1, \dots, x_n)$
$= \langle \psi | \Pi^1_{a_1|x_1} \otimes \cdots \otimes \Pi^n_{a_n|x_n} | \psi \rangle$

(set of quantum behaviors)

Supra-quantum nonlocality detection

GPT: QUANT-world

Operations

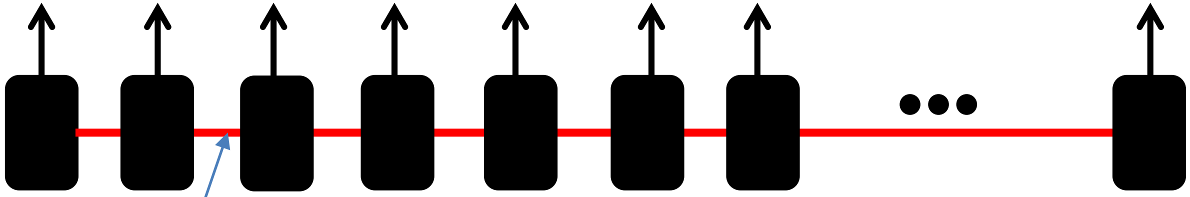


$$N(P) \geq N(\Omega(P))$$

Supra-quantum nonlocality detection

Applications

P

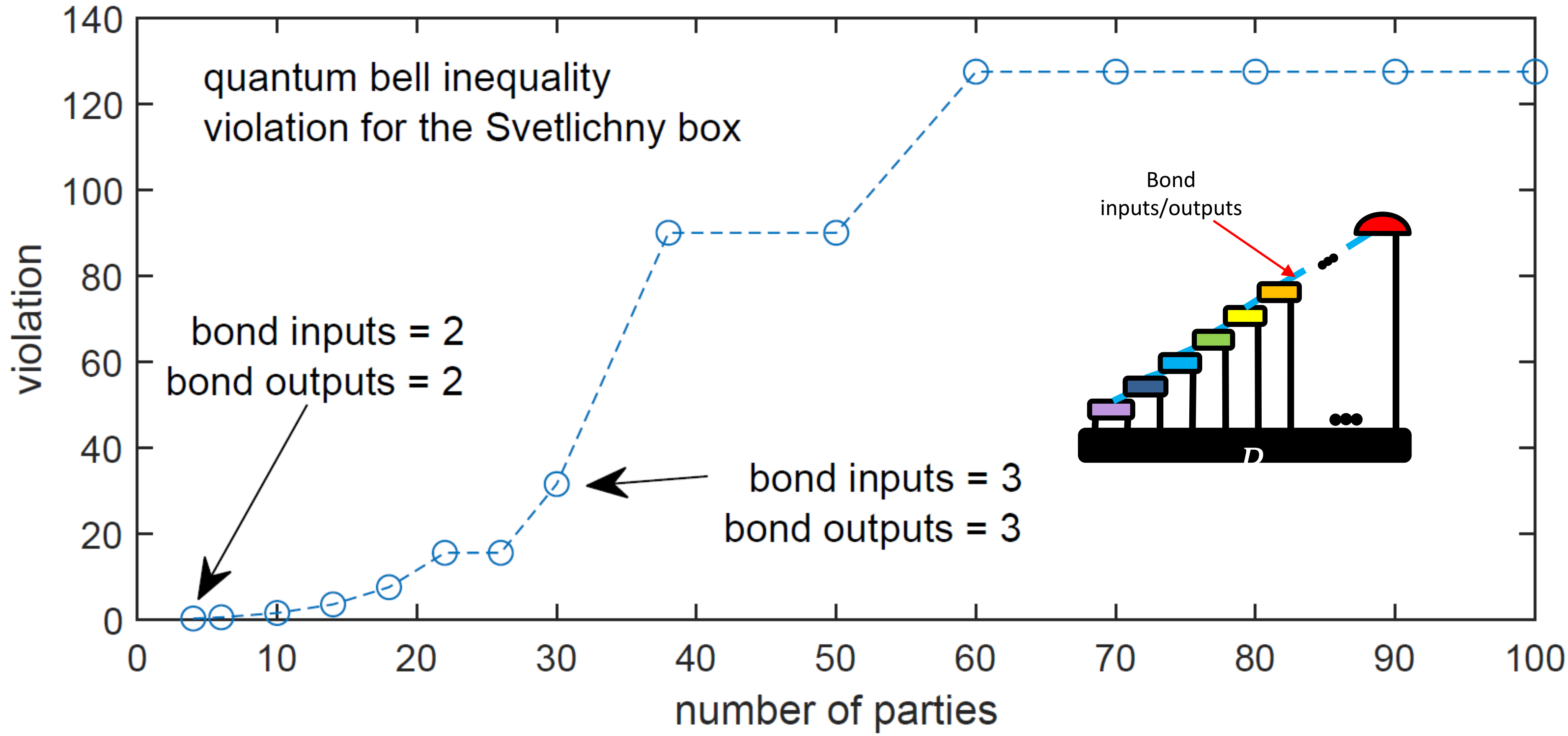


Bond dimension=16

$$P(a_1, \dots, a_n | x_1, \dots, x_n) = \frac{1}{2^{n-1}} \delta(a_1 \oplus \cdots \oplus a_n, f(x))$$

