MY
REVIEW
DYNAMICS OF
QUANTUM ENTANGLEMENT
FROM CREATION TO ANNIHILATION

ENTANGLEMENT EVOLUTION

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ENTANGLED WITH EVOLUTION OF RESEARCH SPECIES

1 HOMO DYNAMIS LOCCALUS
2 HOMO DYNAMIS CALCULUS
3 HOMO DYNAMIS BREAKENS
4 HOMO DYNAMIS SUDDENTROCUS
5 HOMO DYNAMIS ANNIHILIS

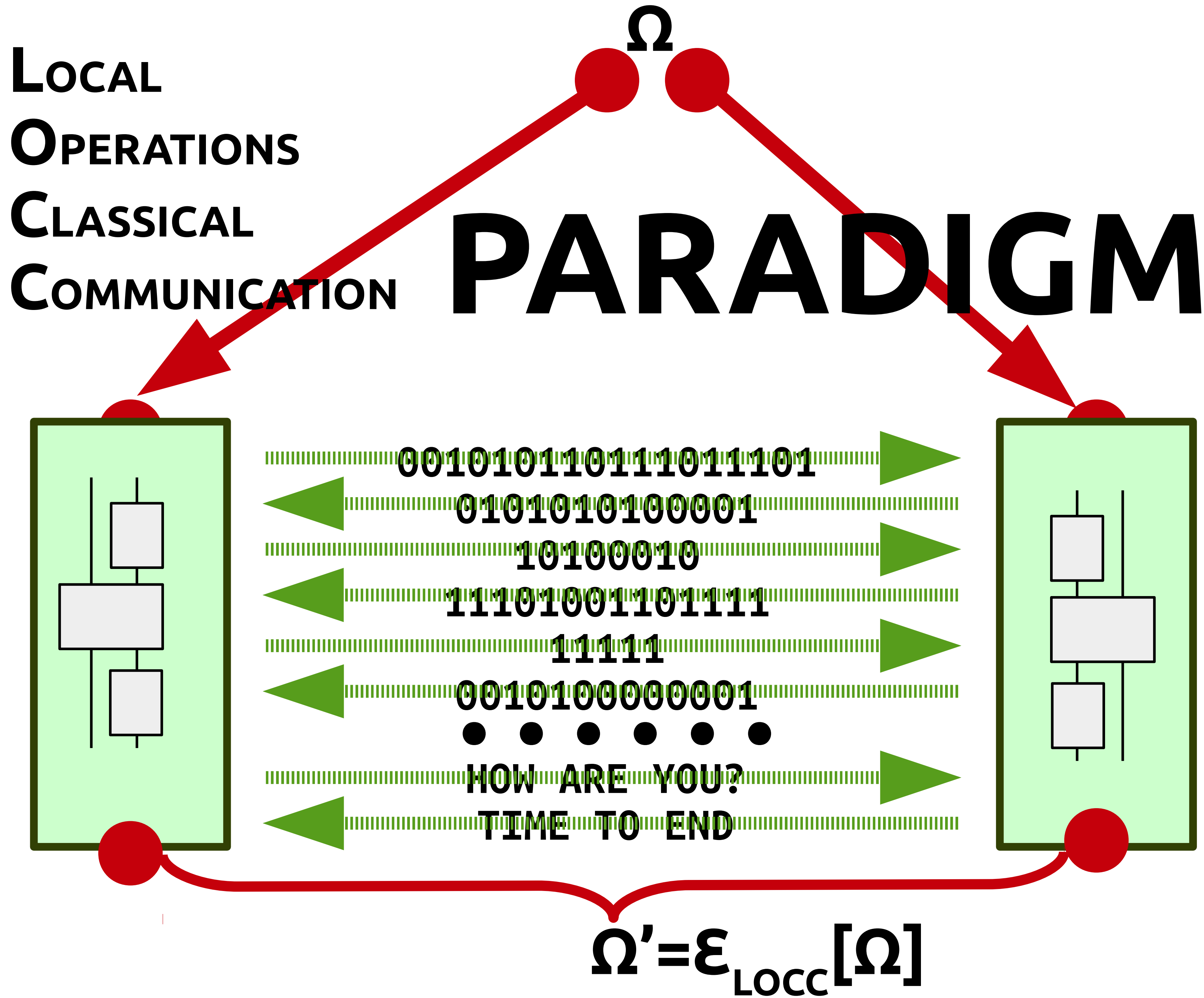
! NO NATURAL SELECTION !

HOMO DYNAMIS LOCCALUS

- development of primitive (but qute) tools

  → nonlocal advantage over his Neilbohrs

- discovery of evolutions beyond L(Q)OCC



LOCAL
OPERATIONS
CLASSICAL
COMMUNICATION
PARADIGM
Ω
00101011011101111101
0101010100001
10100010
1110100110111
11111
001010000001
HOW ARE YOU?
TIME TO END
Ω'=Ɛ_LOCC[Ω]

HOW THE DYNAMICS DEFINE ENTANGLEMENT?

