My QM18 talk focused on adding another exp observable, dijet acoplanarity, to Jinfeng Liao's QM17 list of correlated soft+hard tests that every model is now required to pass



My interest in dijet acoplanarity was rekindled by Peter Jacobs' astute question at INT2017 Related to my talk on

Consistency of Perfect Fluidity and Jet Quenching in semi-Quark-Gluon-Monopole-Plasmas (sQGMP) [within the CUJET3.0 framework]

Jiechen X<u>u</u> , J.Liao, MG, Chin.Phys.Lett. 32 (2015) and JHEP 1602 (2016) 169 **Shuzhe Shi**, J.Xu, J.Liao, MG, QM17 **Shuzhe Shi**, J.Liao, MG: arXiv:1804.01915

Probing the Color Structure of the Perfect QCD Fluids via Soft-Hard-Event-by-Event Azimuthal Correlations [via our recent CIBJET= <u>ebe</u> VISHNU+CUJET3.1 framework]

My paraphrase of Peter Jacobs' question :

Can <u>future</u> higher precision dijet acoplanarity measurement be used to **falsify** sQGMP or wQGP or AdS-BH models of jet-medium interactions in near perfect (unitarity bound) QCD fluids ?

<u>Or</u> are dijet observables limited to the extraction of only one effective BDMS medium saturation parameter that is insensitive to the microspcopic color structure of QCD fluids??

$$Q_s^2(a) \equiv \left\langle q_{\perp}^2 \frac{L}{\lambda} \right\rangle_a \equiv \int dt \, \sum_b \hat{q}_{ab}(x(t), t) \equiv \sum_b \int dt d^2 q_{\perp} \, q_{\perp}^2 \Gamma_{ab}(q_{\perp}, t)$$

Can acoplanarity <u>distribution shapes</u> help to extract more information on the color d.o.f in the near Perfect QCD fluids and on the microscopic differential scattering rates, Γ_{ab} , near T ~ T_c?

$$\Gamma_{ab}(q_{\perp},T) = \rho_b(T) d^2 \sigma_{ab}(T) / d^2 q_{\perp}$$



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3

Jet-hadron acoplanarity azimuthal distribution from Chen,Qin,Xiao,Zhang PLB773, 2017 A+A Vacuum Sudakov + BDMS(Qs) model compared to current RHIC and LHC data

State of the "acoplanarity art"

L. Chen et al. / Physics Letters B 773 (2017) 672-676



Fig. 1. Normalized dihadron angular correlation compared with PHENIX [51] and STAR [52] data.



Fig. 2. Normalized hadron-jet angular correlation compared with STAR [53] and the ALICE [54] data. A factor of 3/2 is multiplied to the charged jet energy for our calculation to account for the energy carried by neutral particles. Two sets of ALICE data are shown: TT(trigger track)[20–50] (GeV) represents the signal and TT[20–50] (GeV)–[8–9] (GeV) subtracts the reference to suppress the contribution from the uncorrelated background.

[MG: Current exp precision does not constrain medium opacity better than RAA(pT), but much higher precision future data could test microscopic structure $n_a(T)$ and $d\sigma_{ab}/dq^2$]

(Acoplanarity of) Jets as a probe of quark-gluon plasmas (has a long history)

David A. Appel

Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York 11794 (Received 29 August 1985)

We investigate the propagation of jets through a quark-gluon plasma. The transverse-momentum imbalance of a jet pair is shown to be sensitive to multiple scattering off the constituents of the plasma for expected values of the plasma temperature and size. This raises the possibility that such transverse-momentum imbalance could be used as a probe of a quark-gluon plasma produced by partonic interactions in ultrarelativistic nucleus-nucleus collisions.

PHYSICAL REVIEW D

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1 NOVEMBER 1986

Jets in expanding quark-gluon plasmas

J. P. Blaizot

Service de Physique Theorique, CEA Saclay, 91191 Gif-sur-Yvette, Cedex, France

Larry D. McLerran

In summary, our analysis supports Appel's conclusion that jets may provide a useful diagnostic tool for studying the quark-gluon plasma. We have shown that in highenergy nuclear collisions, the effects of jet rescattering do in fact appear in the acoplanarity distribution. The cross section for scattering from the plasma may be inferred.

