Genuine Multipartite Entanglement Detection Without Reference Frames

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Entanglement Detection without (stable) Reference Frames

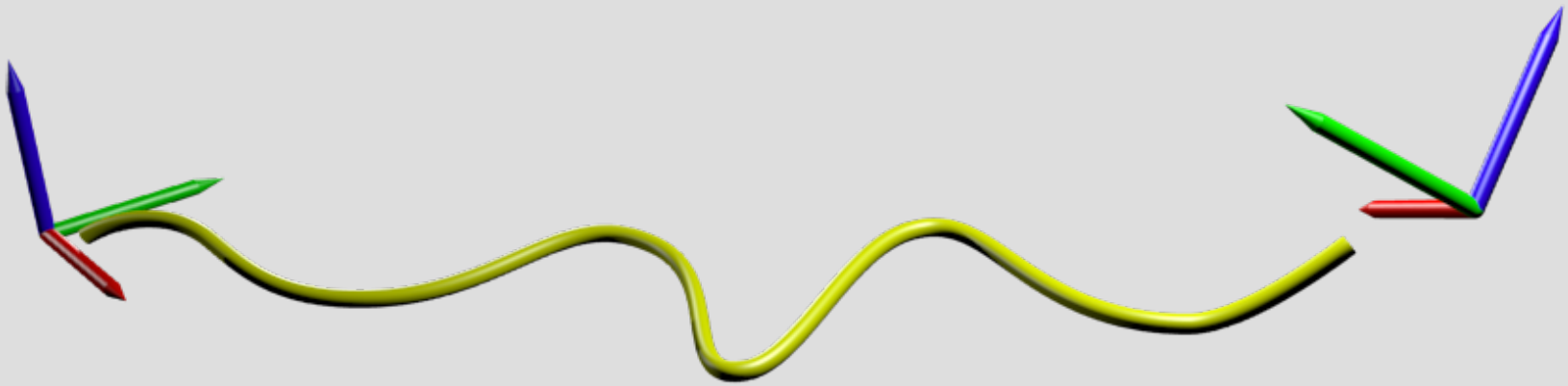
genuine multipartite entanglement (GME) detection

- entanglement witnesses
- (partly) reconstructed density matrix
- Bell inequalities
- ...
  - specific measurement settings needed
  - at least: knowledge of current setting
  - require repeatability of measurement settings

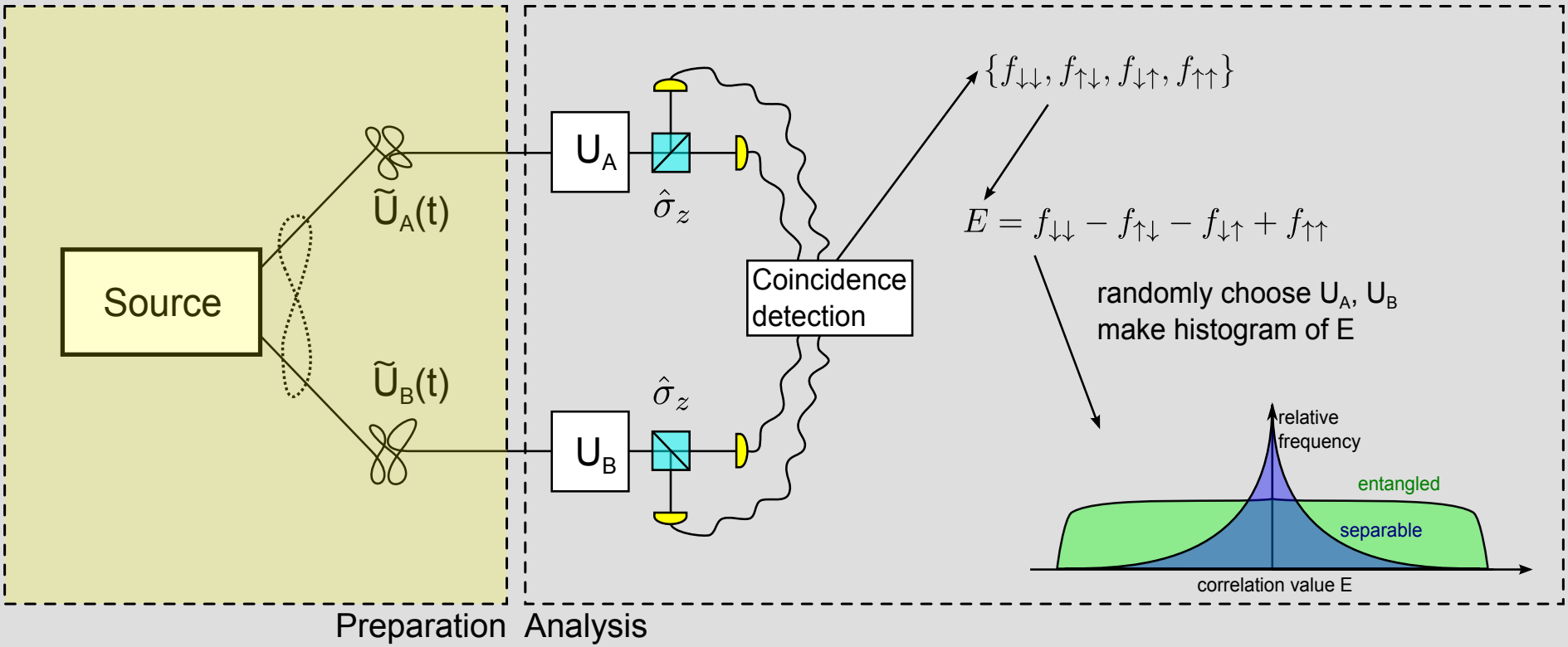
$\rightarrow$ here: detection of GME without control nor knowledge about settings

Uncontrollable Measurements

- local reference frames may be inaccessible or drifting over time
- no usual tomography feasible
- no Bell test, etc. feasible
- Ansatz: measure observers' correlations in randomly sampled bases

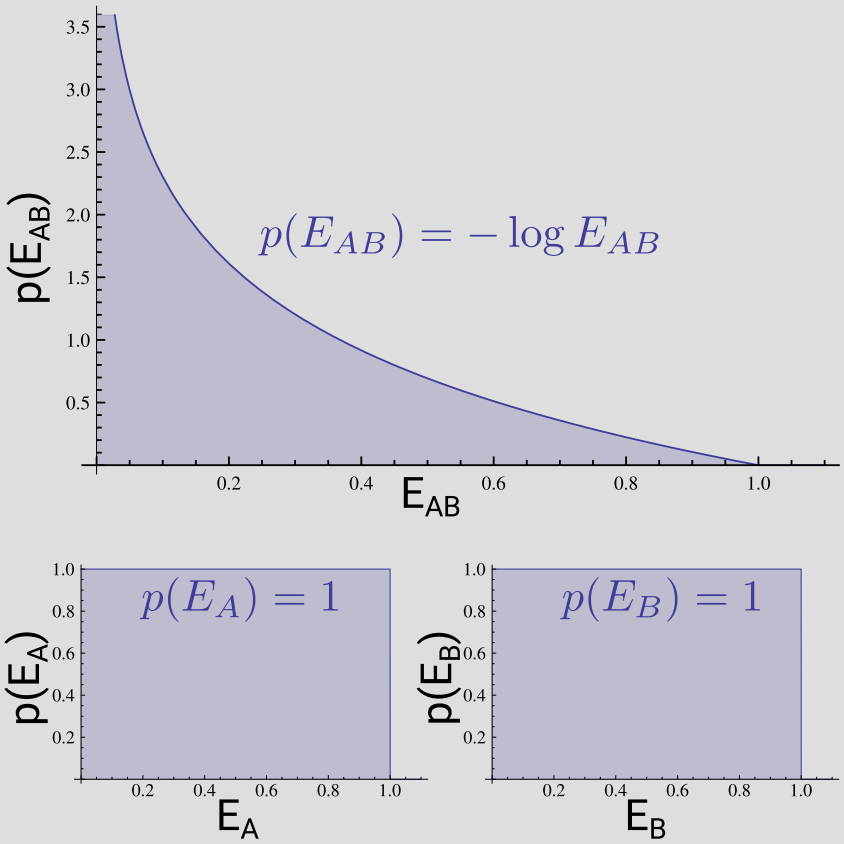


Random Measurements: Concept

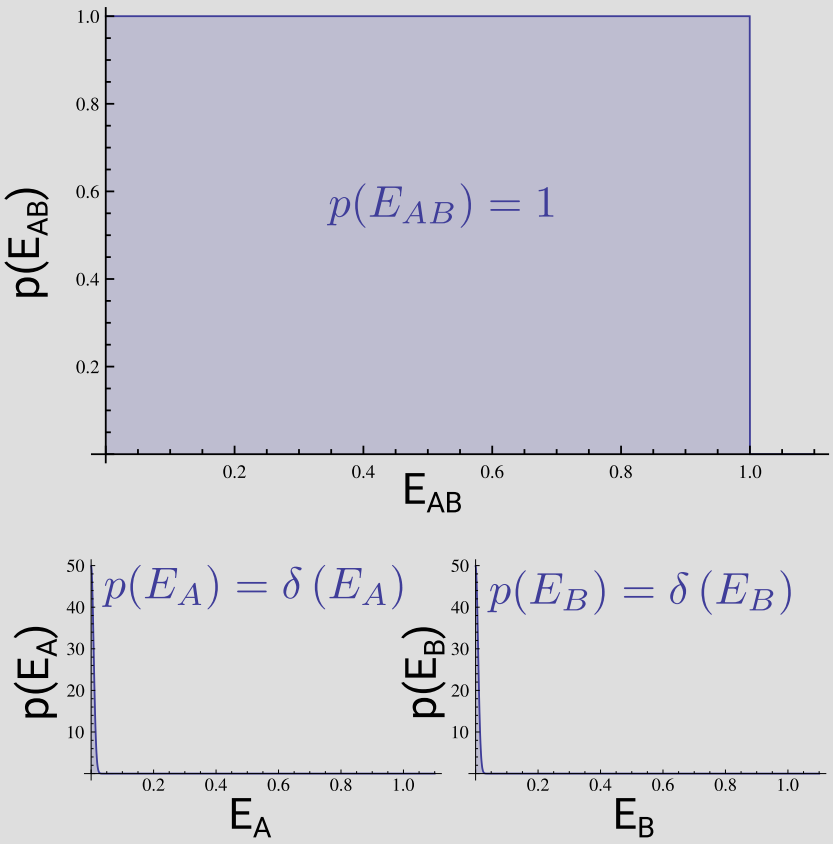


- adding (*Haar uniform*) random unitary canceling *biased* noise
- cf. biased random number sequence XORed with good sequence
- symmetric distribution
- generalizable to multiqubit systems

Product state



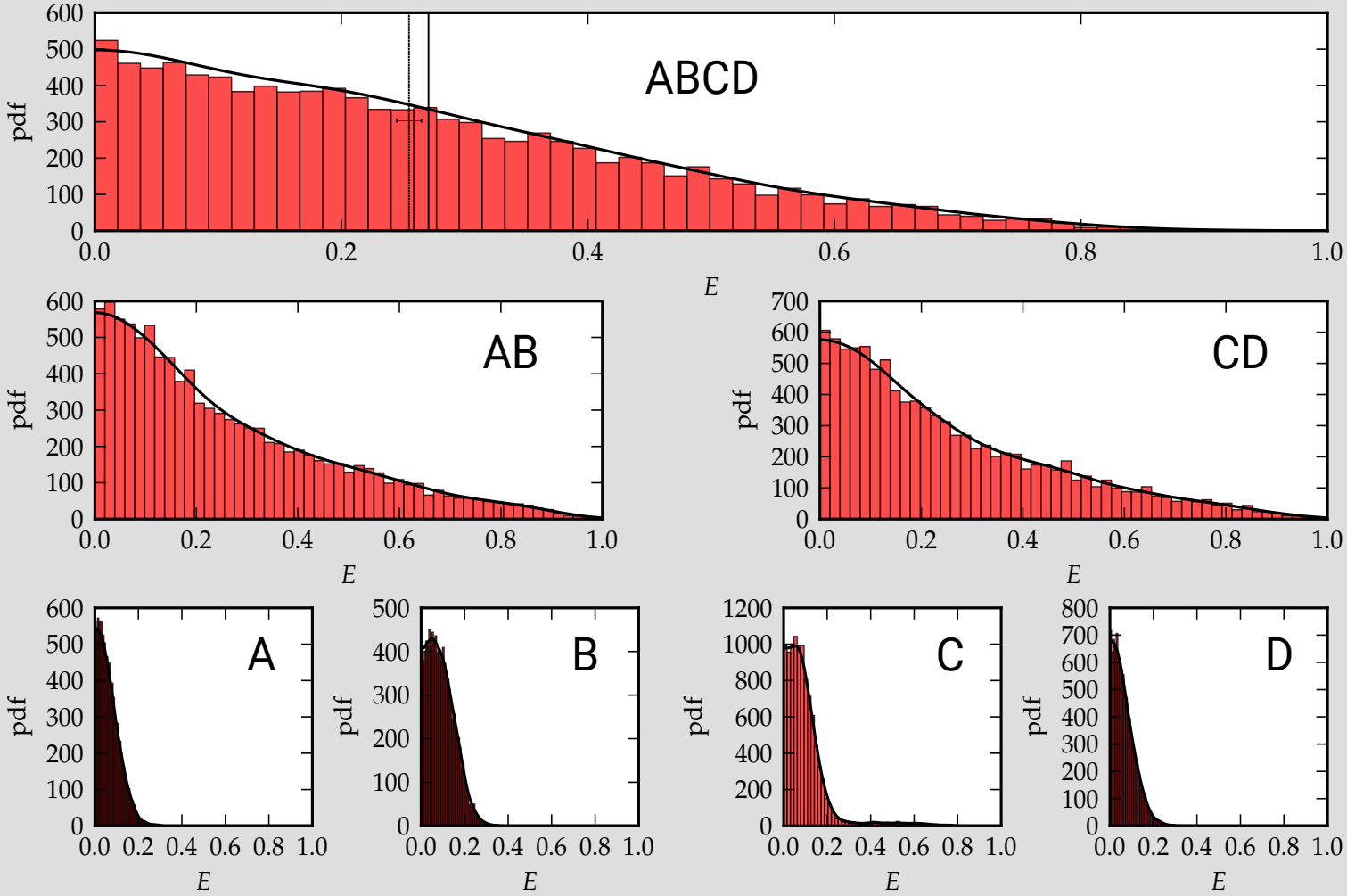
Bell state



- full and lower orders: $2^n - 1$ distributions
- directly retrieve some LU invariants (e.g., purity)
- characterizing distributions using moments
- tomographically complete (up to local unitaries)? No!