OPERATIONAL DEFINITION

LOCC ORDERING

$$\boldsymbol{\Omega' < \Omega \quad \text{if} \quad \Omega' = \mathcal{E}_{LOCC}[\Omega]}$$

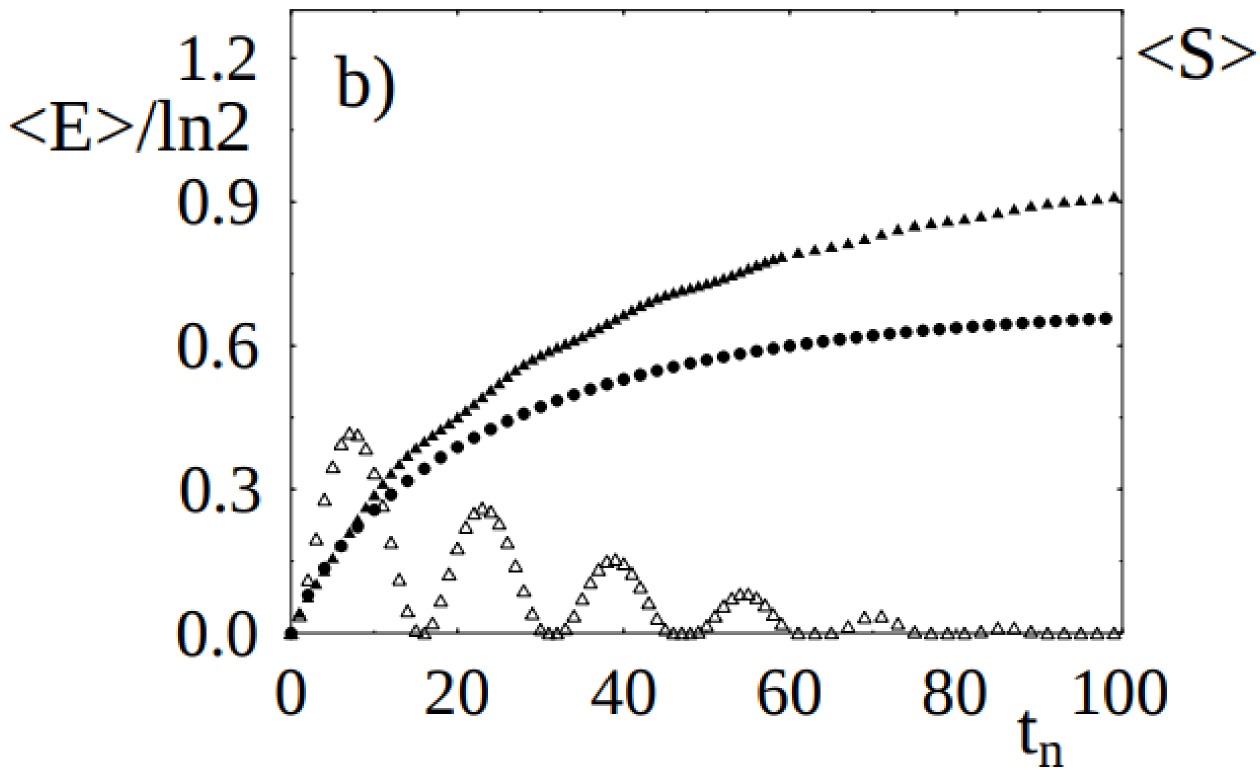
$[\Omega_{min}]$ = separable states

$\rightarrow S(H)\backslash[\Omega_{min}]$ = **entanglement**

$[\Omega_{max}]$ = maximally entangled states* does not hold in multipartite case

$\rightarrow \boldsymbol{\Omega' < \Omega_{max}}$ for all $\boldsymbol{\Omega'}$

HOMO DYNAMIS CALCULUS
→ learns to calculate (and shut up)



Karol Życzkowski, Paweł Horodecki, Michał Horodecki, and Ryszard Horodecki, Dynamics of quantum entanglement, Phys. Rev. A 65, 012101 (2001)

Ryszard Horodecki, Paweł Horodecki, Michał Horodecki, and Karol Horodecki, Quantum entanglement, Rev. Mod. Phys. 81, 865 (2009)

ENTANGLEMENT MEASURES

1. $E = 0$ if(f) state is separable
2. E is non-increasing under LOCC
3. $E \leq E(\Omega_{max}) = E(\Omega_{+})$

...

"cannonical" maximally entangled state

$$\Omega_{+} = 1/\sqrt{d} \sum_{j,k} |jk\rangle\langle jk|$$

ENTANGLEMENT MEASURES
LOCC ordering under local channels

Preservation of LOCC ordering?

$E(\Omega_1) < E(\Omega_2)$ **? then ?**

$E((\mathcal{E}_A \otimes \mathcal{E}_B)[\Omega_1]) \leq E((\mathcal{E}_A \otimes \mathcal{E}_B)[\Omega_2])$

ENTANGLEMENT MEASURES
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IMPOSSIBLE

neither if Ω_2 is maximally entangled

$E((\mathcal{E}_A \otimes \mathcal{E}_B)[\Omega_+]) < E((\mathcal{E}_A \otimes \mathcal{E}_B)[\Omega_1])$

Mário Ziman and Vladimír Bužek, Entanglement-induced state ordering under local operations, Phys. Rev. A 73, 012312 (2006)

HOW DO THE MEASURES EVOLVE? under local channels

$$C((I\otimes\mathcal{E})[\Omega]) \leq C(\Omega)\, C((I\otimes\mathcal{E})[\Omega_+])$$
$$\leq C(\Omega)\, C(\Omega_{\mathcal{E}})$$

Choi-Jamiolkowski operator

$$C((\mathcal{E}_A\otimes\mathcal{E}_B)[\Omega]) \leq C(\Omega)\, C(\Omega_{\mathcal{E}_A})\, C(\Omega_{\mathcal{E}_B})$$

C is concurrence

Chang-shui Yu, X. X. Yi, and He-shan Song, Evolution of entanglement for quantum mixed states, Phys. Rev. A 78, 062330 (2008)
Markus Tiersch, Fernando de Melo, Thomas Konrad, Andreas Buchleitner, Equation of motion for entanglement, Quant. Inf. Proc. 8, 523 (2009)

