Momentum-space entanglement and Loschmidt echo in Luttinger liquids after a quantum quench

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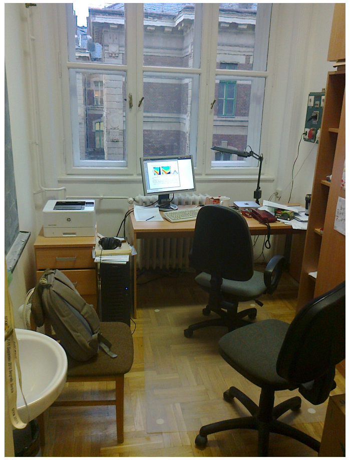
Outline:

- Entanglement: why in momentum space?
- Luttinger liquids
- Loschmidt echo
- Momentum space entanglement in LLs
- Exact diagonalization of spinless fermions (XXZ chain)



Entanglement

Wavefunction: $|\Psi\rangle \longrightarrow$ density matrix: $\rho = |\Psi\rangle\langle\Psi|$.

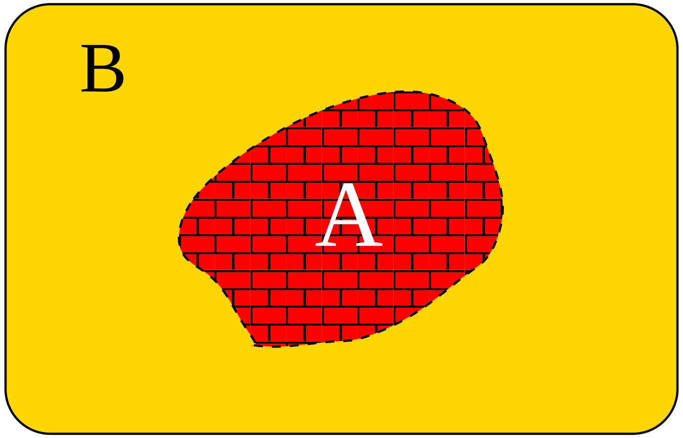
Tracing out B: reduced density matrix $\rho_A = \mathrm{Tr}_B \rho \sim \exp(-H_E)$ entanglement Hamiltonian

entangled state: reduced density matrix does not follow from wavefunction

Universal quantities: entanglement entropy $S = -\mathrm{Tr}\rho_A \ln(\rho_A)$, diagnoses critical points

Entanglement spectrum from H_E: probably not universal

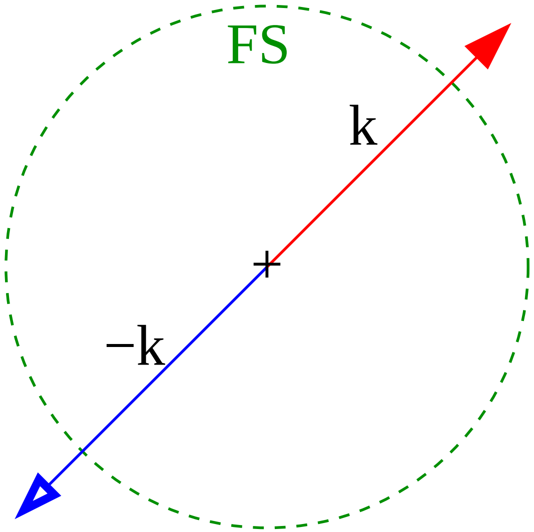
1D critical systems, spatial partitioning: $S = \frac{c}{3}\ln(L)$ in equilibrium (area law) and $\sim t$ after a quench

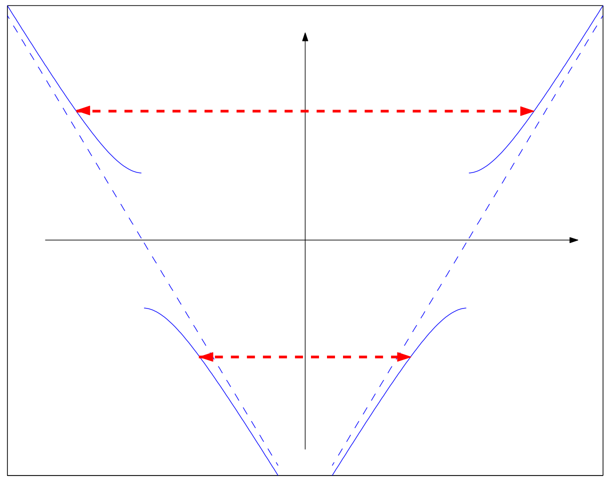


Momentum space entanglement

Many instabilities occur in momentum space rather than in real space:

- Cooper pairs: particles with opposite momentum, superconductivity





- Superfluid Bose systems: particles with opposite momentum, sound

- Density waves: electron-hole pairs with a finite wavevector difference

- Luttinger liquids: coupling right- and left-moving fermions together

A momentum-space partition offers a unique perspective on the structure of many-particle wavefunctions!