We should be careful to note, however, that the existence of acoplanarity does not by itself alone give evidence for a quark-gluon plasma, and may in fact be generated by scattering from a hadronic gas. The jet acoplanarity is therefore not a signal for the plasma, merely a diagonstic

M. Rammerstorfer and U.Heinz, (1990):" We find serious hadronic background effects from the surrounding nuclear matter in nuclear collisions, which severely limit the usefulness of jet acoplanarity as a quark-gluon-plasma probe. "

Ancient History of Acoplanarity

J.P.Blaizot, L.McLerran(1986); M. Greco,(1985); ... V. Sudakov (1956) D.Appel 1986 In the parton model there are no bremsstrahlung ef-Acoplanarity in fects, so we have simply $dP/dK_n = \delta(K_n)$. With pertur-Vacuum is due to bative QCD, multiple gluon emission from the hard Gluon radiation scattering can be resummed in perturbation theory,14 and from dijet antenna for the one-dimensional normal momentum density has the form $\frac{1}{\sigma_0(p,p_T)} \frac{1}{p_T} \frac{d\sigma}{d\phi} = \frac{dP}{dK_n} = \frac{1}{\pi} \int_0^\infty db \cos(K_\eta b) \exp[\widetilde{B}(b)] .$ In Double leading log $\widetilde{B}(b) = -\int_{(b_0/b)^2}^{Q^2} \frac{dq^2}{q^2} \left| \ln \left| \frac{Q^2}{q^2} \right| A'(\alpha_s(q)) + B'(\alpha_s(q)) \right|$ Sudakov approx

Additional acoplanarity in A+A arises from Jet-medium multiple scattering probability

 $F(\ell_T) \propto d\sigma/d^2\ell_T$

$$\frac{dP}{dK_{\eta}} = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \left[\frac{1}{n!} \prod_{i=1}^{n} \int d^{2}k_{Ti} B(\mathbf{k}_{Ti}) \frac{1}{m!} \prod_{j=1}^{m} \int d^{2}l_{Tj} F(l_{Tj}) \delta\left[K_{\eta} - \sum_{i=1}^{n} (\mathbf{k}_{Ti})_{\eta} - \sum_{j=1}^{m} (l_{Tj})_{\eta} \right] \right]$$

 $\int_{-\infty}^{+\infty} dK_{\eta} \exp(iK_{\eta}b) \frac{dP}{dK_{\eta}} = \exp[\widetilde{B}(b) + \widetilde{F}(b)]$

Exact trans mom conservation is easiest To enforce in conjugate b-space



Leading Double Log Approximation <u>Vanishes</u> at kT=0 and at kT=Q

Momentum conservation via b-space Leads to finite q=0 limit

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and R_1 is the region of integration for $S(\theta)$. The complementary region R_2 is the range of integration for $T(\theta)$ and $S(\theta) + T(\theta) = 1$.

Fig. 3.2 and to arrive at what will turn out to be the leading double logarithm we can use the LPA approximation in (3.4.10),

$$\frac{1}{\sigma_0} \left(\frac{d\sigma}{dz d\hat{t}} \right)_{\text{LPA}} = \frac{2\alpha_s}{3\pi} \frac{1+z^2}{(1-z)\hat{t}}.$$
(3.8.7)

This term dominates since the range of integration for $S(\theta)$ is from (3.4.6) given by

$$\frac{k_T^2}{Q^2} = z(1-z)\hat{t} \le \frac{\theta^2}{4},\tag{3.8.8}$$

D.Appel 1986 Jet Scattering in multi-component q+qbar+g plasmas was considered

For $F(l_T)$, the probability density for scattering elastically off the plasma constituents with transverse-momentum transfer l_T , we propose the following form:

$$F(l_T) = \sum_{\mathbf{x}} n_{\mathbf{x}} R \frac{d^2 \sigma_{\mathbf{x}}}{d^2 l_T} , \qquad (11)$$

where x runs over the different particle types comprising the plasma $(x = g, q_i, \overline{q_i})$, with n_x their number density. This equation essentially relates the plasma mean free path to the available distance for scattering (R) for each particular l_T .



$$F(l_T) = 9aRT^3 \left[1 + \frac{N_F}{4} \right] \frac{\alpha_s^2(l_T)}{l_T^4}$$

Cut off soft divergence below pQCD Debye mass $\ell_\perp \sim gT$

"Based on this, one is encouraged to conjecture that someday jet behavior could be used as an effective thermometer of a QCD plasma."