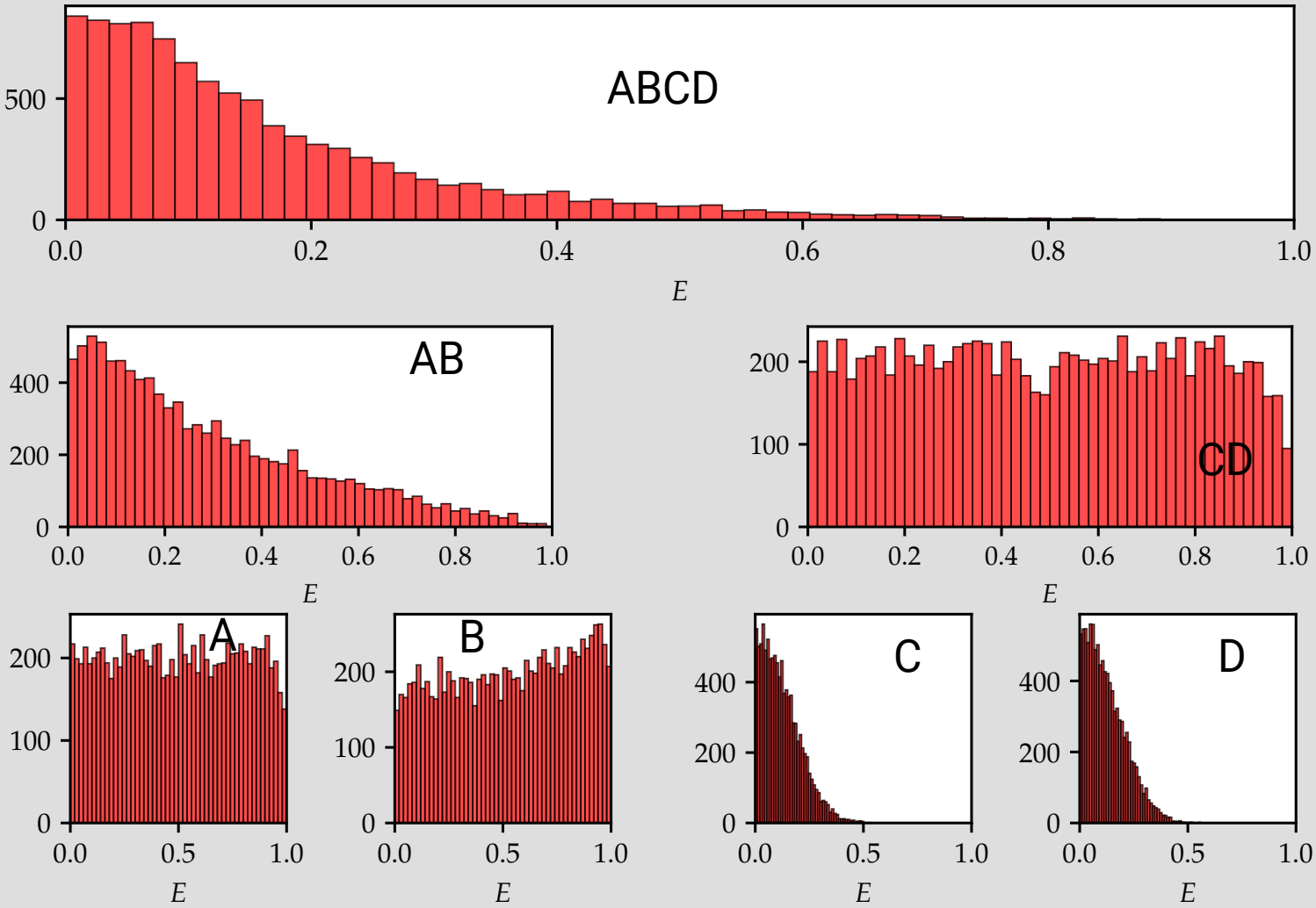
GHZ state

$$\frac{1}{\sqrt{2}}\left(|0000\rangle + |1111\rangle\right) \tag{1}$$



Triseparable state

$$\frac{1}{\sqrt{2}}|0\rangle \otimes |0\rangle \otimes (|00\rangle + |11\rangle) \tag{2}$$



Detecting Entanglement

- consider second moments: $m_{AB} \equiv \mathbb{E}\left[E_{AB}^2\right]$
- product state: $m_{AB} = m_A m_B$
- experimental states never pure
- *how much can full distribution be described by marginals?*
- $f(\varrho) = m_{AB} - m_A m_B$
- $f(\varrho_{\text{exp}}) > f_{\text{max}} \equiv \max_{\varrho_{\text{bs}}} f(\varrho_{\text{bs}}) \Rightarrow \varrho_{\text{exp}}$ is GME

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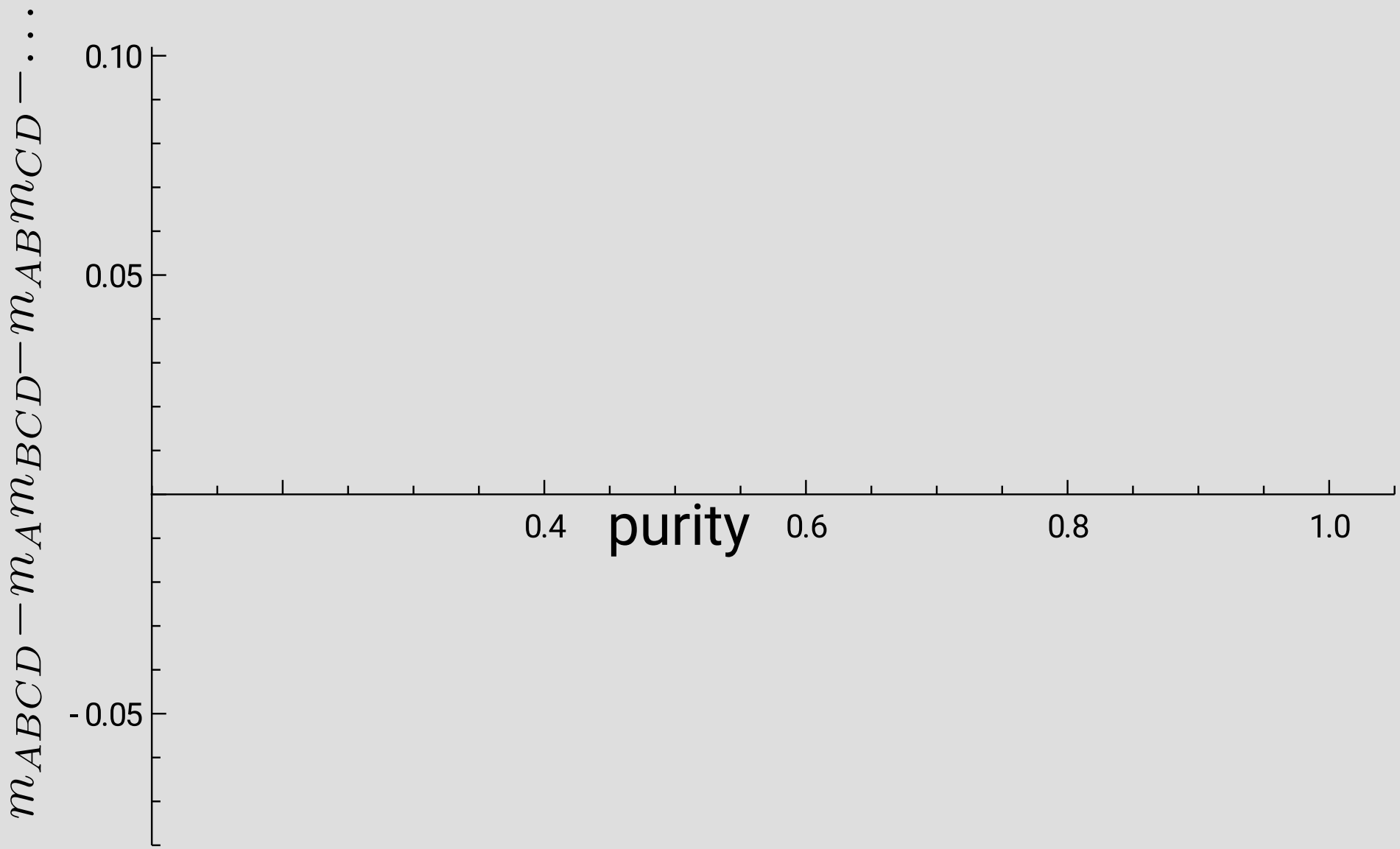
n=4



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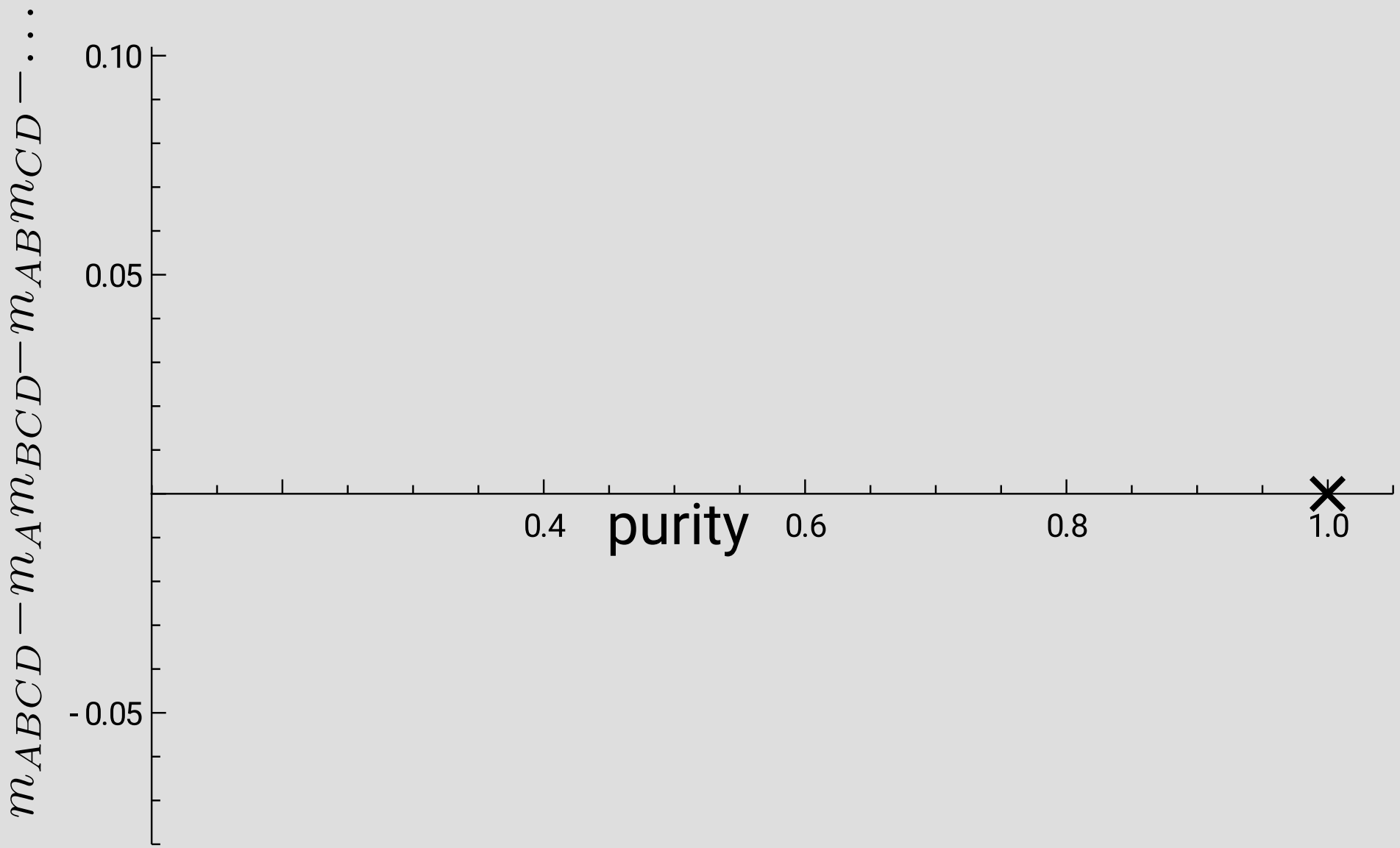
preliminary