ENTANGLING POWER of unitary gates

$$e_p(U) = \langle S_{lin}(U(P_\psi \otimes P_\varphi)U^*) \rangle_{\psi\otimes\varphi}$$

$$S_{lin}(P) = 1 - tr_1[[tr_2 P]^2]$$

$d_1 \leq d_2$

$$0 \underset{U_{local},\, U_{swap}}{\leq} e_p(U) \underset{U_{ctrl}}{\leq} \frac{d_2 - d_2/d_1}{d_2+1} \overset{d_1=d_2=d}{=} \frac{d-1}{d+1}$$

U_{local}, U_{swap} $\quad$ U_{ctrl} $\quad$ *(not for 2x2, 2x3 open for 6x6, etc)

Paolo Zanardi, Christof Zalka, and Lara Faoro, Entangling power of quantum evolutions, Phys. Rev. A 62, 030301(R) (2000)

ENTANGLING POWER of unitary gates

entangled Ψ^{in}

$$e^{\uparrow}_{p}(U) = \sup_{\psi^{in}} [E(\psi^{out})-E(\psi^{in})]$$

$$e^{\downarrow}_{p}(U) = \sup_{\psi^{in}} [E(\psi^{in})-E(\psi^{out})]$$

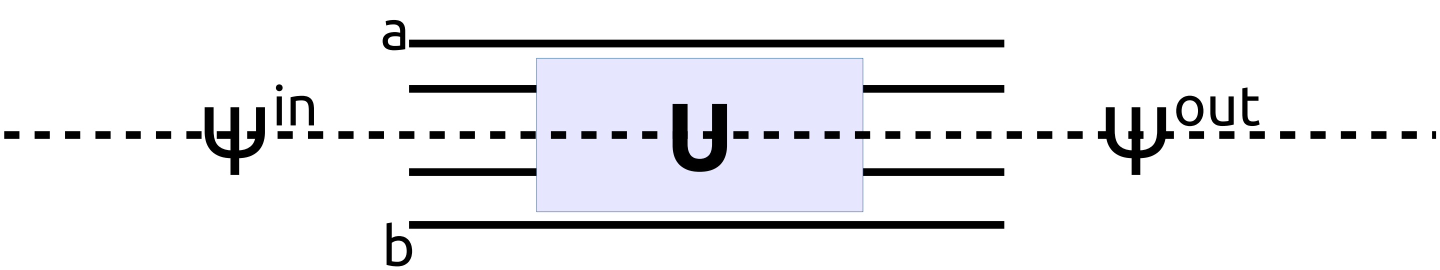
$$e^{\uparrow}_{p}(U) \neq^{*} e^{\downarrow}_{p}(U) = e^{\uparrow}_{p}(U^{*})$$

*[equality holds for 2x2]

C. H. Bennett, A. W. Harrow, D. W. Leung, and J. A. Smolin, On the capacities of bipartite Hamiltonians and unitary gates, IEEE Trans. Inf. Theory 49, 1895 (2003)
M. S. Leifer, L. Henderson, and N. Linden, Optimal entanglement generation from quantum operations, Phys. Rev. A 67, 012306 (2003)
Noah Linden, John A. Smolin, and Andreas Winter, Entangling and Disentangling Power of Unitary Transformations Are Not Equal, Phys. Rev. Lett. 103, 030501 (2009)
Lin Chen and Li Yu, Entangling and assisted entangling power of bipartite unitary operations, Phys. Rev. A 94, 022307 (2016)

ANCILLA-ASSISTED ENTANGLING POWER of unitary gates

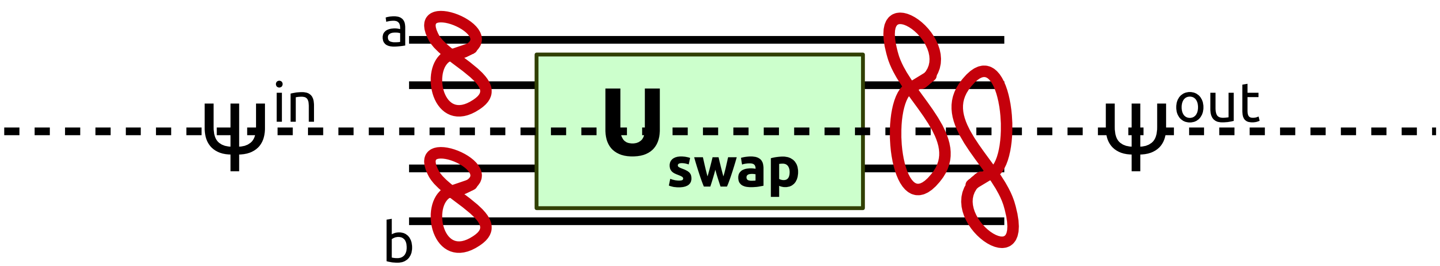
$$\boxed{\mathbf{E}^{\uparrow}_{p}(U) = \sup_{a,b}\, e^{\uparrow}_{p}(I_a \otimes U \otimes I_b)}$$



$$0 \le e^{\uparrow}_{p} \le \mathbf{E}^{\uparrow}_{p} \le \log_2 d_1 d_2$$
$$\le \log_2 d_{min}$$

ANCILLA-ASSISTED ENTANGLING POWER of unitary gates

$$\mathbf{E}^{\uparrow}_{p}(U) = \sup_{a,b}\, e^{\uparrow}_{p}(I_a \otimes U \otimes I_b)$$



$$0 = e^{\uparrow}_{p}(U_{swap}) < \mathbf{E}^{\uparrow}_{p}(U_{swap}) = 2 \log_2 d$$

! SWAP is NONLOCAL !