Confirmed by J.P.Blaizot, L.McLerran(1986) In more realistic detail



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ample we use $\mu^2 = 0.25$ GeV².





10% Percent level precision needed <u>even to resolve BDMS Qs</u> from Sudakov $\sim lpha/q^2$



One parameter, Q_s, BDMS medium convoluted with Sudakov dijet transverse distributions



For realistic Sudakov fits to p+p need lower $\alpha \approx 0.09$ and next to leading corr. Requires very high precision to resolve GLV finite (χ,μ) from BDMS(Qs) medium effects



Ratio dN(Vac+GLV)/dN(Vac) (red) vs dN(Vac+BDMS)/dN(Vac) (blue) vs q



Ratio dN(Vac+GLV)/dN(Vac) Qs²=9.6 (red), 16 (blue) vs q for Q=20, α=0.09, (μ,χ)=(0.5,6&10)sol, (0.75,3.1&5.1)dash, (0.25,20&33)dot



Cosmic Inspiration for pushing toward a future high precision era of A+A

1 part per 100,000 fluctuations can and have been observed to constrain cosmological models

https://en.wikipedia.org/wiki/Cosmic_microwave_background

Graph of cosmic microwave background spectrum measured by the FIRAS instrument on the COBE, the most precisely measured black body spectrum in nature.[7] The error bars are too small to be seen even in an enlarged image, and it is impossible to distinguish the observed data from the theoretical curve.



CMS Studies of dijet transverse momentum balance and pseudorapidity distributions in pPb collisions at 5.02 TeV have alreadyachieved great precision

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Eur. Phys. J. C (2014) 74:2951



Very high precision has (after 30 years) been reached at LHC in pp and pA that constrain vacuum Sudakov acoplanarity due to jet gluon showers. Thus Sudakov A, B and *non-perturb* D factors can now be tuned to high accuracy and to higher NN..LO α_s^n



FIG. 6: (Color online) Angular distribution of γ -jet in central (0–30%) Pb+Pb (red) and p+p collisions (blue) at $\sqrt{s} = 2.76$

Exp should focus in "sweet spot"

 $2.4 < \Delta \phi < \pi$

To reduce large distortion due to the quenching of multiple medium minijets unrelated to the dijett

Multiple jets and y-jet correlation

in high-energy heavy-ion collisions

Luo,Cao,He,Wang CCNU arXiv:1803.06785 [hep-ph]

High pT~ 100 GeV makes small angle Deviations from pi nearly independent Of medium effect and are dominated by Vaccum Sudakov effects.

At large angles < 2 there is a predicted suppression of gam-jet correlations due to multiple induced jet suppression complementary to RAA(pT) Sensitive to qhat(E,T).

"Dominance of the Sudakov form factor in y-jet correlation from soft gluon radiation in large pT hard processes pose a challenge for using y-jet azimuthal correlation to study medium properties via large angle parton-medium interaction." High precision needed to map out the temperature and jet energy dependence of

the microscopic composition and rates

$$\Gamma_{ab}(q_{\perp},T) = \rho_b(T) d^2 \sigma_{ab}(T) / d^2 q_{\perp}$$

Will need "multi-messenger" precision experimental constraints to get beyond simple Qs phenomenlogy and try to deconvolute Γ_{ab} from soft+jet and soft+dijet observables These rates have so far been hidden inside

$$Q_s^2(a) \equiv \left\langle q_\perp^2 \frac{L}{\lambda} \right\rangle_a \equiv \int dt \; \sum_b \hat{q}_{ab}(x(t), t) \equiv \sum_b \int dt d^2 q_\perp \; q_\perp^2 \Gamma_{ab}(q_\perp, t)$$

Qs is a path integral functional over ensemble averaged over evolving e-by-e fluctuating local temperature and flow velocity fields T(x,t), u(x,t) and limited to the second moment in qT space

From our extensive global CIBJET=ebe IC + VISHNU + CUJET3.1 Analysis of RHIC+LHC1+LHC2 data on light and heavy single jet RAA & vn There is strong indication for highly nontrivial nonperturbative physics near Tc That can be captured by the semi-Quark+Gluon+Monopole Plasma model Of the QCD perfect fluid consistent not only with Exp data but also Lattice QCD data As well as providing a microscopic picture of how near unitarity bound eta/s can arise Through emergent color magnetic monopole d.o.f. in the cross over temperature range

Precision acoptanarity <u>distribution shapes can test such models</u> on the color d.o.f in the near Perfect QCD fluids by constrining the microscopic differential scattering rates, Γ_{ab} , near T ~ T_{c} ?