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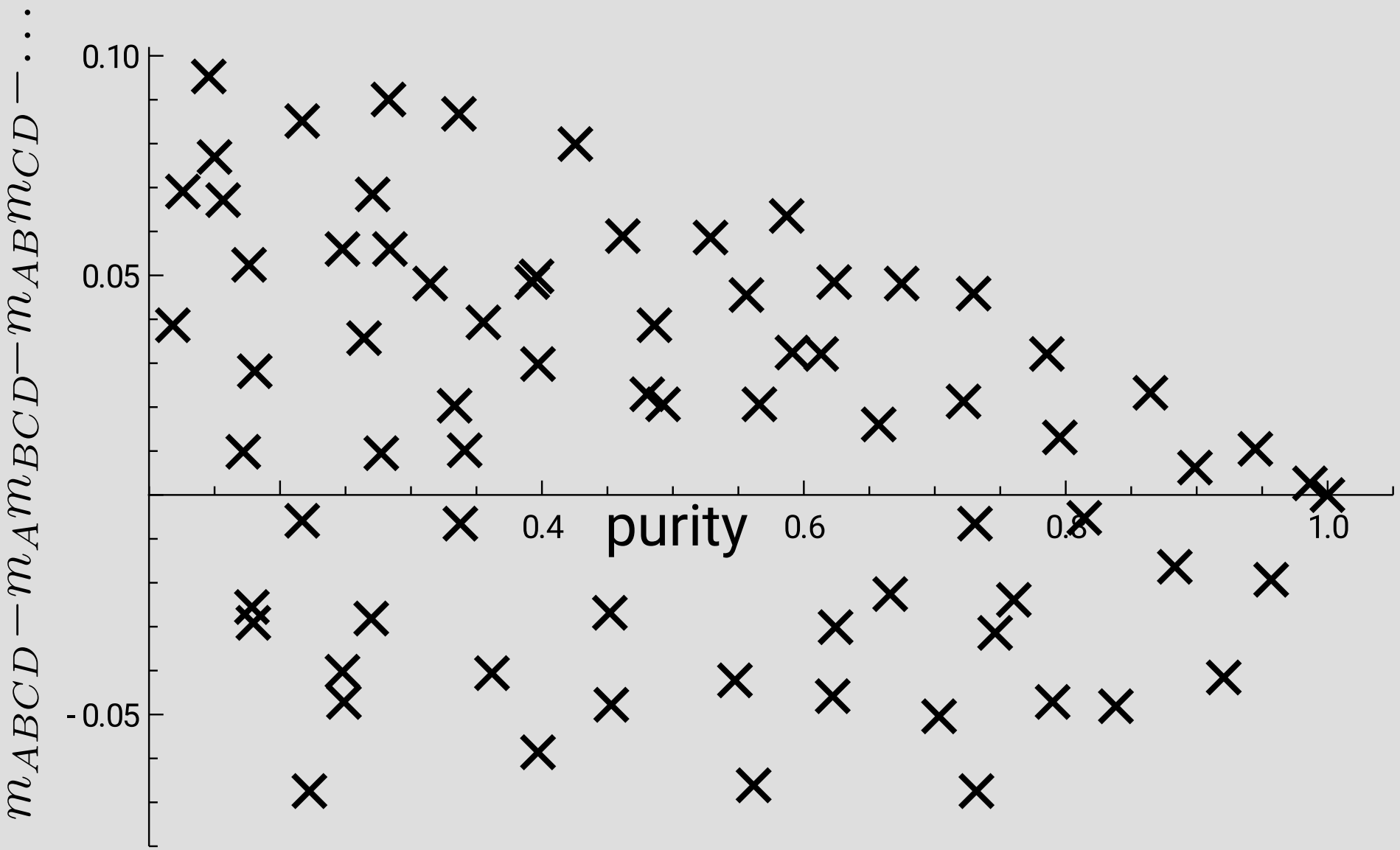

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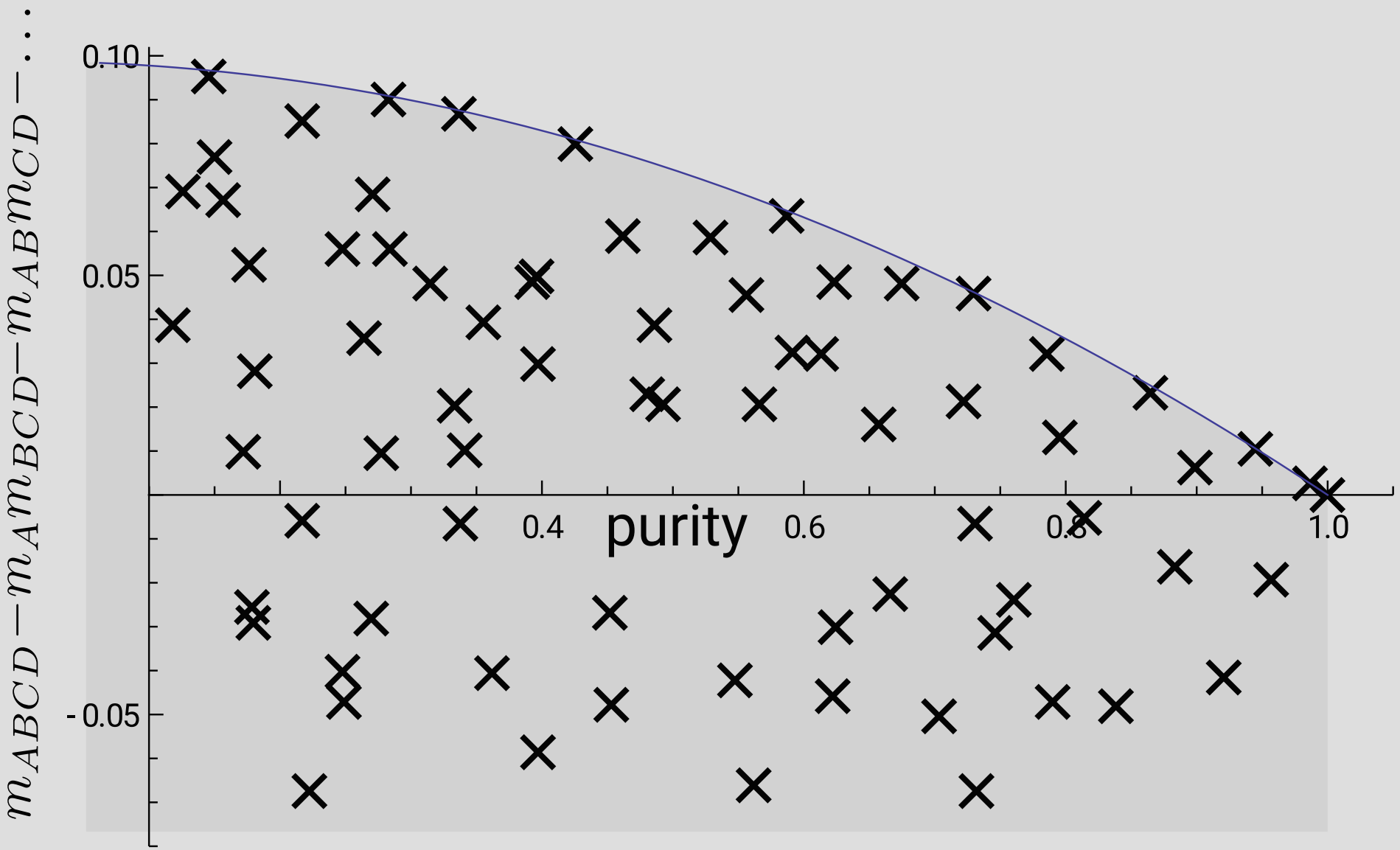

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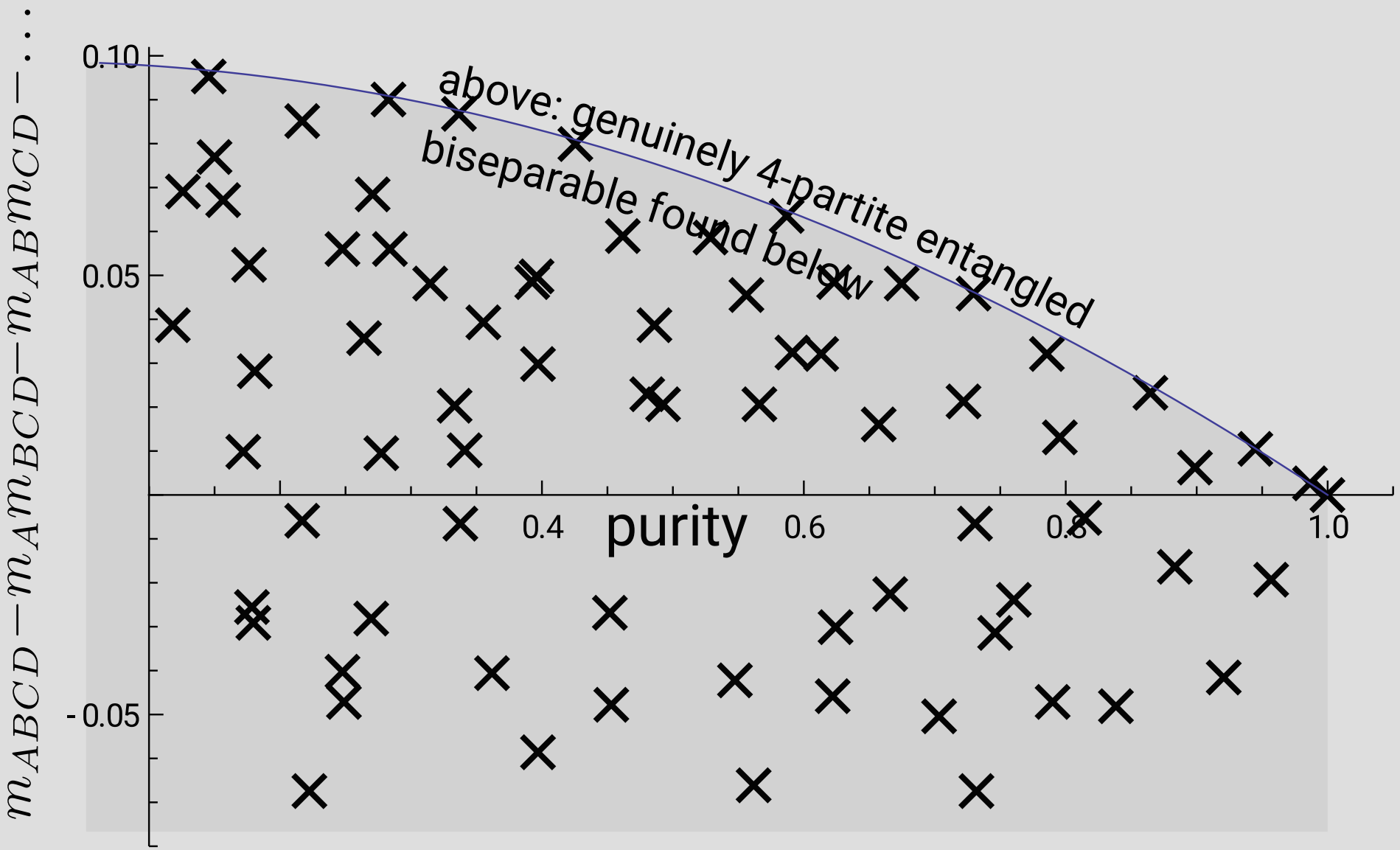