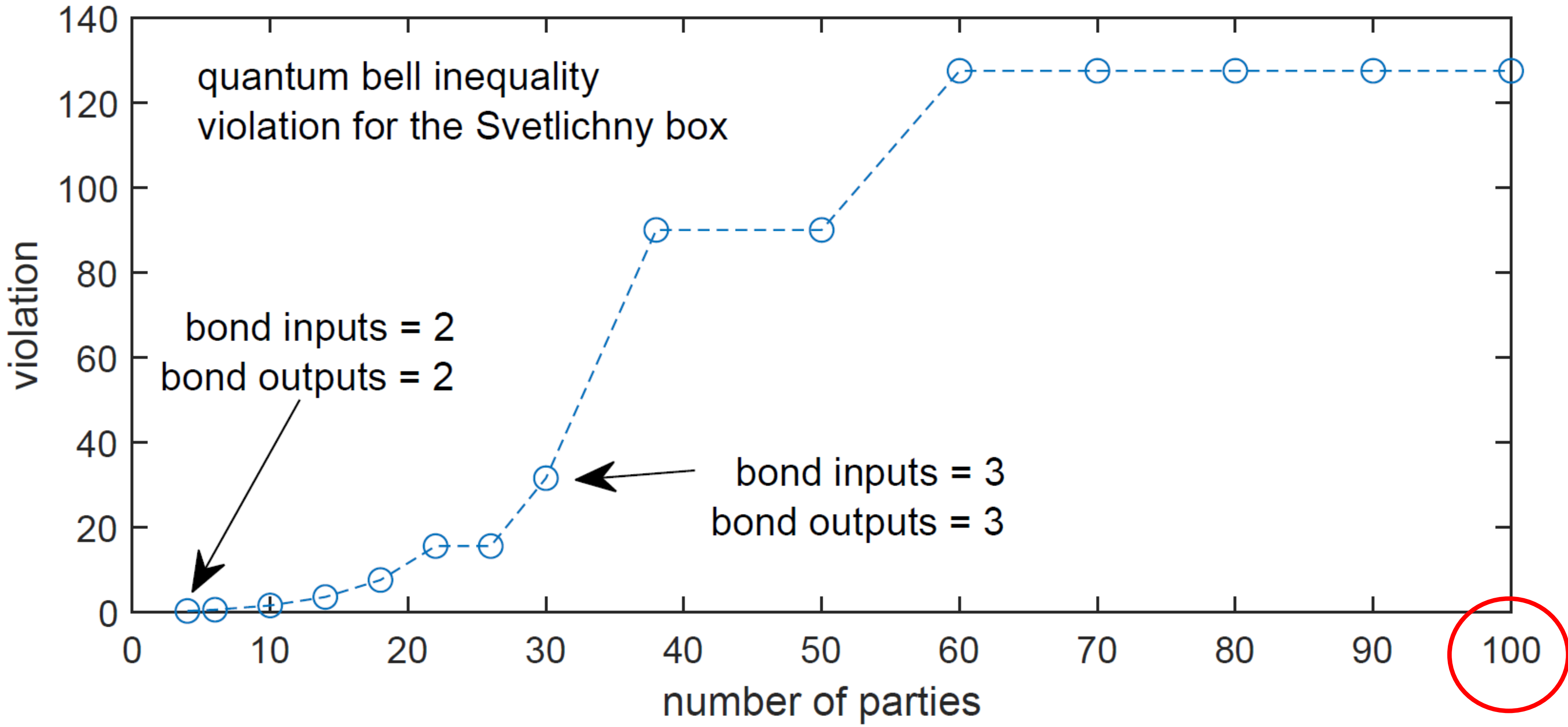
$$f(x) = x_1 x_2 \oplus x_2 x_3 \oplus \cdots \oplus x_n x_1$$

Supra-quantum nonlocality detection



$(0 \leq B_n(P) \leq 1$, for all P, quantum)

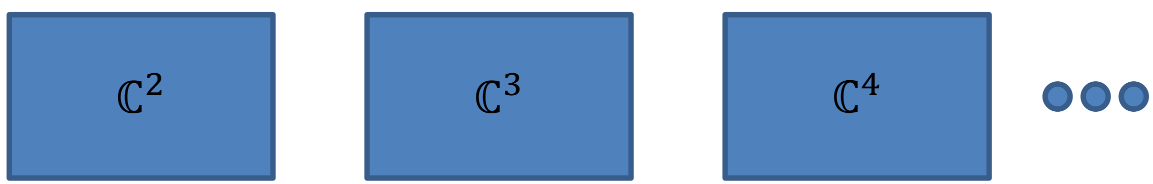
Supra-quantum nonlocality detection



Entanglement detection

Entanglement detection

GPT: SEP-world



“Basic” system types: same as in quantum theory

Entanglement detection

GPT: sep-world

$\mathbb{C}^{(D_1,\dots,D_n)}$

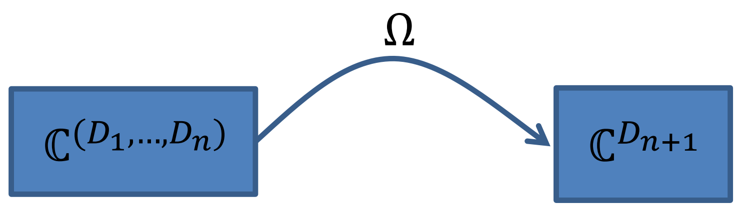
State space $\Rightarrow$ $B(\mathbb{C}^{D_1\dots D_n})$

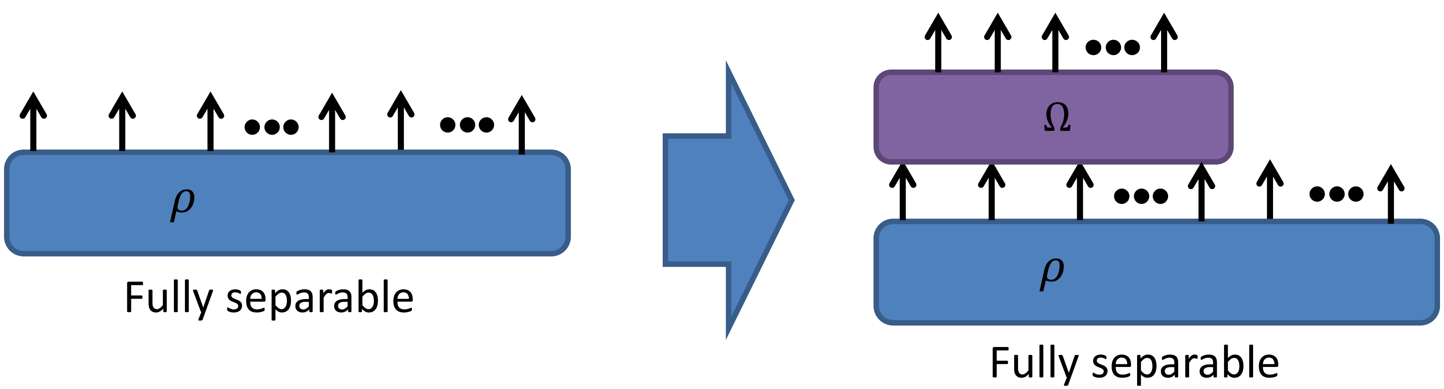
Set of states $\Rightarrow$ $\rho = \sum_i p_i |\psi_i^1\rangle\langle\psi_i^1| \otimes |\psi_i^2\rangle\langle\psi_i^2| \otimes \dots \otimes |\psi_i^n\rangle\langle\psi_i^n|$