CIBJET was developed by A. Buzzatti, J.Xu, and Shuzhe Shi to quantitatively test this _{MGyulassy Wigner 5/25/18} idea with global chi² analysis of SPS, RHIC and LHC RAA, v2, v3 data

$$\hat{q}_{F}(E,T) = \int_{0}^{6ET} dq_{\perp}^{2} \frac{2\pi}{(\boldsymbol{q}_{\perp}^{2} + f_{E}^{2}\mu^{2}(\boldsymbol{z}))(\boldsymbol{q}_{\perp}^{2} + f_{M}^{2}\mu^{2}(\boldsymbol{z}))}\rho(T) \\ \times \left\{ \left[C_{qq}f_{q} + C_{qg}f_{g} \right] \cdot \left[\alpha_{s}^{2}(\boldsymbol{q}_{\perp}^{2}) \right] \cdot \left[f_{E}^{2}\boldsymbol{q}_{\perp}^{2} + f_{E}^{2}f_{M}^{2}\mu^{2}(\boldsymbol{z}) \right] + \left[C_{qm}(1 - f_{q} - f_{g}) \right] \cdot \left[1 \right] \cdot \left[f_{M}^{2}\boldsymbol{q}_{\perp}^{2} + f_{E}^{2}f_{M}^{2}\mu^{2}(\boldsymbol{z}) \right] \right\},$$
(14)

$$\hat{q}_{g}(E,T) = \int_{0}^{6ET} dq_{\perp}^{2} \frac{2\pi}{(q_{\perp}^{2} + f_{E}^{2}\mu^{2}(\boldsymbol{z}))(q_{\perp}^{2} + f_{M}^{2}\mu^{2}(\boldsymbol{z}))}\rho(T) \\
\times \left\{ \begin{bmatrix} C_{gq}f_{q} + C_{gg}f_{g} \end{bmatrix} \left[\alpha_{s}^{2}(q_{\perp}^{2}) \right] \cdot \left[f_{E}^{2}q_{\perp}^{2} + f_{E}^{2}f_{M}^{2}\mu^{2}(\boldsymbol{z}) \right] + \\
\begin{bmatrix} C_{gm}(1 - f_{q} - f_{g}) \end{bmatrix} \cdot \left[1 \right] \cdot \left[f_{M}^{2}q_{\perp}^{2} + f_{E}^{2}f_{M}^{2}\mu^{2}(\boldsymbol{z}) \right] \right\}, \quad (15)$$

$$\hat{q}_{m}(E,T) = \int_{0}^{6ET} dq_{\perp}^{2} \frac{2\pi}{(q_{\perp}^{2} + f_{E}^{2}\mu^{2}(\boldsymbol{z}))(q_{\perp}^{2} + f_{M}^{2}\mu^{2}(\boldsymbol{z}))}\rho(T) \\
\times \left\{ \begin{bmatrix} C_{mq}f_{q} + C_{mg}f_{g} \end{bmatrix} \cdot \left[1 \right] \cdot \left[f_{L}^{2}q_{\perp}^{2} + f_{E}^{2}f_{M}^{2}\mu^{2}(\boldsymbol{z}) \right] + \\
\begin{bmatrix} C_{mm}(1 - f_{q} - f_{g}) \end{bmatrix} \cdot \left[\alpha_{s}^{-2}(q_{\perp}^{2}) \right] \cdot \left[f_{M}^{2}q_{\perp}^{2} + f_{E}^{2}f_{M}^{2}\mu^{2}(\boldsymbol{z}) \right] \right\}. \quad (16)$$