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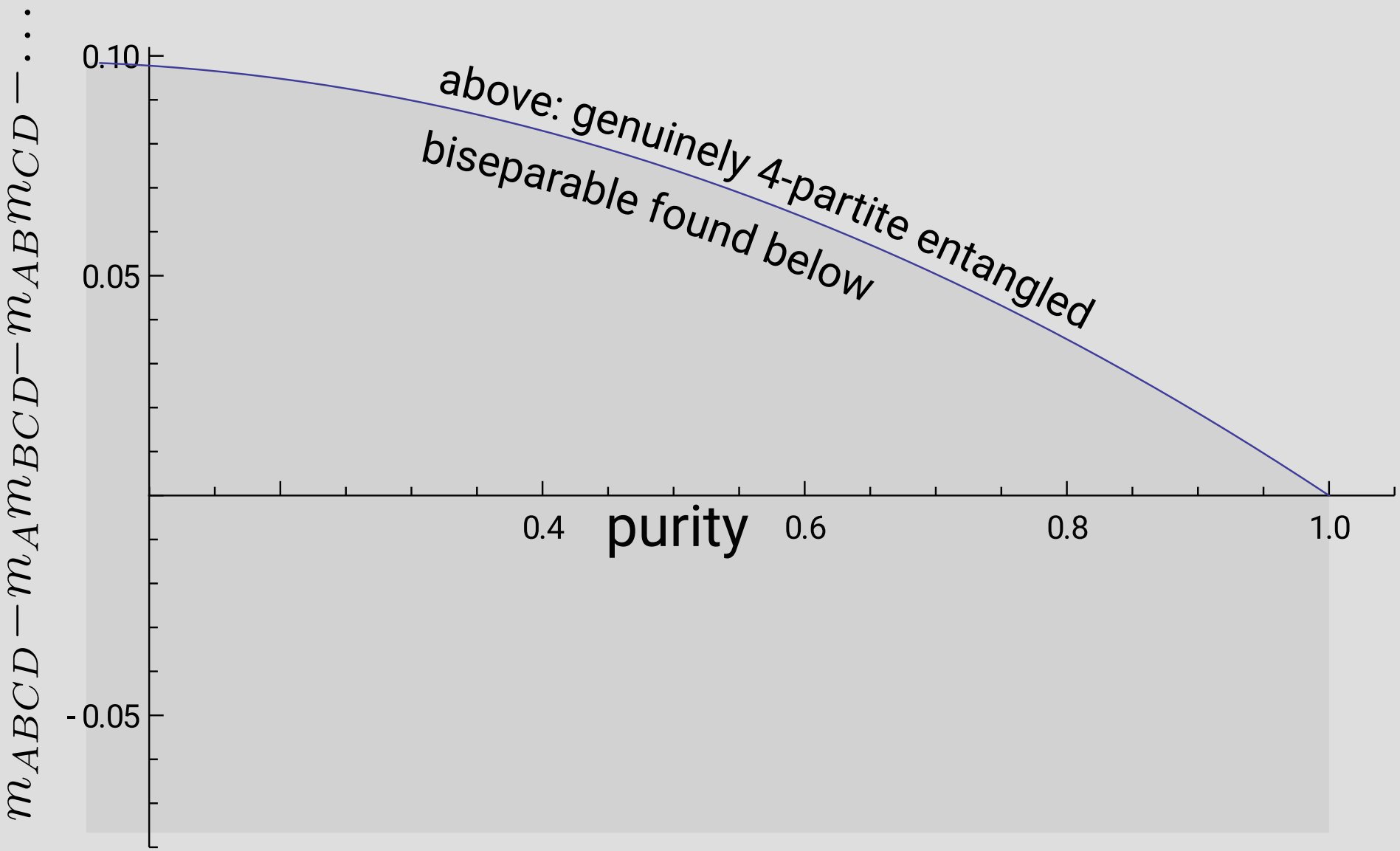

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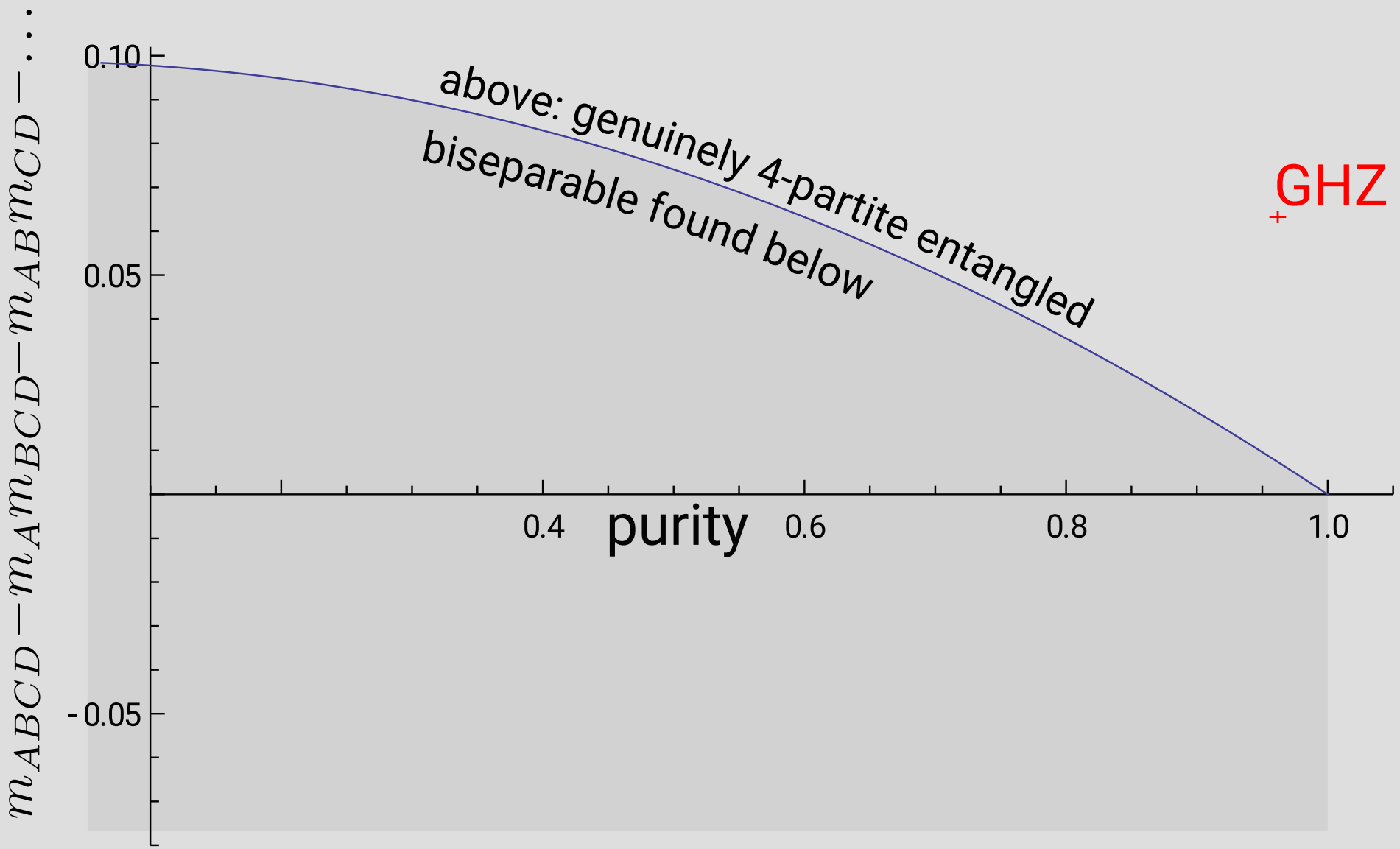

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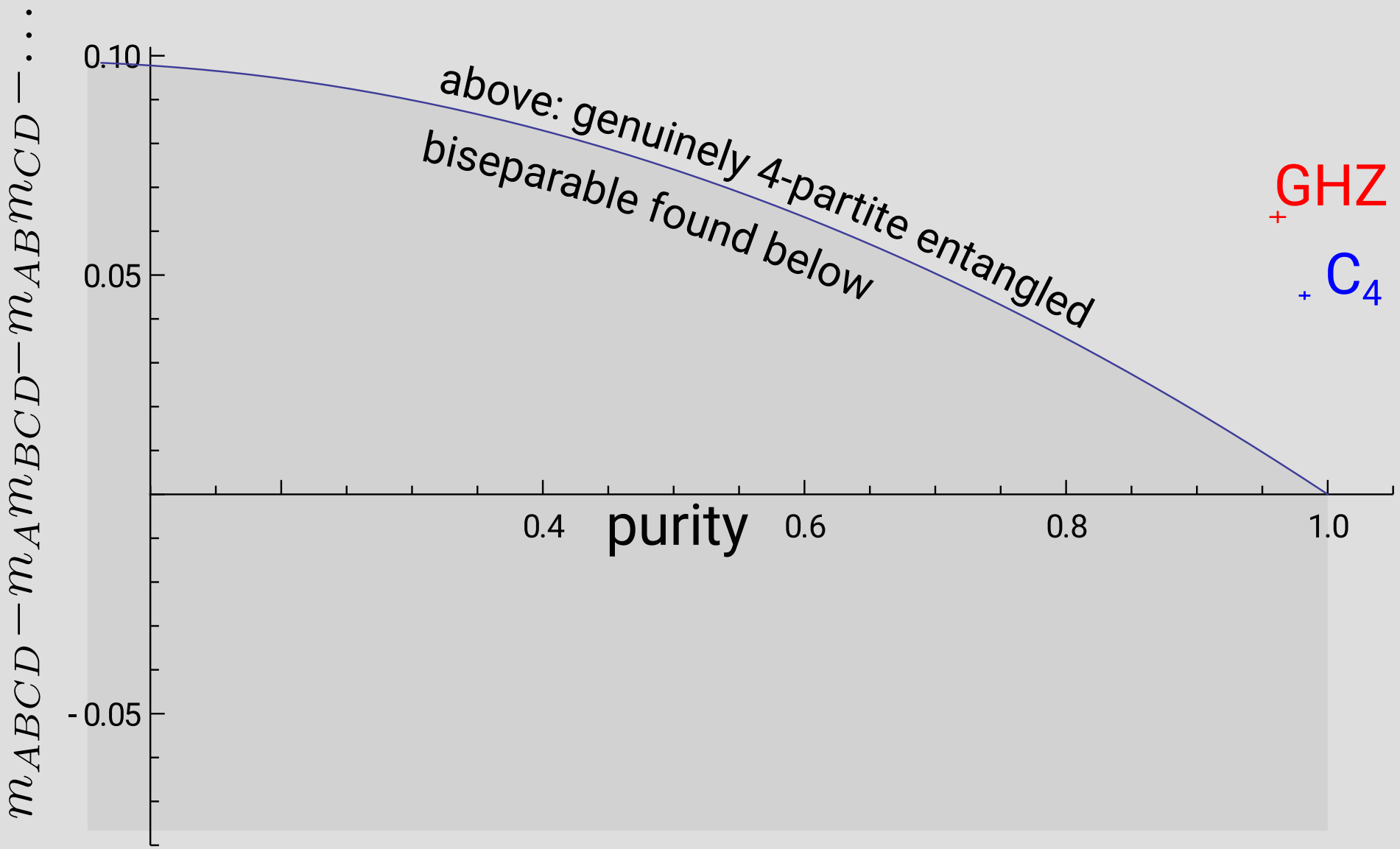

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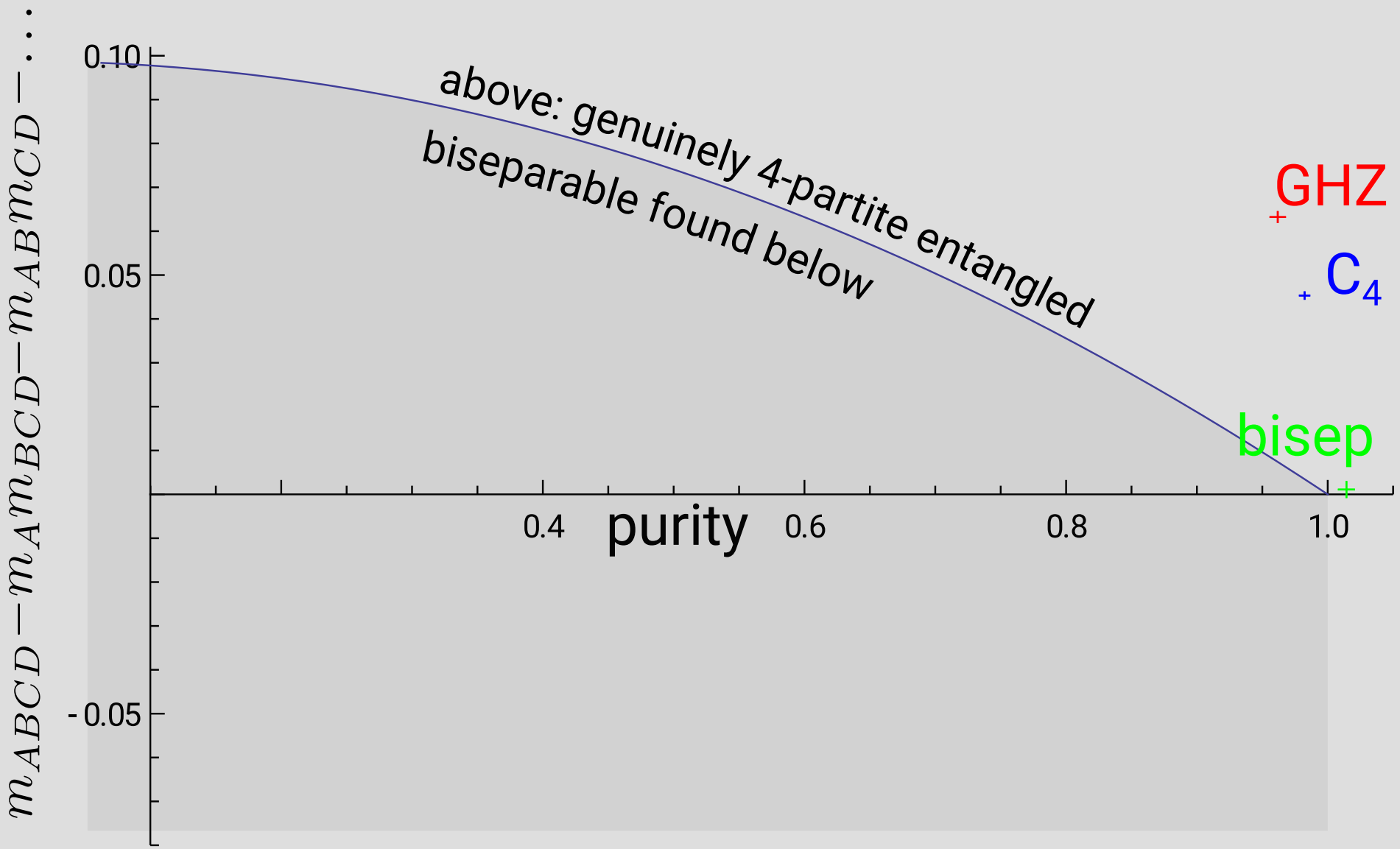