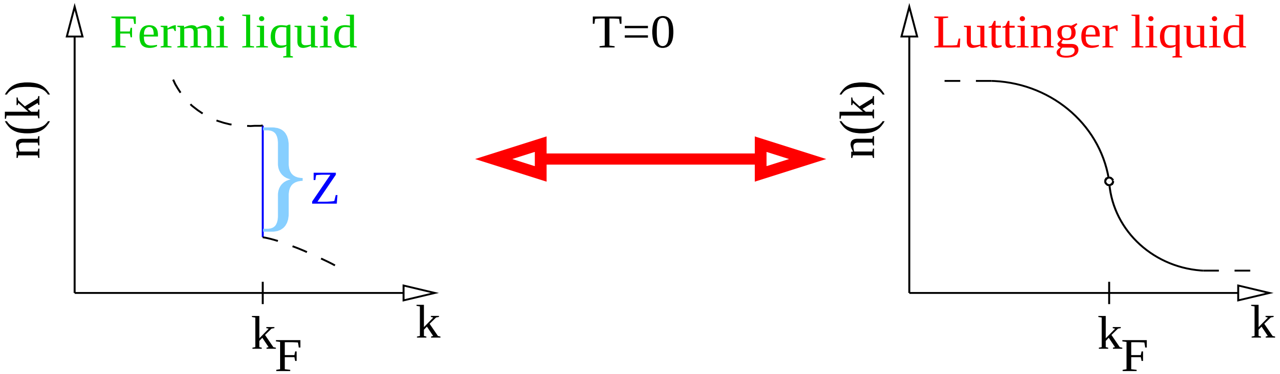
Disentangles quantumness and correlations.

Luttinger liquid in equilibrium

Interacting 1D electron gas: metallic or gapped, Fermi liquid picture breaks down.

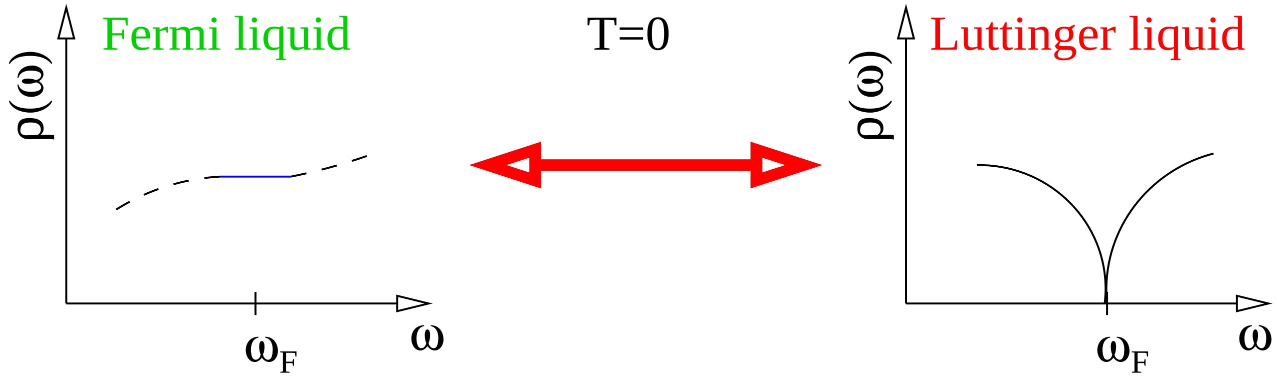
$$\langle \Psi(x,t)\Psi^{+}(0,t)\rangle \sim \quad \frac{Z}{x} \quad \Longleftrightarrow \quad \frac{1}{x^{1+\gamma^2}}$$

momentum distribution:



$$\langle \Psi(x,t)\Psi^{+}(x,0)\rangle \sim \quad t^{-1} \quad \Longleftrightarrow \quad t^{-(1+\gamma^2)}$$