(set of quantum fully separable states)

Entanglement detection

Connectors for SEP-world

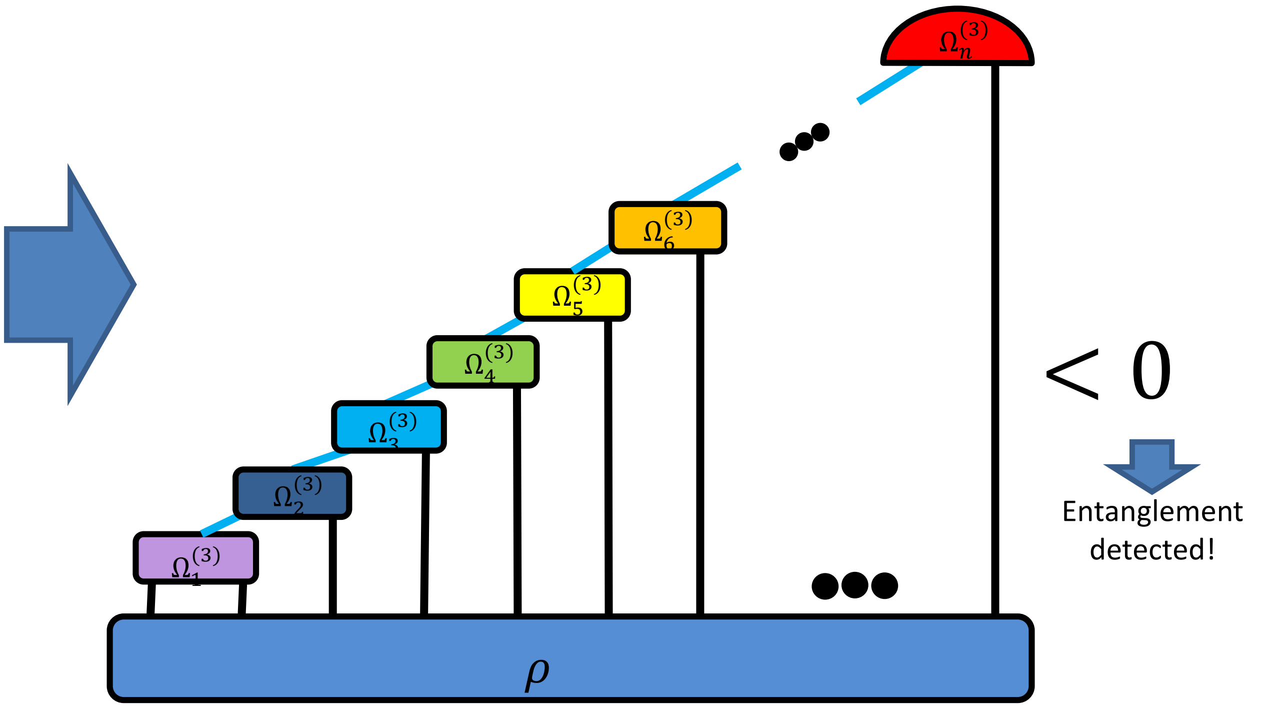




Entanglement detection

Applications

PPT entangled states from n-qubit UPBs, $n = 3, 4, 5, 6$



N. Johnston, J. Phys. A: Math. Theor. 47:424034 (2014).

Applications

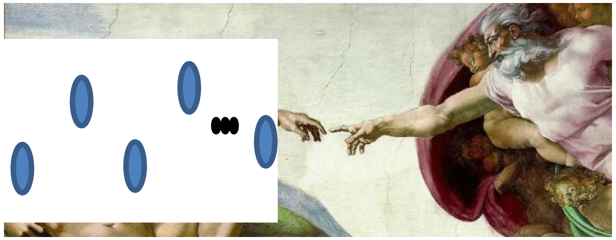
Entanglement detection in condensed matter states?

For mixed states, we get mixed results

Applications

Entanglement detection in condensed matter states?

Quantum states are not God-given

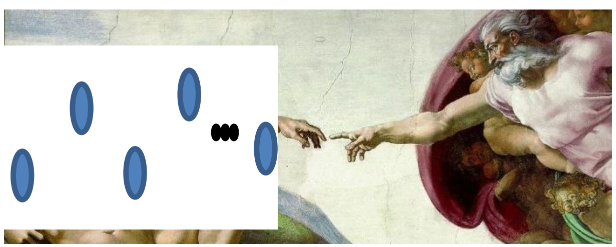


Non-trivial witnesses for finitely correlated states

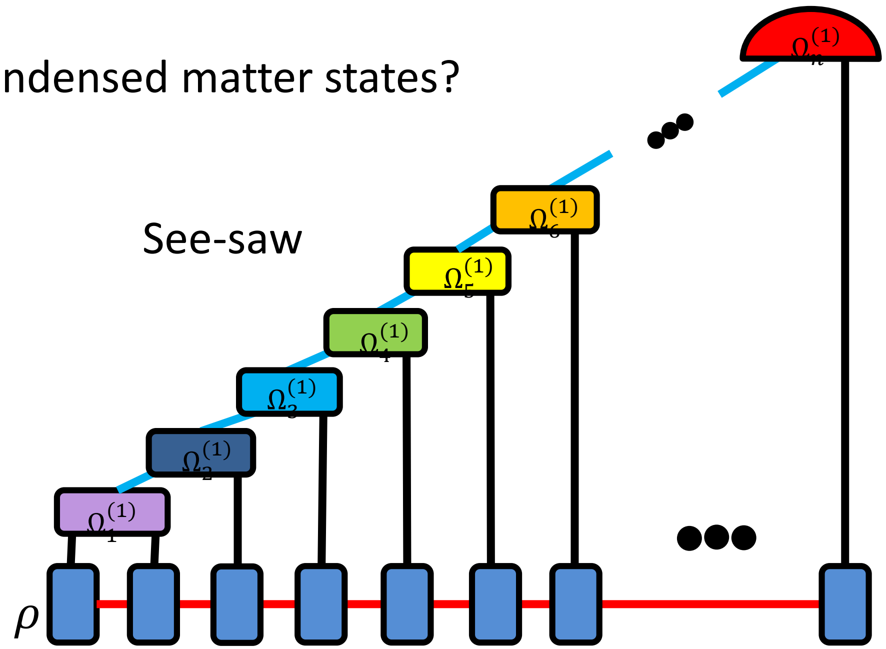
Applications

Entanglement detection in condensed matter states?