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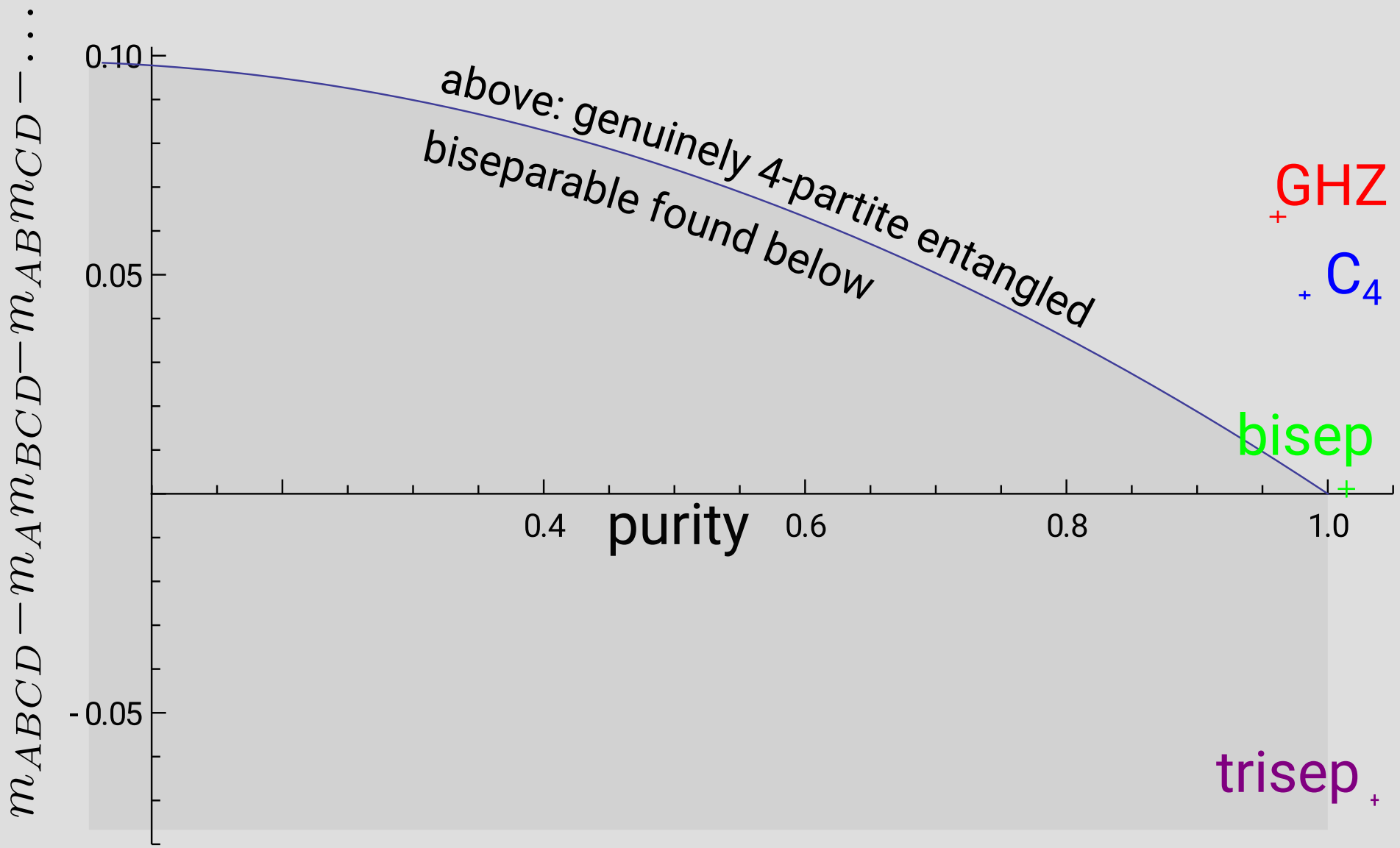

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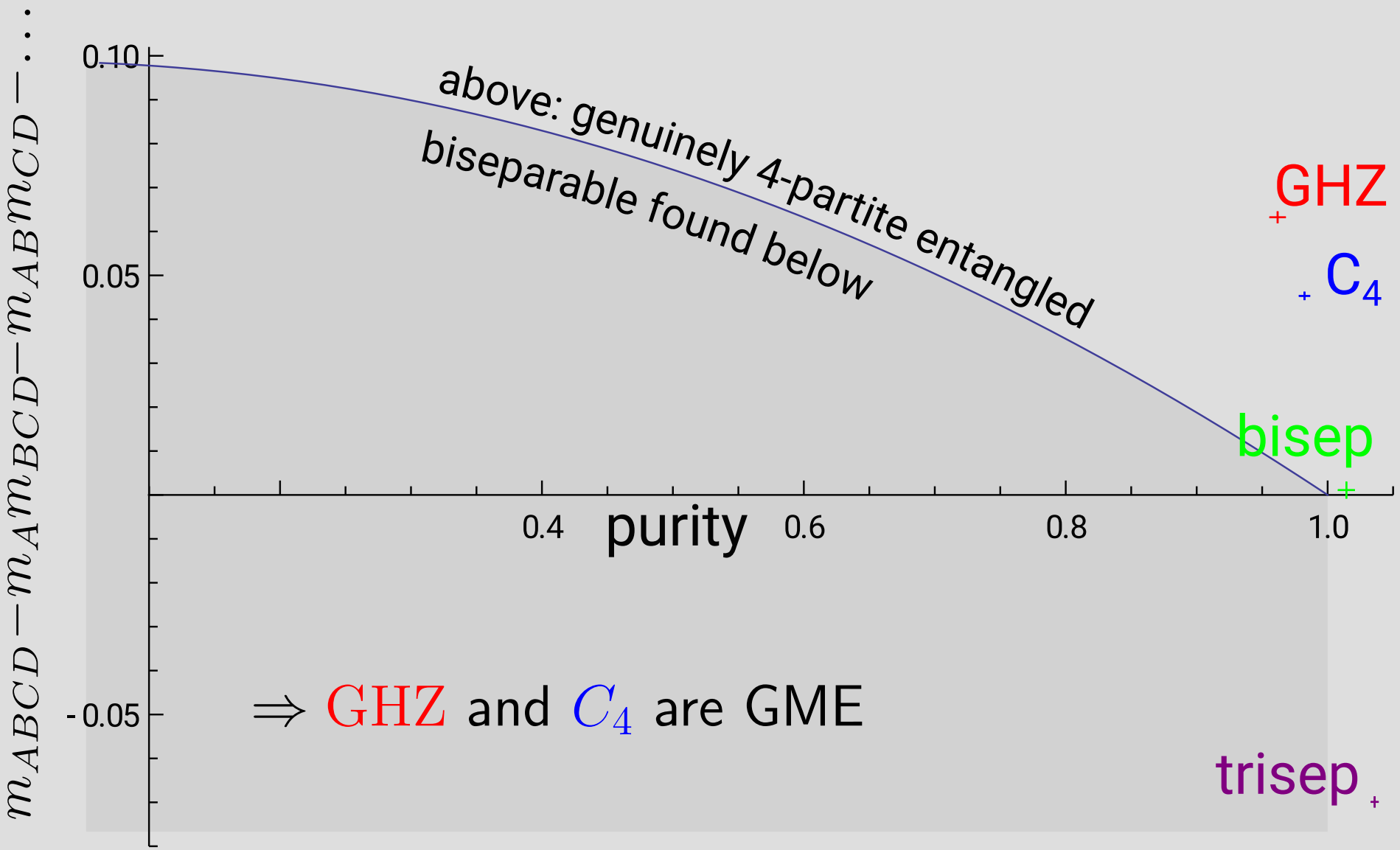
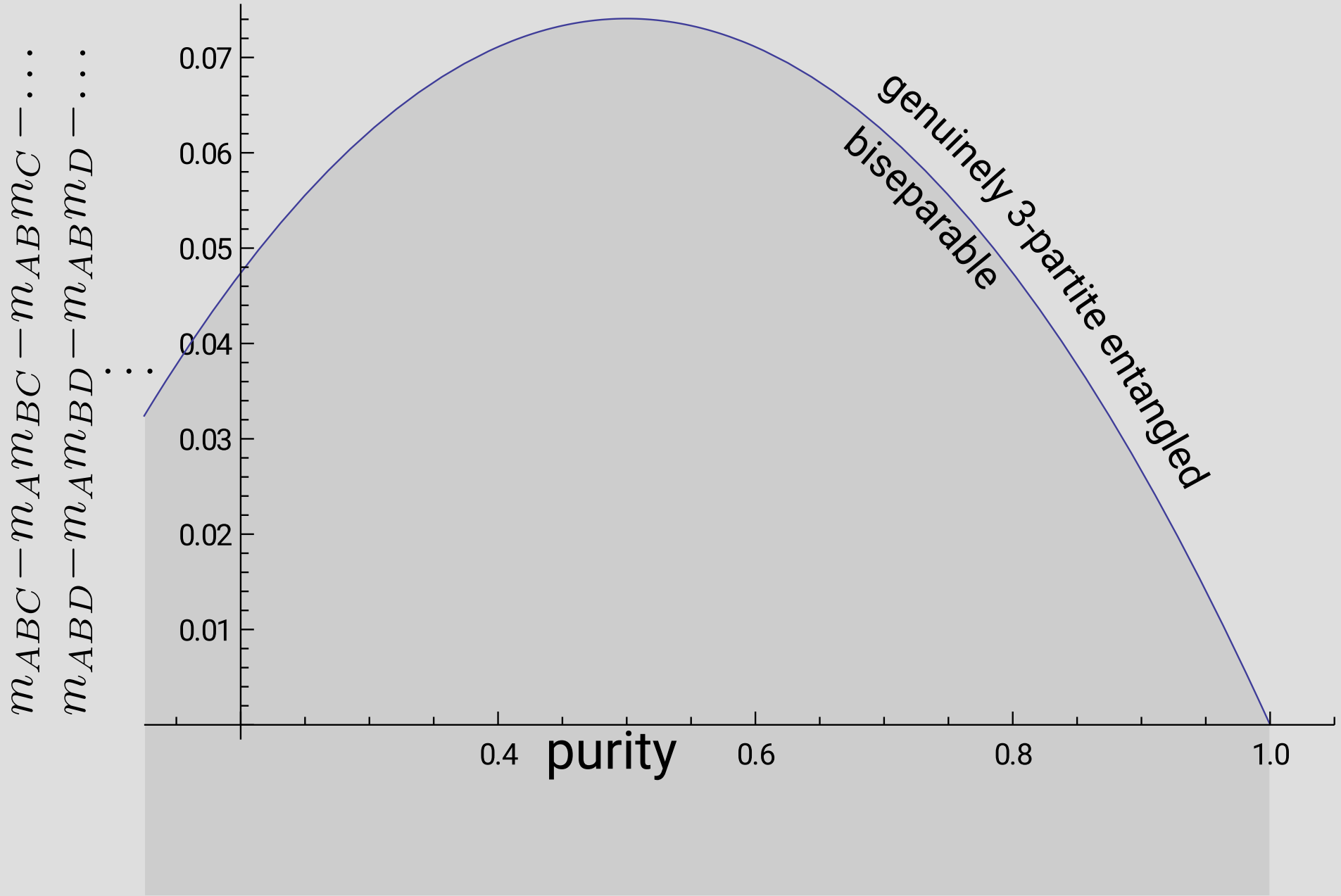


n=3


$m_{ABC} - m_A m_{BC} - m_{AB} m_C - \ldots$
$m_{ABD} - m_A m_{BD} - m_{AB} m_D - \ldots$
$\ldots$
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.4
purity
0.6
0.8
1.0
genuinely 3-partite entangled
biseparable