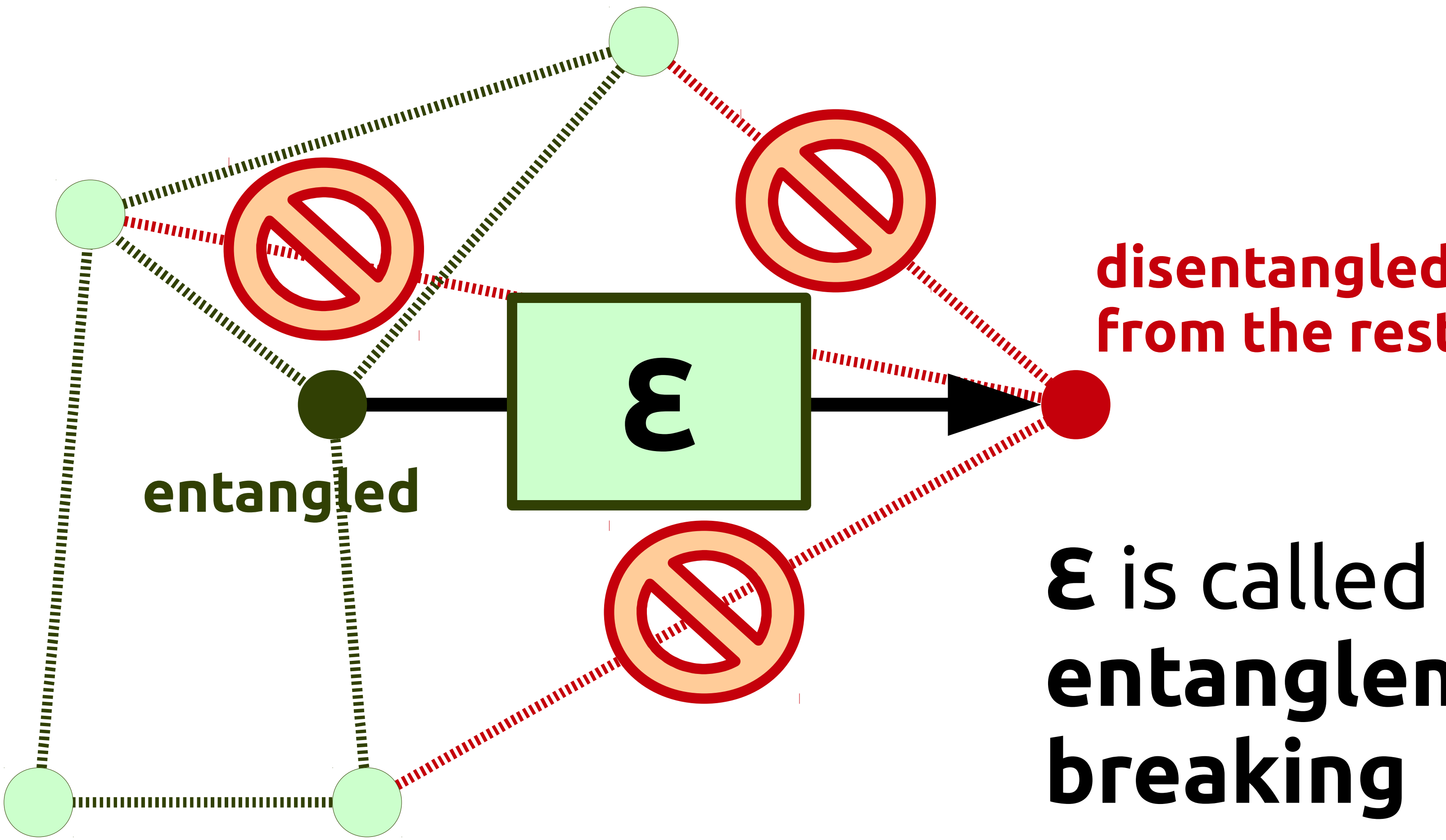
HOMO DYNAMIS BREAKENS
→ master the art of (local) disentangling

Mary Beth Ruskai, Qubit Entanglement Breaking Channels, Reviews in Mathematical Physics 15, 643-662 (2003)

Michael Horodecki, Peter W. Shor, Mary Beth Ruskai, General Entanglement Breaking Channels, Rev. Math. Phys 15, 629--641 (2003)

A. S. Holevo, Entanglement-breaking channels in infinite dimensions, Problems of Information Transmission 44:3 (2008) 3-18