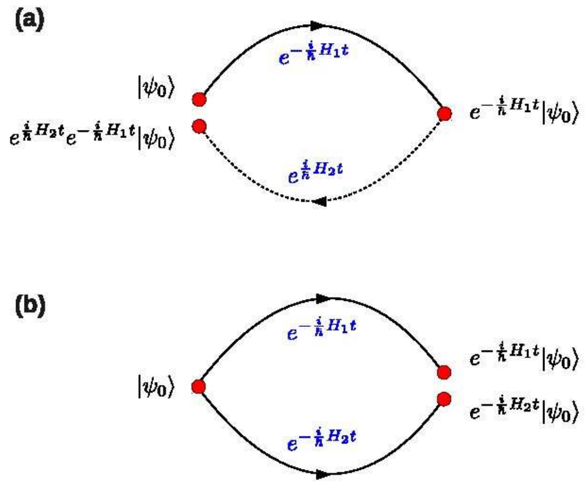
density of states:



Loschmidt echo

$$\mathcal{L}(t) \equiv \left|\langle\Psi_0(t)|\Psi(t)\rangle\right|^2 = \left|\langle\Psi_0|U_2^+(t)U_1(t)|\Psi_0\rangle\right|^2$$

- measures the "distance" between two quantum states, characterizes non-equilibrium time evolution,



- serves to quantify irreversibility and chaos in quantum mechanics,
- diagnoses quantum phase transitions,
- generalized orthogonality catastrophe,
- contains all higher moments of energy, work statistics, $P(W)$,
- measures how small changes during a time evolution cause decoherence and are detrimental for quantum information processing and storage, NMR.

Momentum space entanglement spectrum

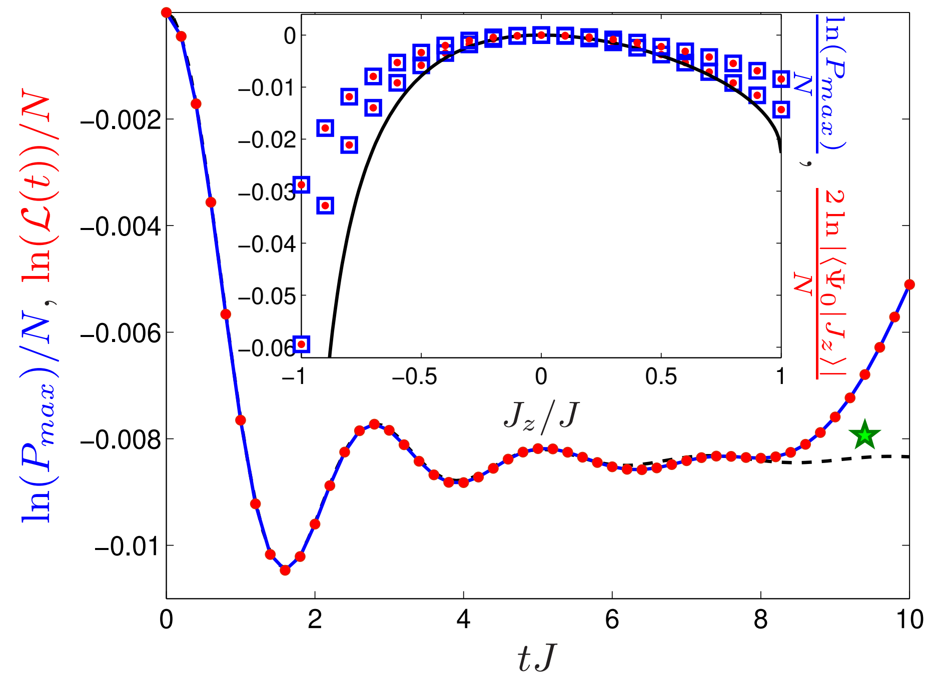
The largest eigenvalue of ρ_A (single copy (or $n = \infty$ Rényi) entropy):

$$P_{max} = |\langle \Psi_0 | \Psi(t) \rangle|^2,$$

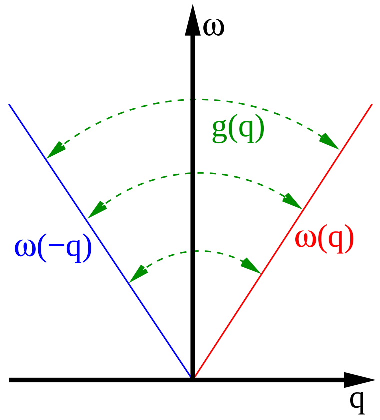
identical to Loschmidt echo and related to work statistics!