Quantum states are not God-given



Non-trivial witnesses for
finitely correlated states



For the termal states of “famous” Hamiltonians (e.g.: Heisenberg), we struggle to detect PPT entanglement

G. Toth, C. Knapp, O. Gühne and H. J. Briegel, Phys. Rev. A 79, 042334 (2009).

A. C. Doherty, P. A. Parrilo, and F. M. Spedalieri, Phys. Rev. A, Vol. 71, 032333 (2005).

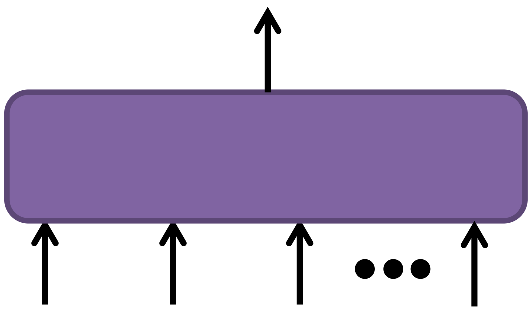
Take-home messages

Take-home messages

Bell inequalities

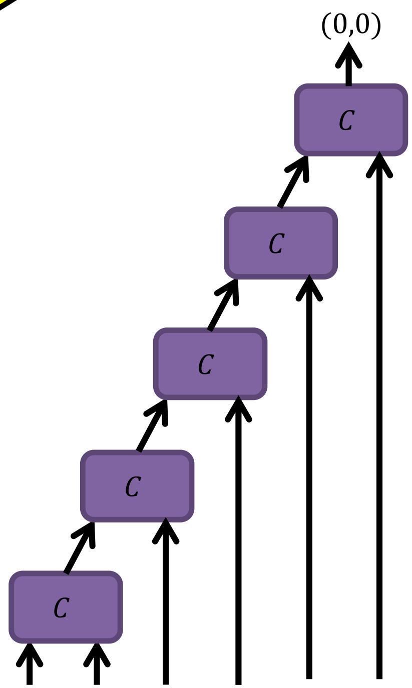
Quantum Bell inequalities

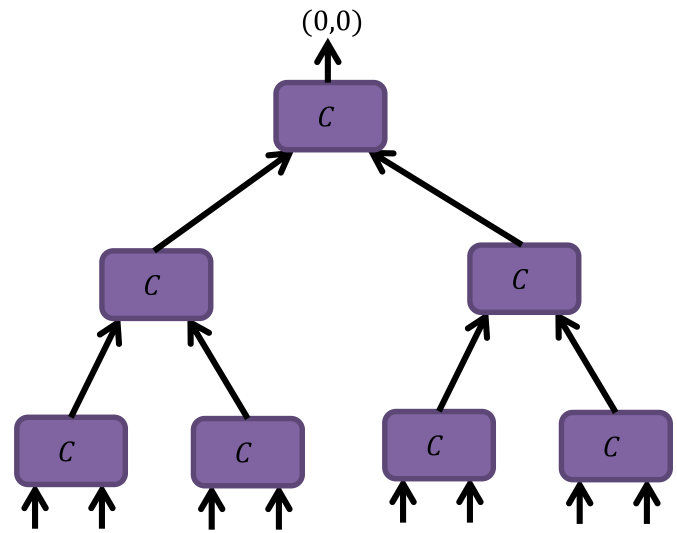
Entanglement witnesses



k-to-1 connectors

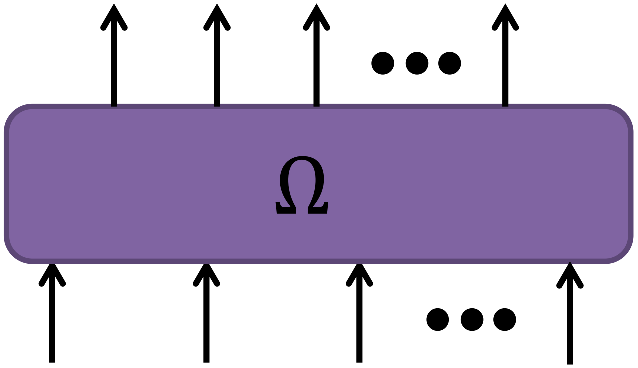
Take-home messages





You can connect them to create new witnesses/Bell inequalities!!