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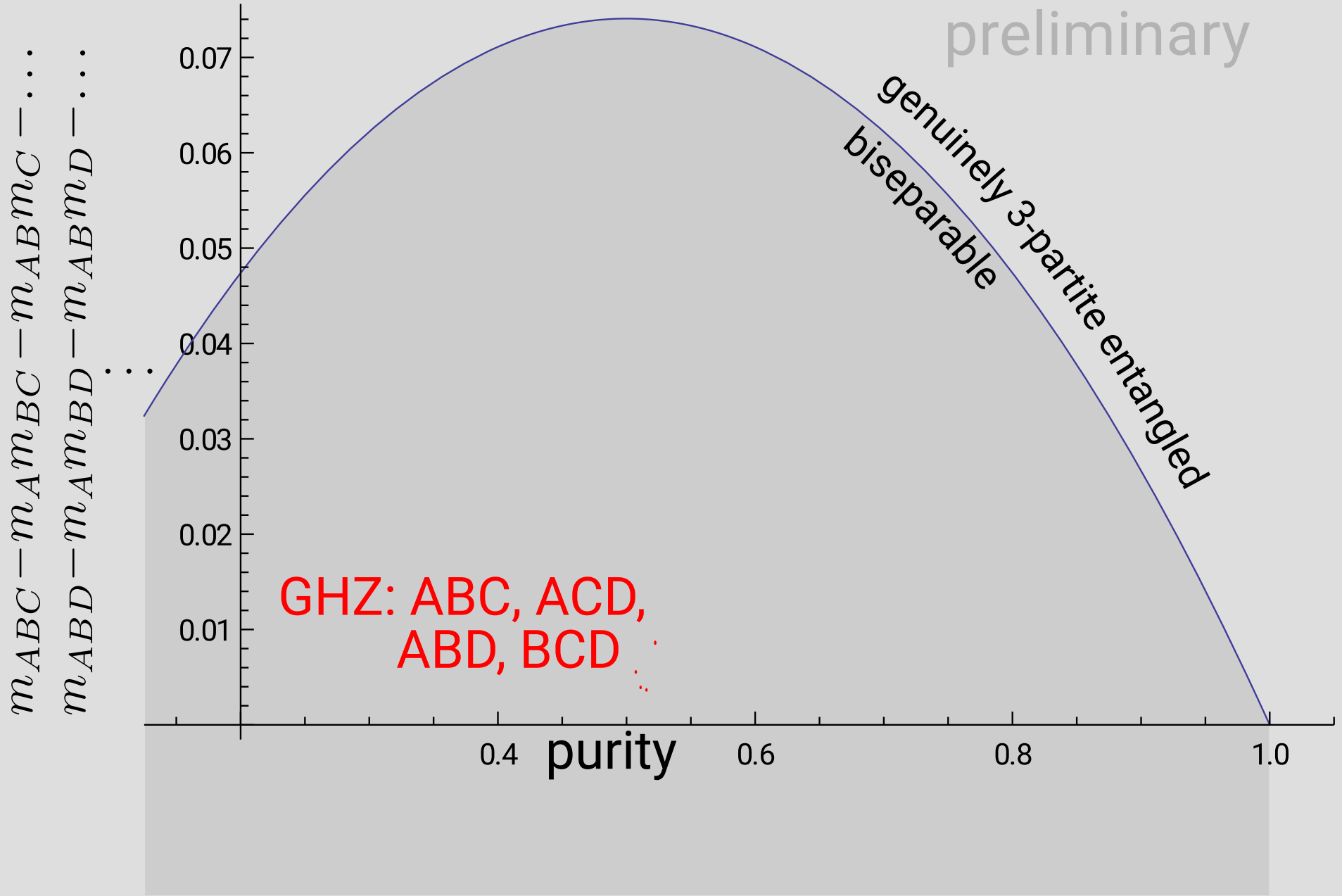
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GHZ: ABC, ACD,
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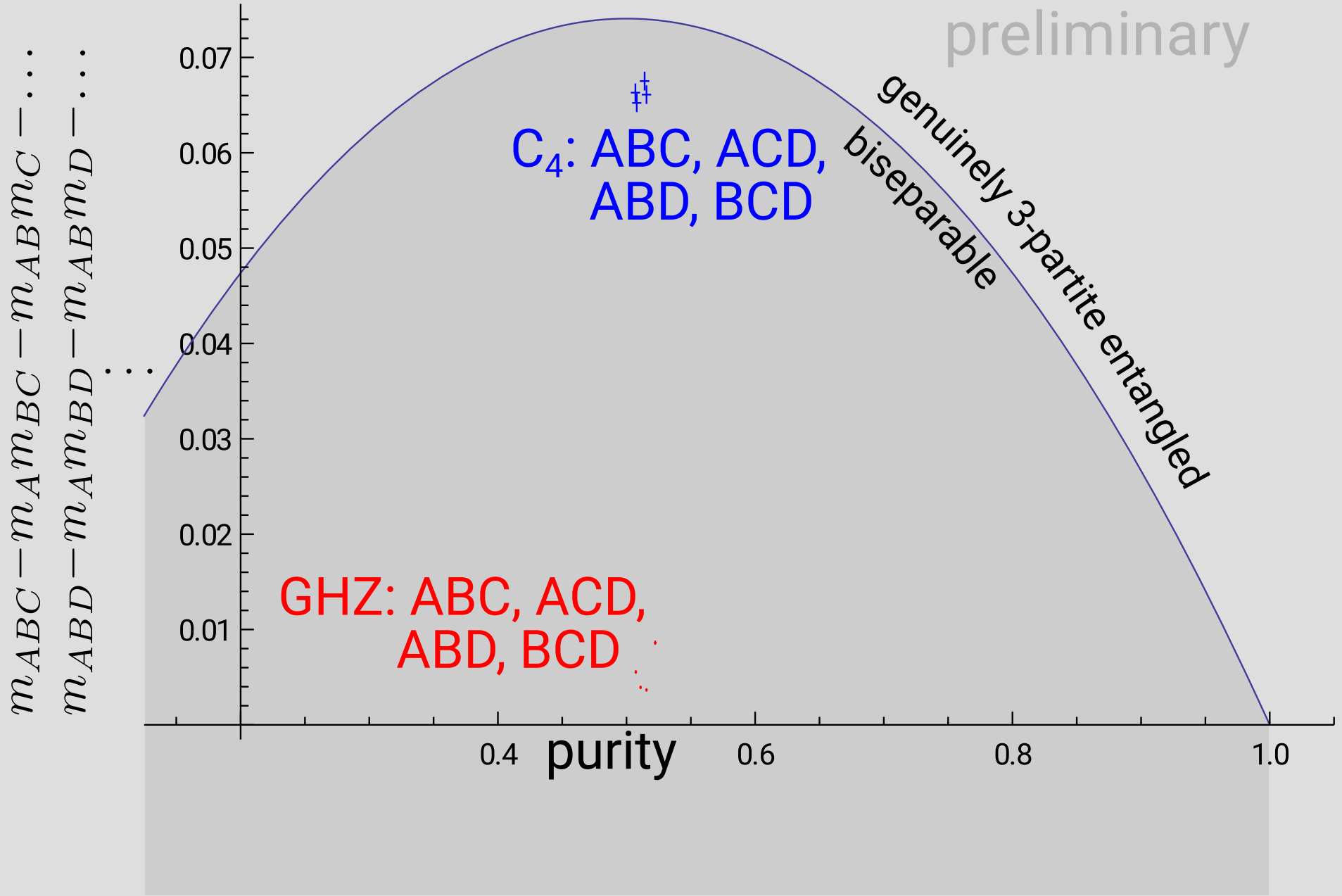
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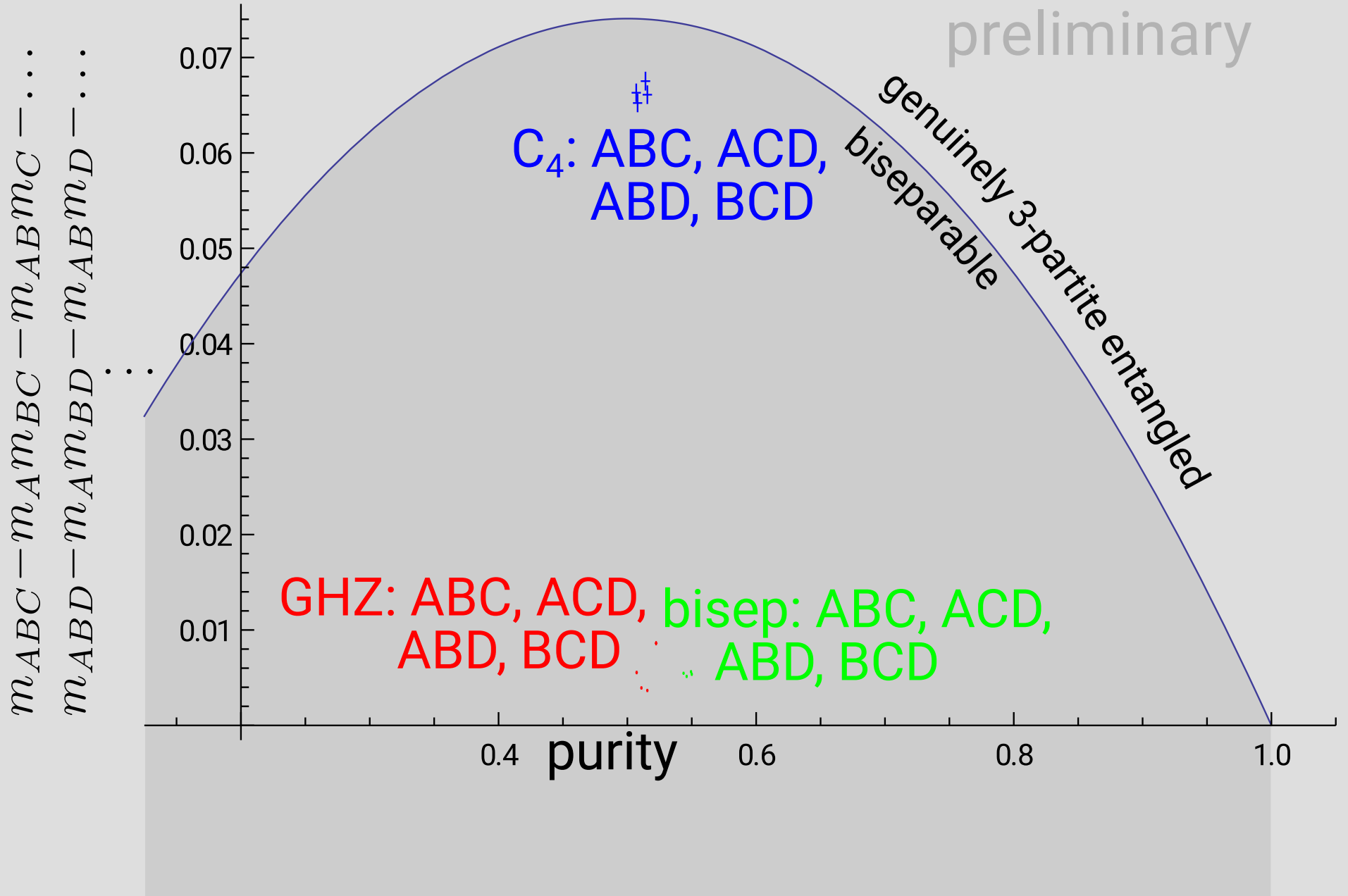
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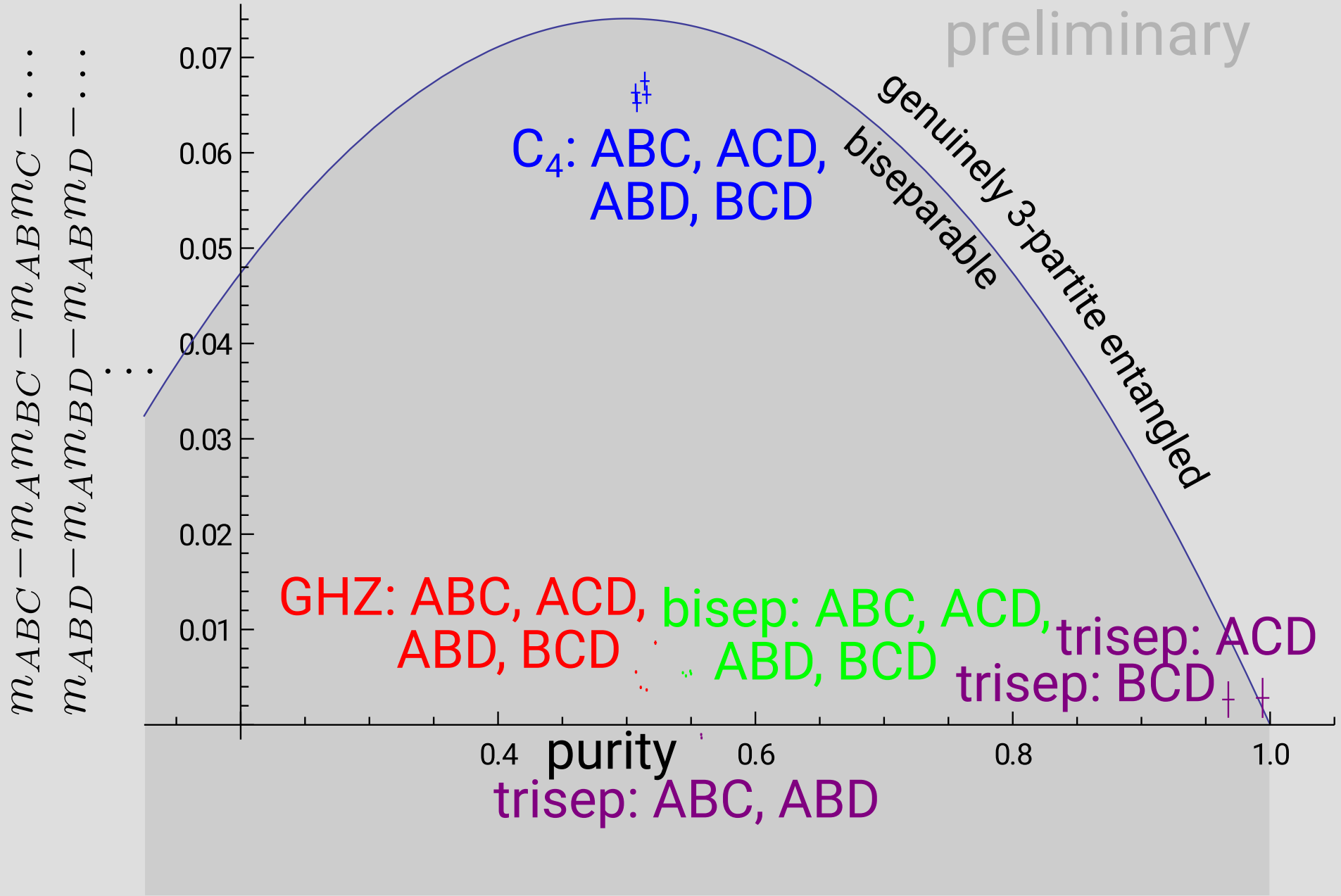
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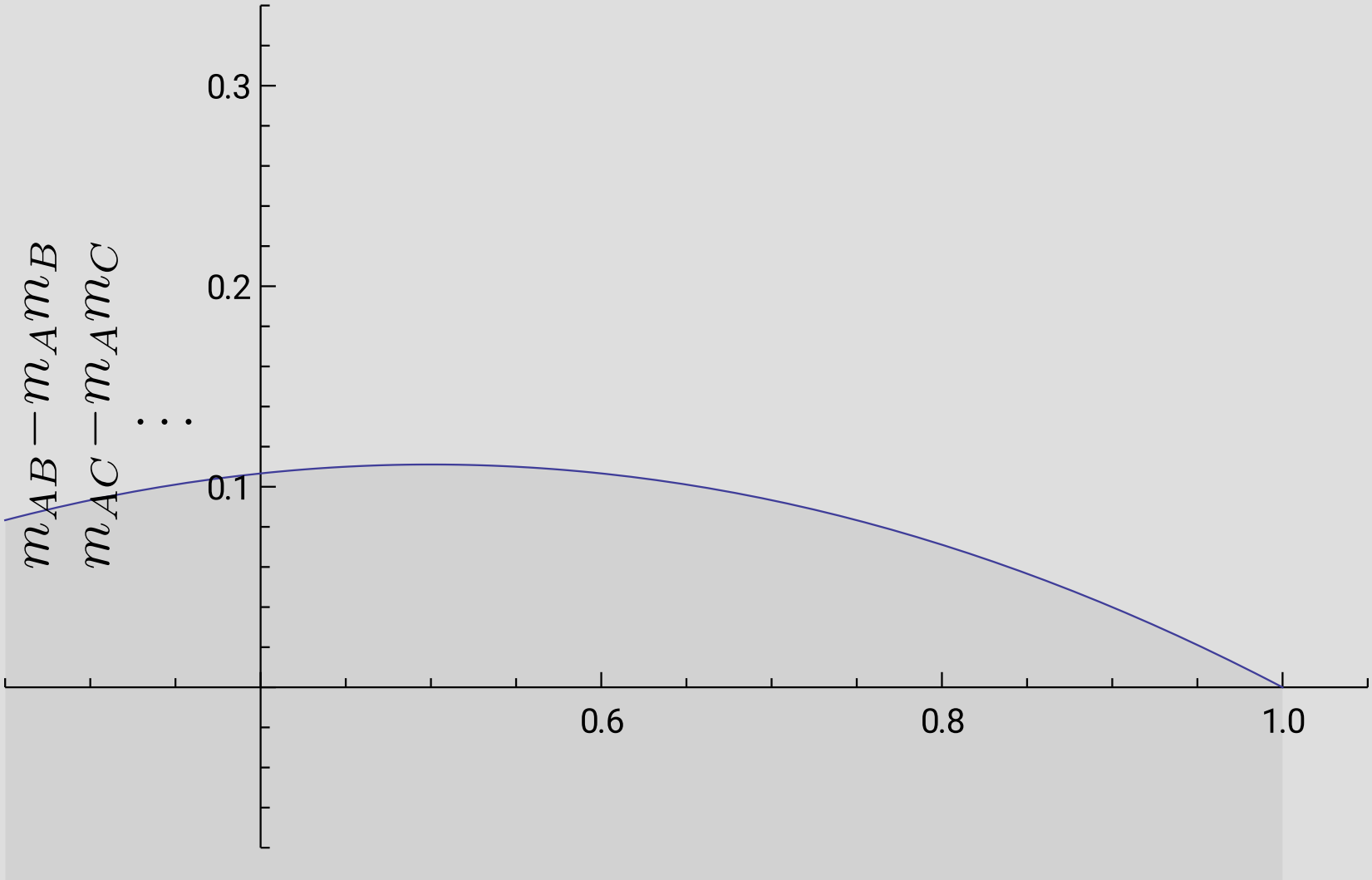
trisep: ACD
trisep: BCD
0.4
purity
0.6
0.8
1.0
trisep: ABC, ABD