Numerics (PBC, $N = 10, 14...26$):

$$H = J \sum_k \cos(k) c_k^\dagger c_k + \frac{J_z}{N} \sum_{k,p,q} \cos(q) c_{p-q}^\dagger c_p c_{k+q}^\dagger c_k,$$

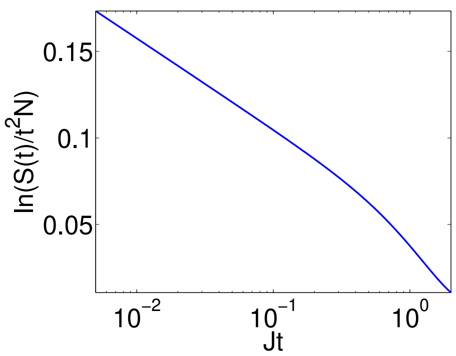


Momentum space entanglement entropy and gap



$$S = 2\sum_{q>0} |u_q(t)|^2 \ln |u_q(t)| - |v_q(t)|^2 \ln |v_q(t)|$$

$S \sim L$, short times: $S(t) \sim Lt^2 \ln(1/t)$

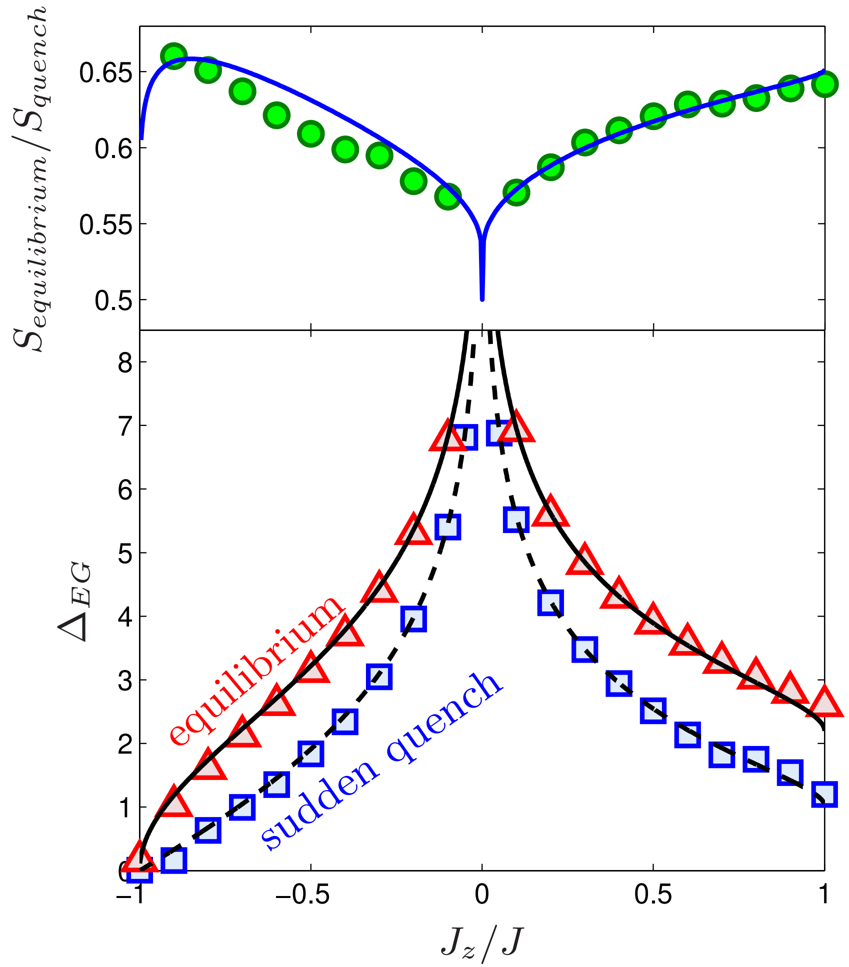


equilibrium:

$$\Delta_{EG} = \ln\left[\frac{(1+K)^2}{(1-K)^2}\right]$$

quench:

$$\Delta_{EG} = \ln\left[\frac{(1+K^2)^2}{(1-K^2)^2}\right]$$



Summary

- Other than real space partitioning is useful for entanglement.
- MSE: disentangles correlations from pure quantumness.
- For a LL: entanglement ground state = Loschmidt echo.
- The EG is universal for LL and stays *finite* at BKT: advantagous for numerics
- Volume law for the EE.
- Measuring ES is difficult, maybe overlaps?