Take-home messages



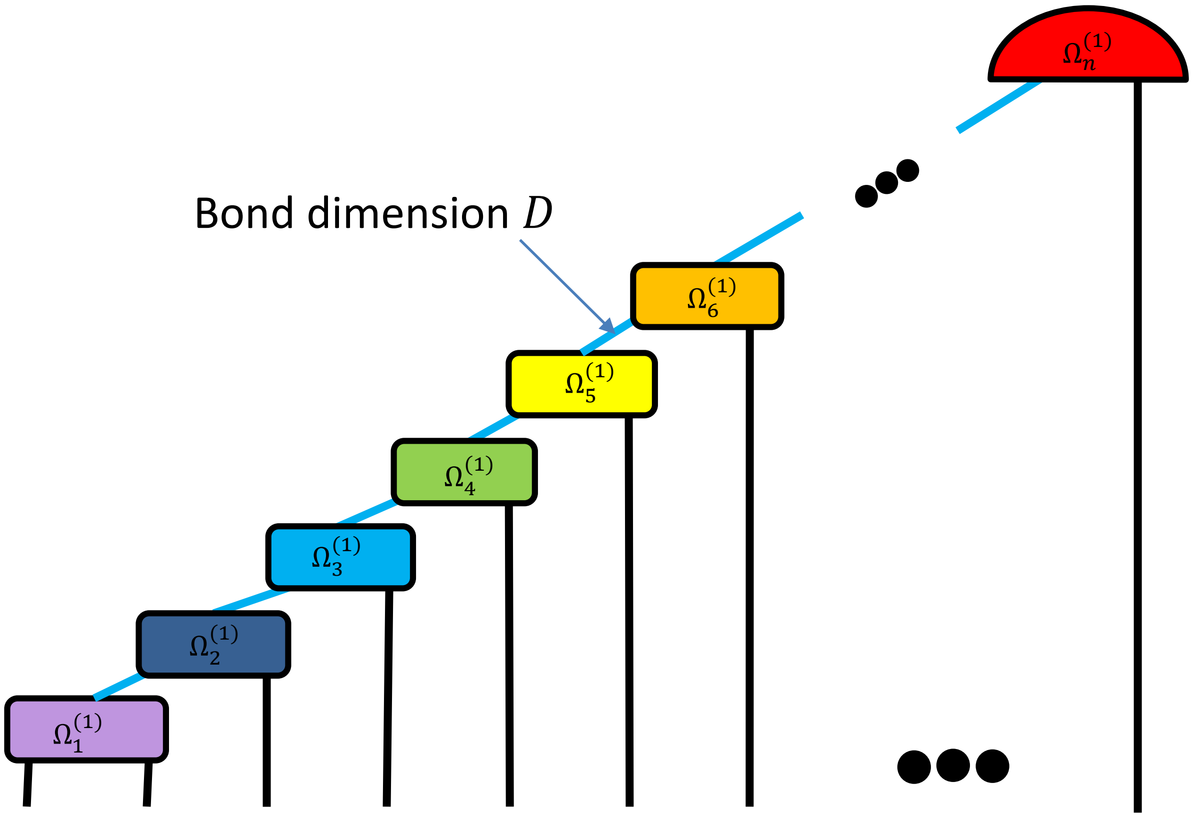
k-to-k' connectors

Proven utility in nonlocality detection
I'm sure they can be applied to detect entanglement as well

Open questions

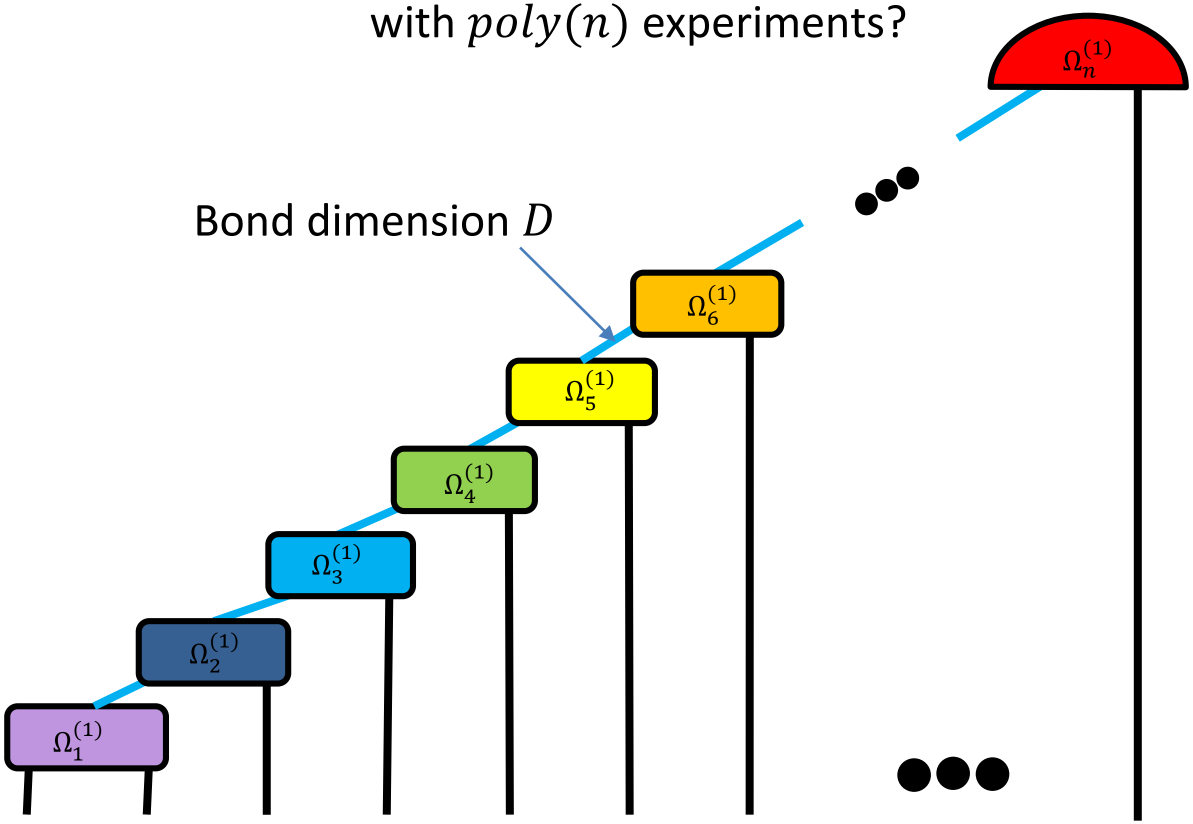
How far can one get with m $-$to$-m$' connectors?

How far can one get with m –to–m' connectors?



E.g.: can one generate all extreme entanglement witnesses just by taking D large enough?

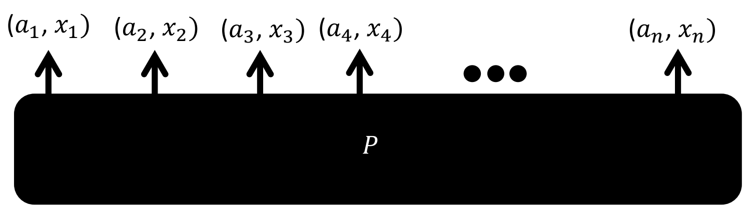
Can one estimate the resulting entanglement witnesses with $poly(n)$ experiments?



Other applications of connector “theory”?