n=2

preliminary



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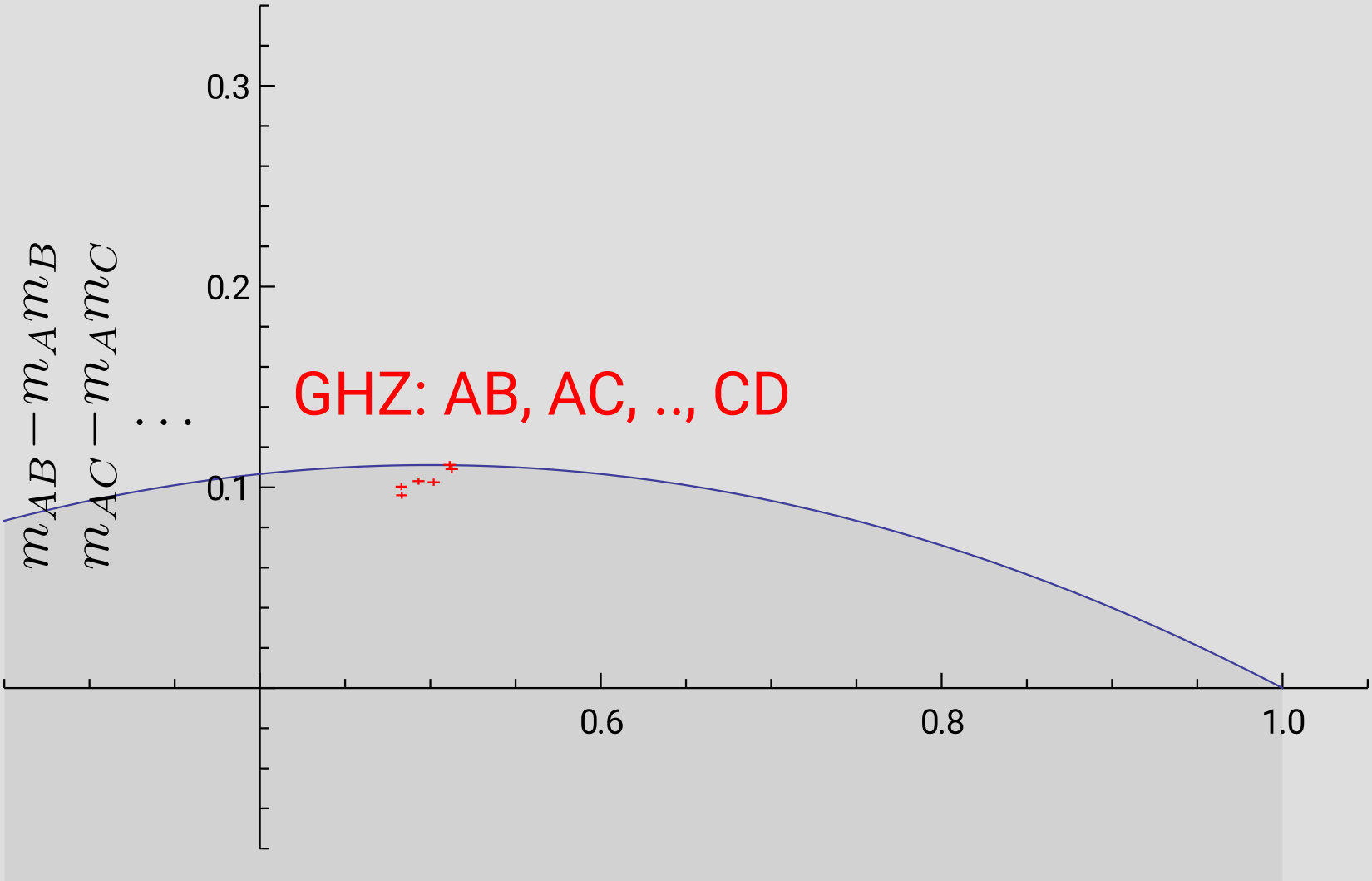

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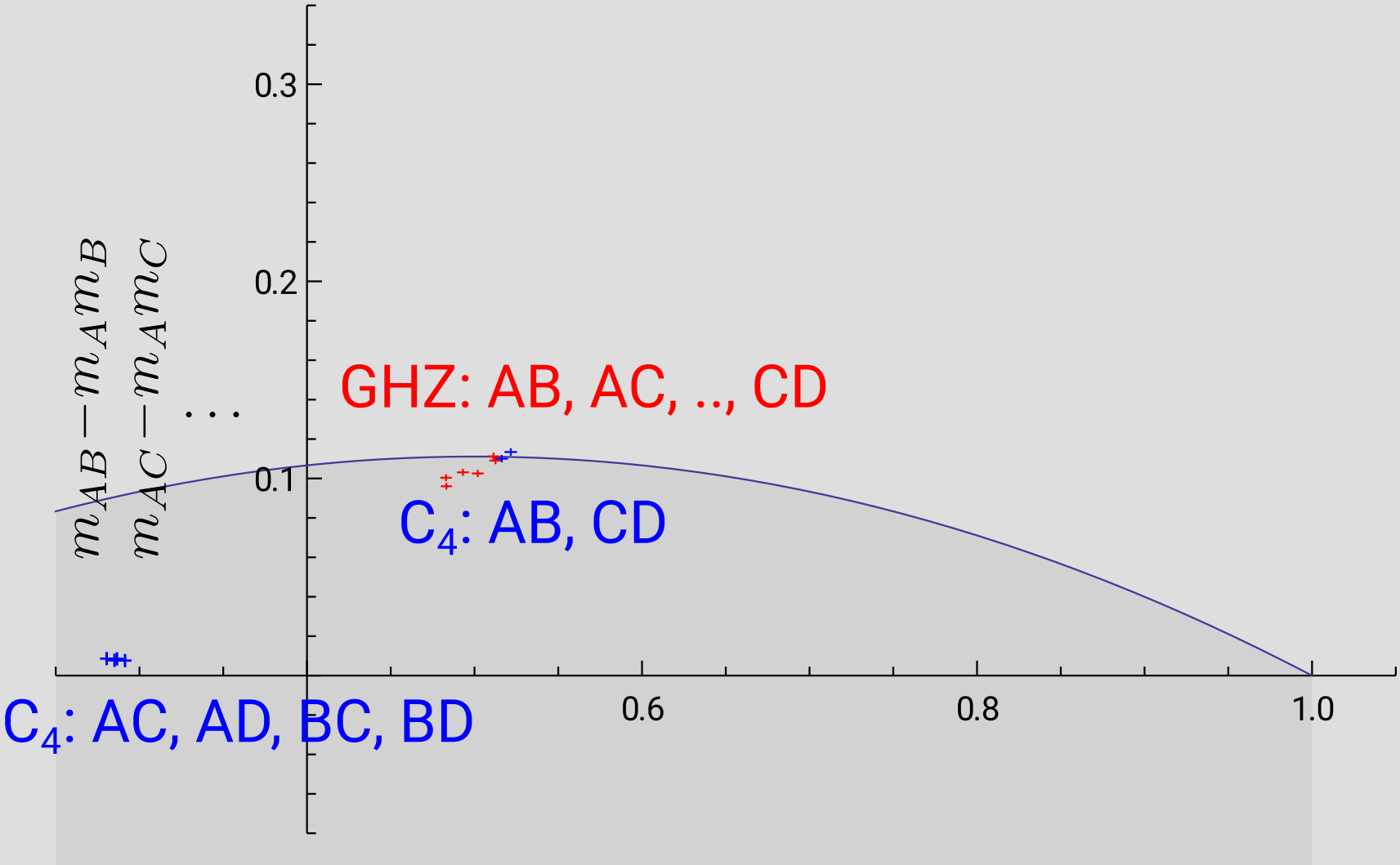