ENTANGLEMENT BREAKING CHANNELS
disentangling system locally



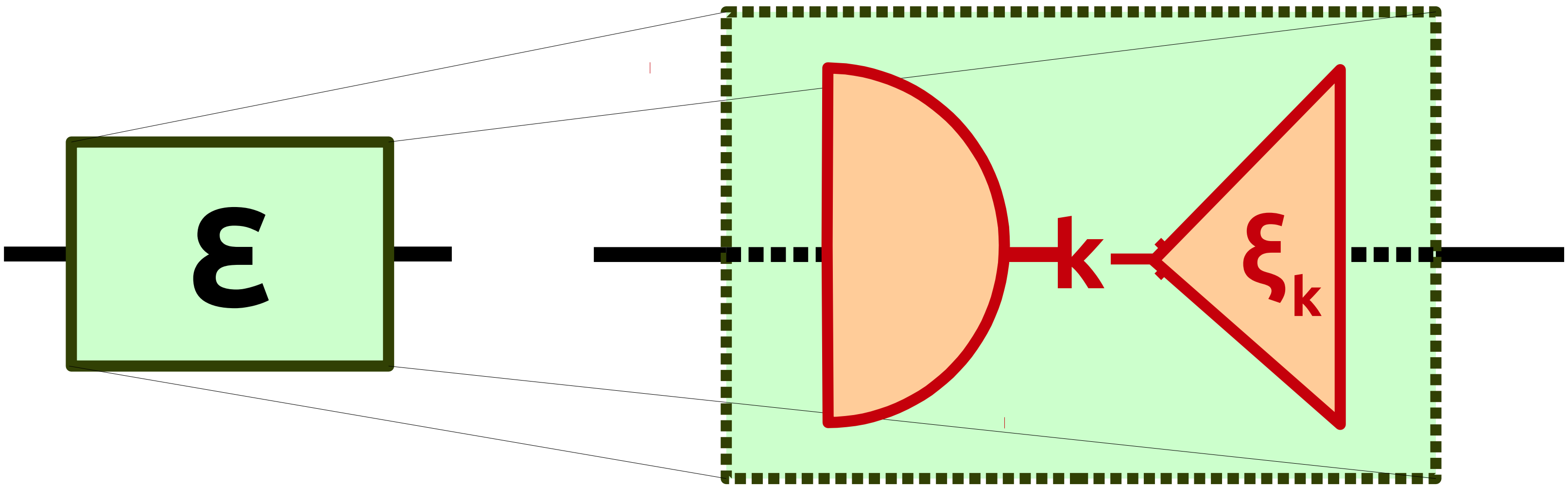
ε is called **entanglement breaking**

ENTANGLEMENT BREAKING CHANNELS verification

$\boldsymbol{\varepsilon}$ is EB **iff** $\boldsymbol{\Omega}_{\boldsymbol{\varepsilon}}$ is separable

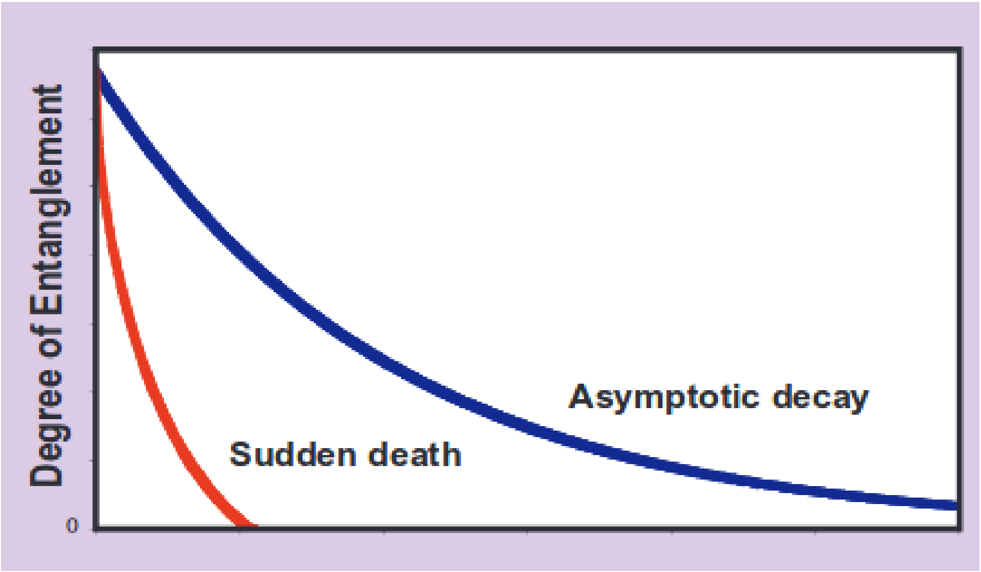
ENTANGLEMENT BREAKING CHANNELS
structure theorem

EB = **measure-and-prepare**



$$\mathcal{E}(\rho)=\sum_k \mathrm{tr}[\rho F_k]\,\xi_k$$

HOMO DYNAMIS SUDDENTROCUS
→ suddenly appeared and disappeared
→ main result



Ting Yu, J.H. Eberly, Sudden Death of Entanglement, Science 323, 598-601 (2009)

HOMO DYNAMIS ANNIHILIS
→ art of system disentanglement

Lenka Moravčíková, Mário Ziman, Entanglement-annihilating and entanglement-breaking channels, J.Phys.A 43, 275306 (2010)

Sergey N. Filippov, Kamil Yu. Magadov, Positive tensor products of qubit maps and n-tensor-stable positive qubit maps, J. Phys. A: Math. Theor. 50, 055301 (2017)

Alexander Müller-Hermes, David Reeb, Michael M. Wolf, Positivity of linear maps under tensor powers, J. Math. Phys. 57, 015202 (2016)