Dimension witnesses

Input:



Output:

$$P(a_1, \dots, a_n | x_1, \dots, x_n) = \langle \psi | \Pi^1_{a_1|x_1} \otimes \cdots \otimes \Pi^n_{a_n|x_n} | \psi \rangle$$

“No”

$$\Pi^k_{a|x} \in B(\mathbb{C}^D), \sum_a \Pi^k_{a|x} = \mathbb{I},$$

$$\Pi^k_{a|x} = \left(\Pi^k_{a|x}\right)^2 = (\Pi^k_{a|x})^\dagger$$

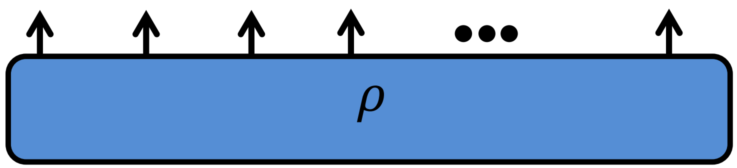
Otherwise

“Yes”

Other applications of connector "theory"?

Bond dimension witnesses

Input:



Output:

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i| \,,$$

"No"

$\{|\psi_j\rangle\}$, MPSs of bond dimension $\leq D$

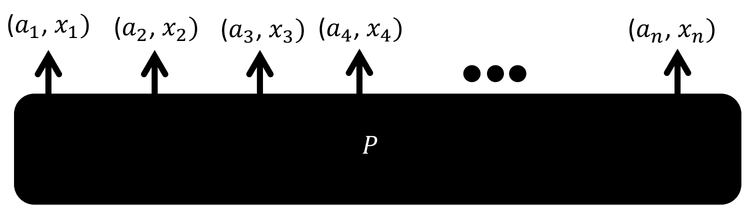
otherwise "Yes"

MN and T. Vértesi, Quantum 2, 50 (2018) .

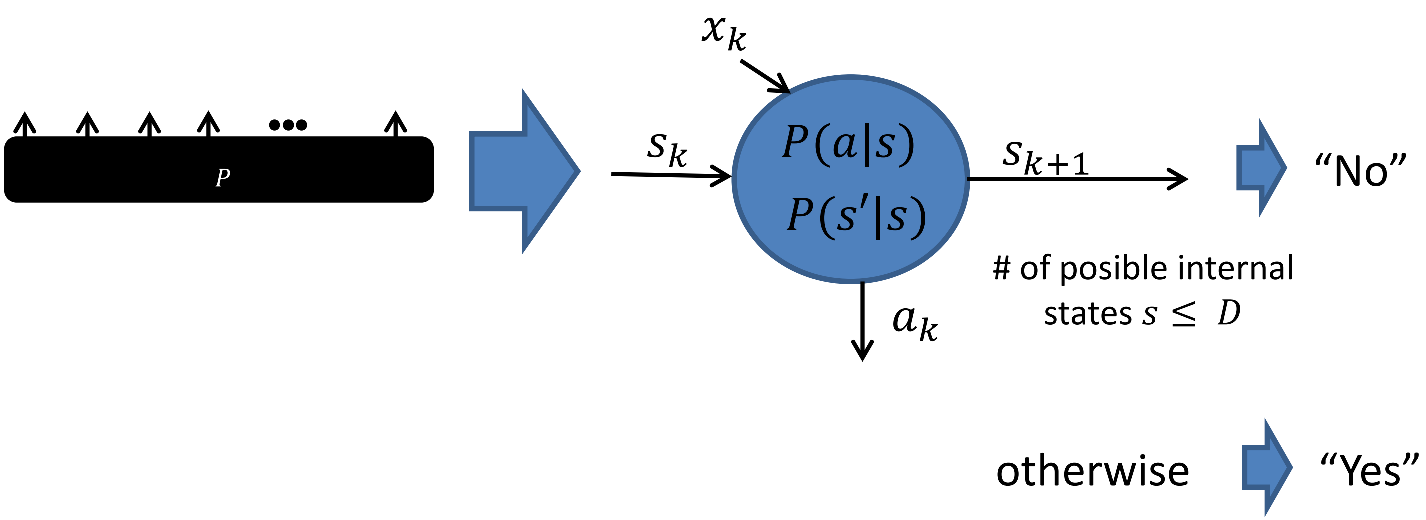
Other applications of connector "theory"?

Number of states of a probabilistic finite-state machine

Input:



Output:



THE END

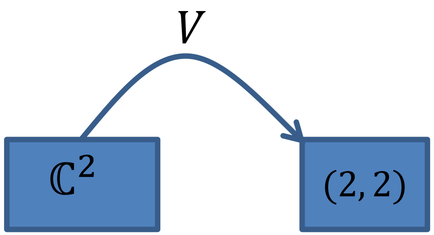
Motivating example

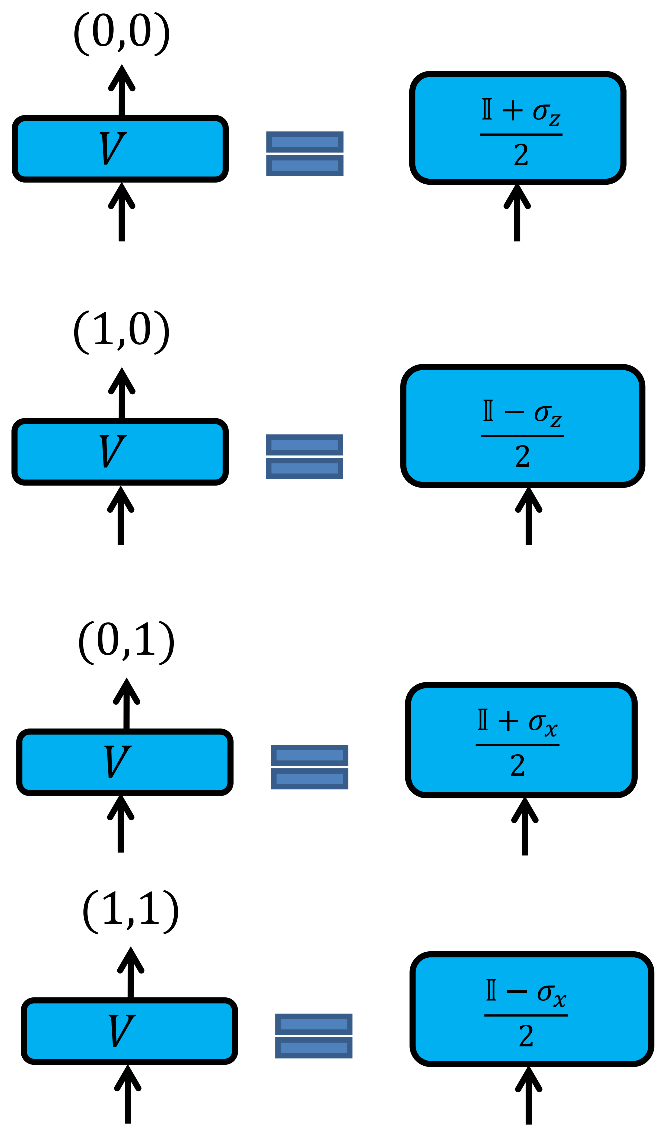
GPT: STEER-world

$\mathbb{C}^D$ $\quad$ (A, X)

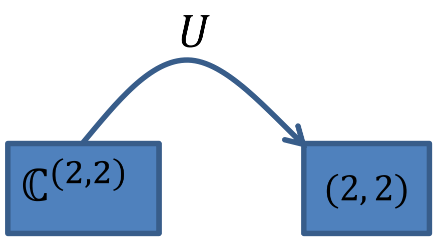
“Basic” system types: same as SEP-world and LOC-world