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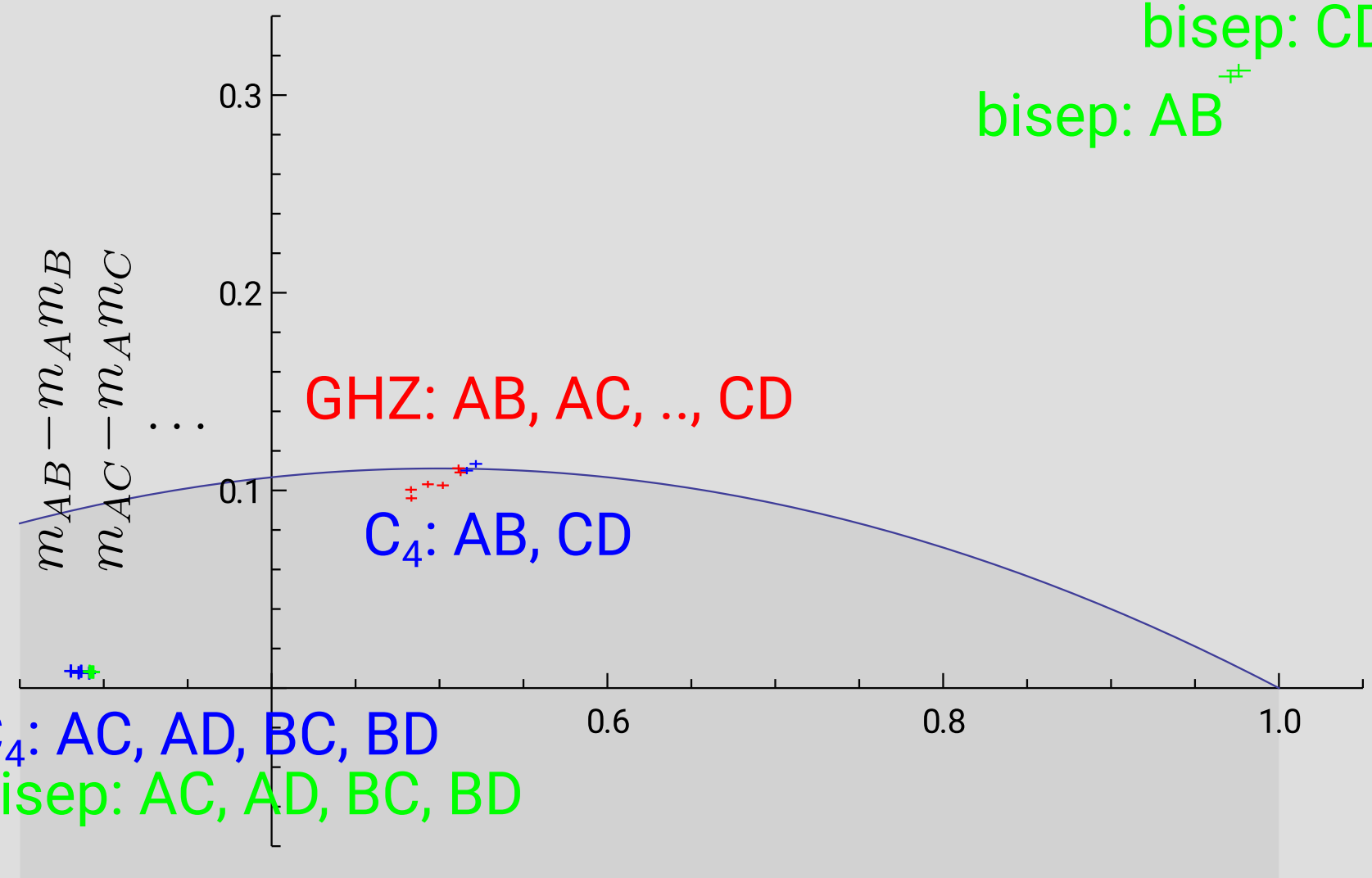

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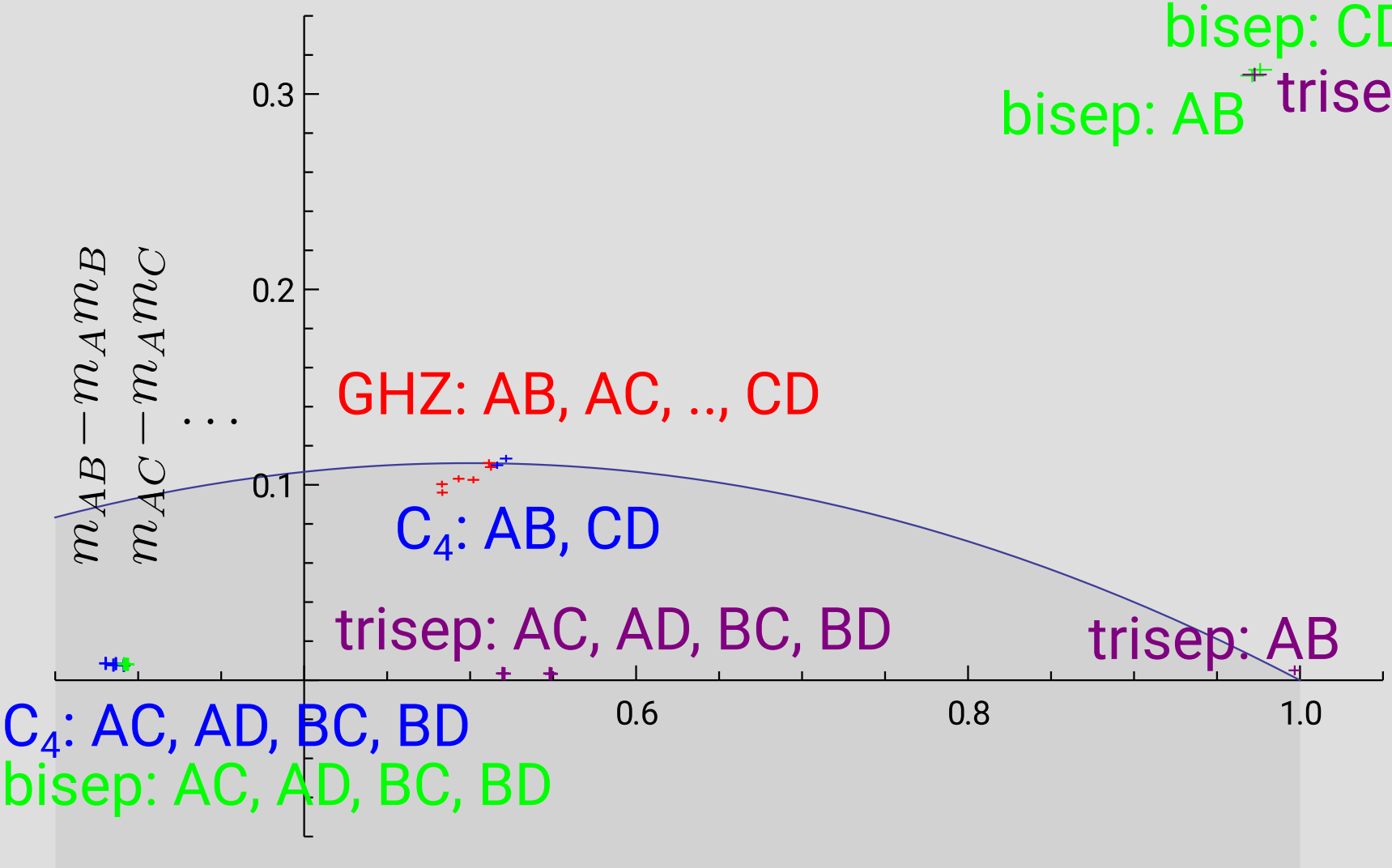

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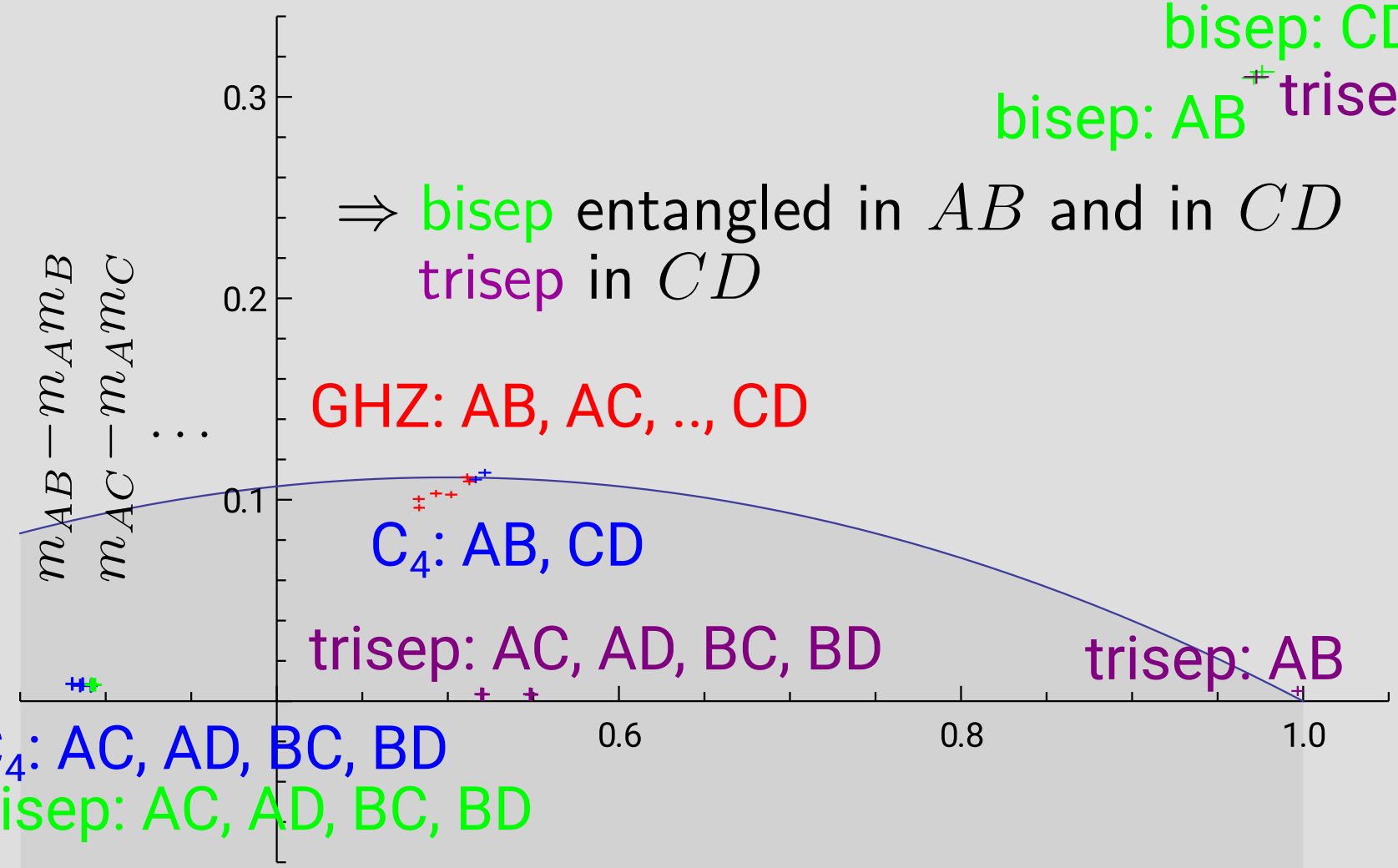
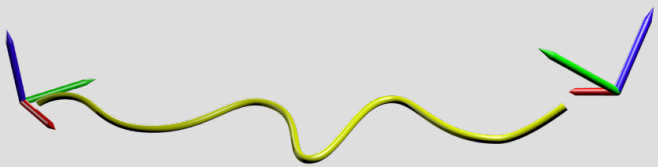
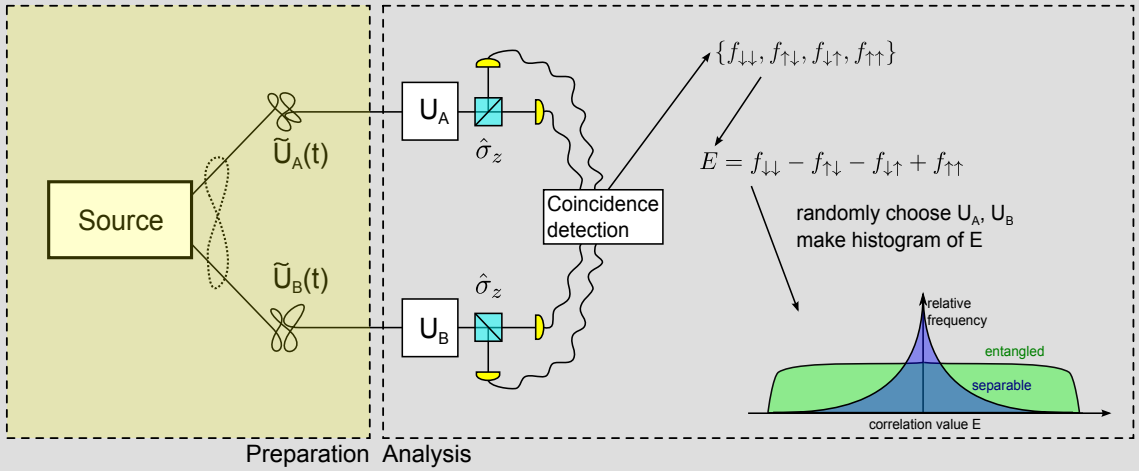


Summary

- inaccessibility of reference frames invalidates usual schemes
- univariate distributions not tomographically complete
- purity dependent bound using second moments
- entanglement structure inspected

Thank you for your attention!





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