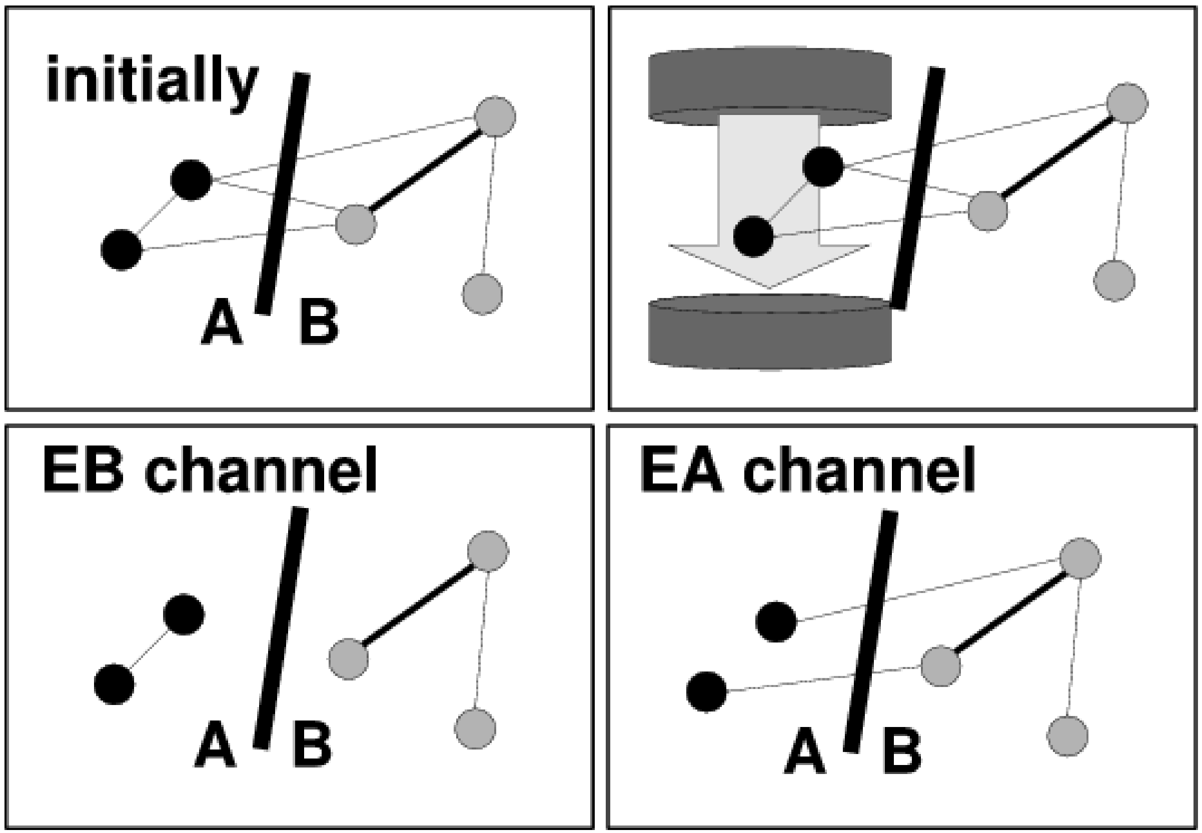
Sergey N. Filippov, Tomáš Rybár, Mário Ziman, Local two-qubit entanglement-annihilating channels, Phys. Rev. A 85, 012303 (2012)

Sergey N. Filippov, Alexey A. Melnikov, Mário Ziman, Dissociation and annihilation of multipartite entanglement structure in dissipative quantum dynamics, Phys. Rev. A 88, 062328 (2013)

Sergey N. Filippov, Mário Ziman, Bipartite entanglement-annihilating maps: necessary and sufficient conditions, Phys. Rev. A 88, 032316 (2013)

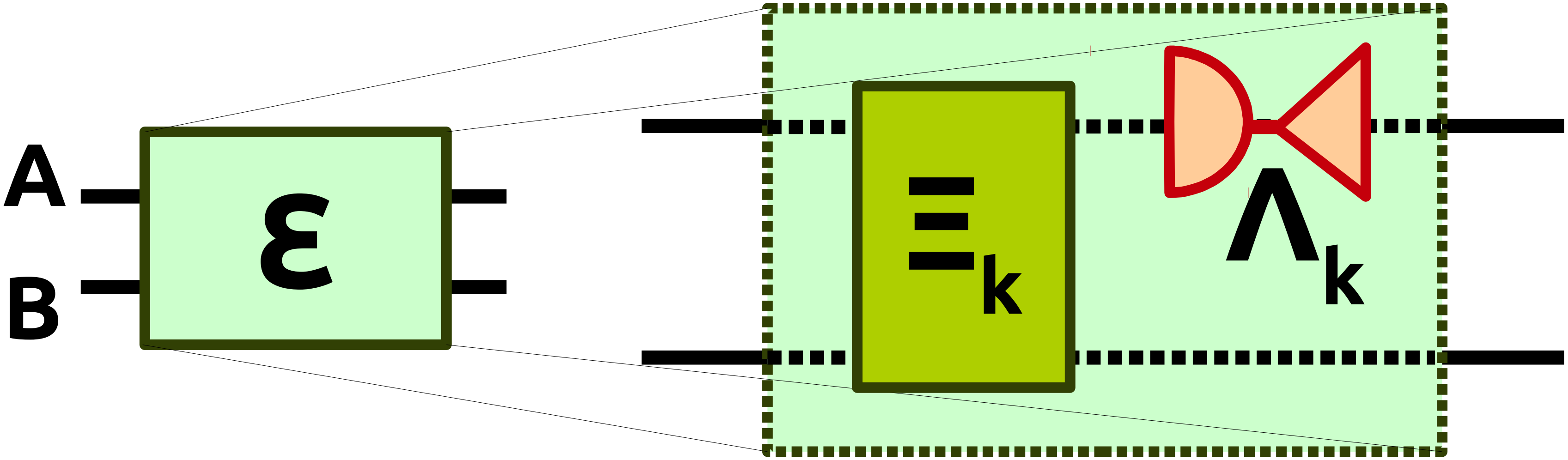
Sergey N. Filippov, Mário Ziman, Entanglement sensitivity to signal attenuation and amplification, Phys. Rev. A 90, 010301(R) (2014)