Entanglement detection via hybrid GPTs





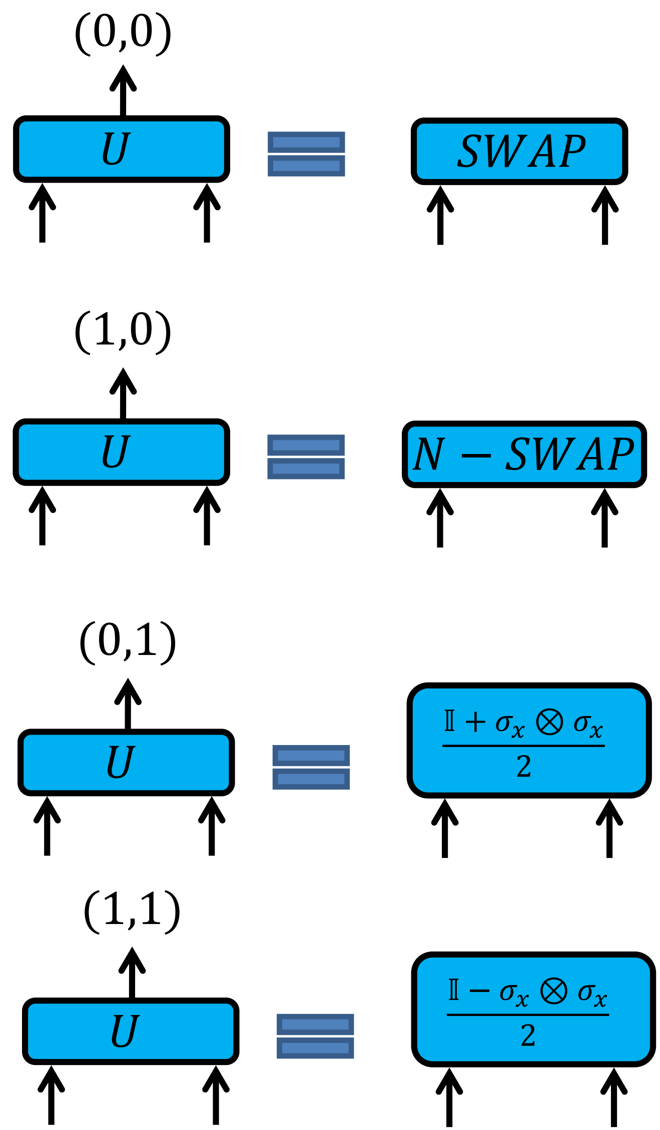
Entanglement detection via hybrid GPTs



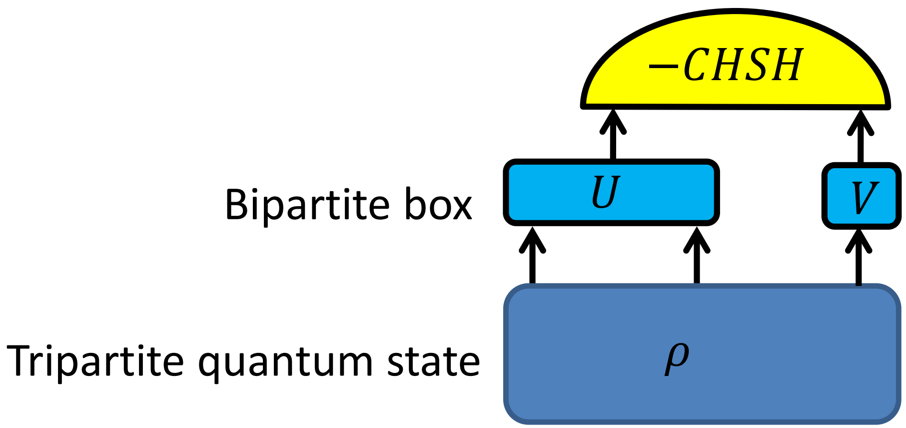
Two-qubit
separable state

$$0 \leq tr(\rho SWAP) \leq 1$$

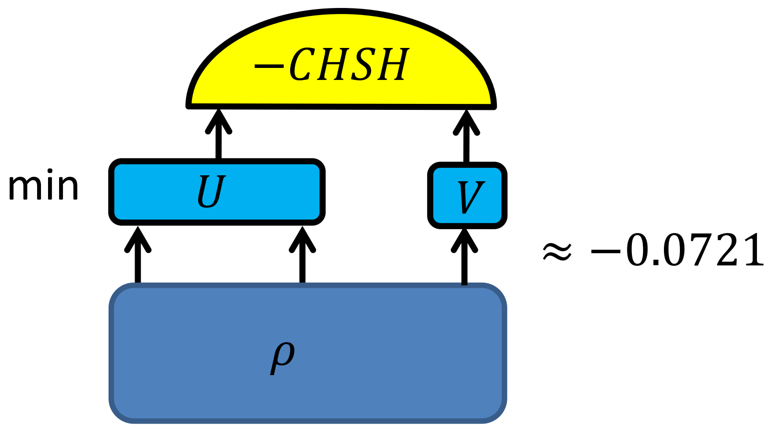
ρ, separable



Entanglement detection via hybrid GPTs

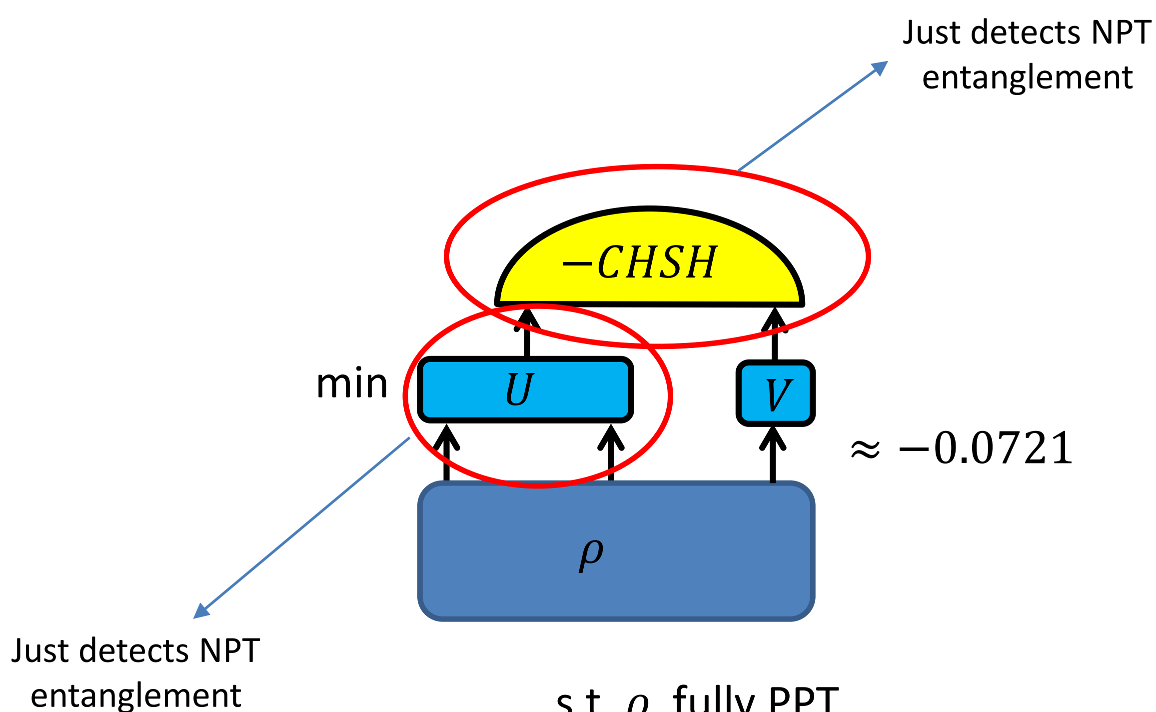


Entanglement detection via hybrid GPTs



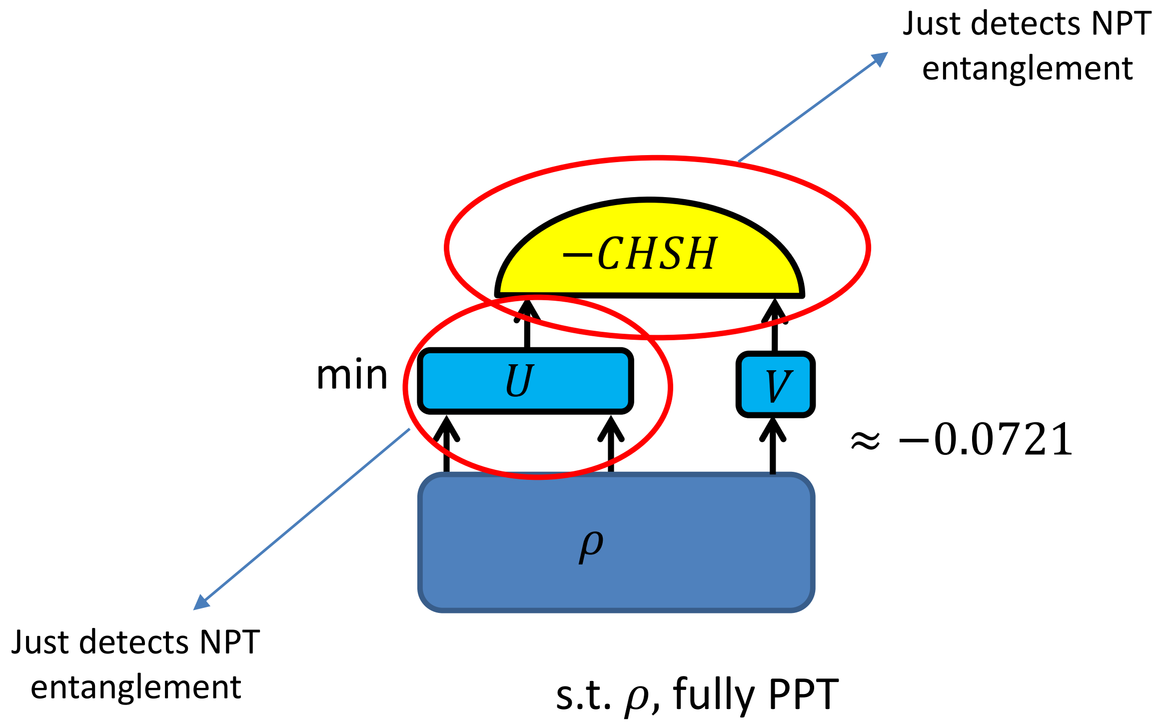
s.t. ρ, fully PPT

Entanglement detection via hybrid GPTs



s.t. ρ, fully PPT

Entanglement detection via hybrid GPTs



The composition of known stuff has created something new!