ENTANGLEMENT ANNIHILATION
internal disentanglement



limits entanglement-enabled technologies

ENTANGLEMENT ANNIHILATION
structure



$$\varepsilon = \sum_k \Xi_k^{AB}\left(\Lambda_k^{A/B} \otimes I^{B/A}\right)$$

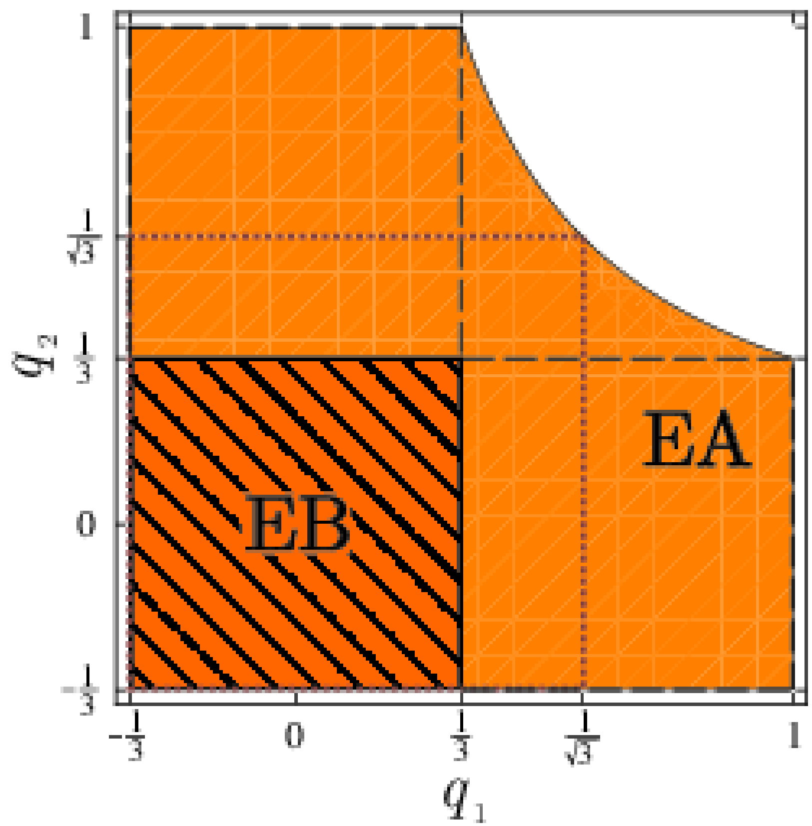
ENTANGLEMENT ANNIHILATION
Choi-Jamiolkowski

$\boldsymbol{\varepsilon}$ is EA **iff** $\boldsymbol{\Omega_\varepsilon}$ is (special) biseparable

$$\boxed{\mathbf{tr}[\boldsymbol{\Omega}_{\boldsymbol{\varepsilon}}^{ABA'B'}\, \boldsymbol{\rho}^{A'B'} \otimes \boldsymbol{\xi}_{bp}^{AB}] \geq \mathbf{0}}$$

non-EA witness $\boldsymbol{\rho}^{A'B'} \otimes \mathbf{W}^{AB}$
(positive $\otimes$ block-positive)

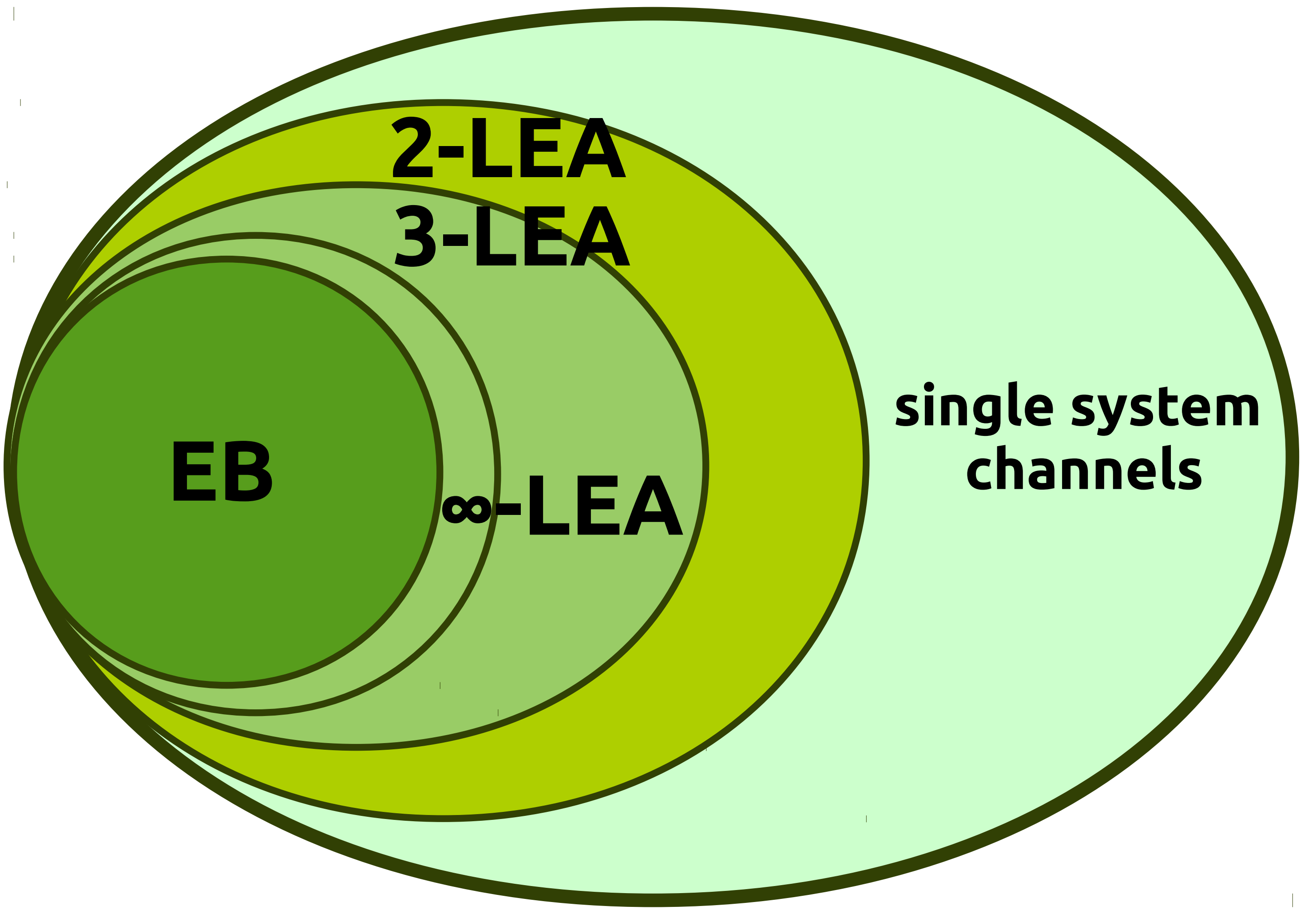
ENTANGLEMENT ANNIHILATION
Local depolarizing channels $\mathcal{E}_{q1} \otimes \mathcal{E}_{q2}$



$$\mathcal{E}_q = q\,\mathbf{Id} + (1-q)\,\mathbf{C}_{mix}$$

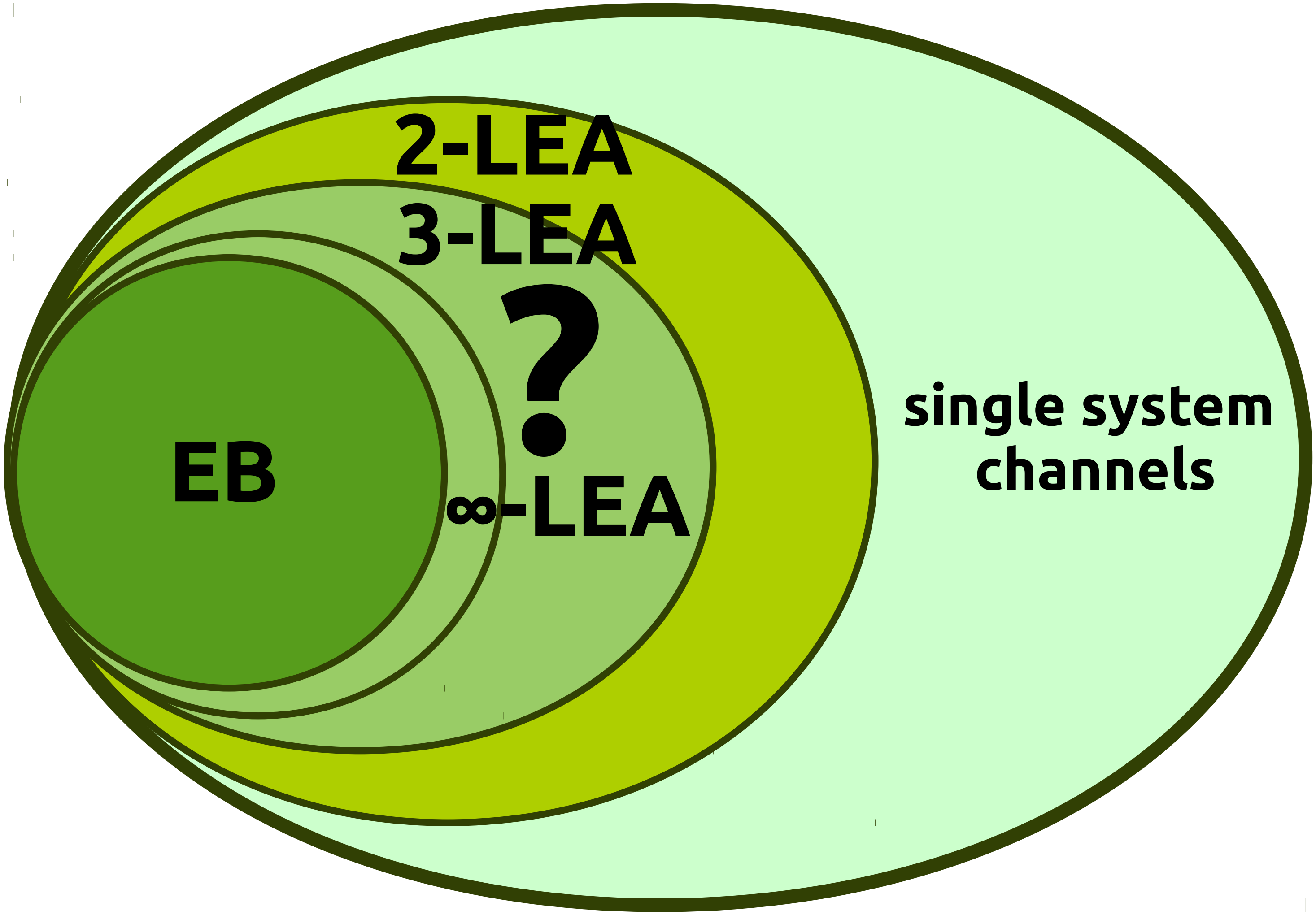
LOCAL ENTANGLEMENT ANNIHILATION

$\mathcal{E}\otimes\mathcal{E}\otimes\mathcal{E}\otimes\mathcal{E}\otimes\mathcal{E}\otimes\ldots\otimes\mathcal{E}\otimes\mathcal{E}\otimes\mathcal{E}\otimes\mathcal{E}\otimes\mathcal{E}\otimes\mathcal{E}\otimes\mathcal{E}$



LOCAL ENTANGLEMENT ANNIHILATION

open question ∞-LEA = EB



Gaussian channels on Gaussian states: **∞-LEA $\neq$ EB**

CONCLUSIONS

Why study dynamical aspects of entanglement?

- relevant for implementation
- there are things to understand

- many open questions (also of medium difficulty)
- even more to be asked …

THANK YOU FOR YOUR ATTENTION