The HTL wQGP model of the perfect QCD fluid is obtained with fE=1 and fM=0 AND setting poly loop L=1 AND chiral suscept =1. Global chi^2 rules out this models <u>And</u> internally it is inconsistent with eta/s near 1/4pi and hence inconsistent with soft observables MGyulassy Wigner 5/25/18 Conclusion: we need to add dijet acoplanarity and strive for higher precision Demanding every model to pass all global soft+hard probes tests consistently In order to extract conclusions about the novel color structure of QCD perfect fluids





<u>Appendix: Review of past and current advances with CUJET and CIBJET</u>

Jiechen Xu, J.Liao, MG, Chin.Phys.Lett. 32 (2015), JHEP 1602 (2016) 169

Shuzhe Shi, J.Xu, J.Liao, MG, QM17, NPA967 (2017) 648-651

Shuzhe Shi, J.Liao, MG: arXiv:1804.01915

The Liao-Shuryak sQGMP Transition Phase



Slide from Jinfeng Liao, APS DNP Hawaii 2014

In CUJET3 we tested 4 models of sQGMP composition compatible with Lattice QCD thermo



Figure 6. (Color online) (a) The effective ideal quasiparticle density, $\rho/T^3 = \xi_p P/T^4$, in the Pressure Scheme (PS, Blue) is compared with effective density, $\rho/T^3 = \xi_p S/4T^3$, in the Entropy Scheme (ES, Red) based on fits to lattice data from HotQCD Collaboration [56]. The difference is due to an interaction "bag" pressure $-B(T)/T^4$ (Green) that encodes the QCD conformal anomaly

VISHNU+CUJET3.0 implemented sQGMP in realistic visc.hydro fit to RHIC & LHC1

$$\chi^2/dof \sim 1.0 - 1.3 \quad \longrightarrow \quad (\alpha_c = 0.9 \pm 0.1, c_m = 0.25 \pm 0.03)$$



J.Xu, J.Liao, MG, Chin. Phys. Lett. 32 (2015)

The combined set of observables

(R_{AA}+v₂)*(RHIC+LHC)*(pion+D+B)

are consistently accounted for in CUJET3.0 using lattice data constrained sQGMP near Tc + pQCD/DGLV jet quenching At QM17 CMS/LHC2 found discrepancies with CUJET3.0 predictions for the centrality dependence of 5ATeV RAA and v2

Shuzhe Shi found 3 bugs in CUJET3.0, now corrected in CUJET3.1

1) Initial parton spectra for 5.02 ATeV were erroneously read in

2) VISHNU hydro fluid grid for 5.02 incorrectly oriented in CUJET3.0

3) The parton spectra range was set too low for the 5.02 run



CMS 5.02 ATeV

v₂{SP}

v₃{SP}

0.2

0.0

5

---v₂ CUJET3.0

--v₂ SHEE, lin.

····v, SHEE, lin.

(Shuzhe Shi et al 2018) CUJET3.1 test of v2 centrality dependence at 5.02ATeV vs CMS data



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FIG. 1: (color online) χ^2 /d.o.f. comparing χ^L_T -scheme CUJET3 results with RHIC and LHC data. Left:

 $\chi^2/d.o.f.$ for R_{AA} only. Middle: $\chi^2/d.o.f.$ for v_2 only. Right: $\chi^2/d.o.f.$ including both R_{AA} and v_2

An open question at QM 2017 was how much would the inclusion of ebe fluctuations of Initial Conditions modify CUJET3.1 results using only event averaged IC geometries. Shuzhe Shi generalized CUJET3.1 to ebe CIBJET = ebe IC+VISHNU+CUJET3.1 framework And found that with CIBJET ebe only makes ~10% changes relative to event ave geom MGyulassy Wigner 5/25/18

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Consistent Soft-Hard Event Engineering in the ebe CIBJET framework





FIG. 2: (color online) A comparison of the azimuthal anisotropy coefficient $v_2(p_T)$ at high transverse momentum, computed from different models for 40-45% Pb+Pb collisions at 2.76ATeV and 5.02ATeV. All models are calibrated with R_{AA} data already. Both CIBJET (red) and CLV+LBT (black) [39] models demonstrate very small difference between their respective average-geometry results (dashed curves) and event-by-event (solid curves) results. Compared with available CMS measurements [37, 41], results from CIBJET model as well as event-by-event vUSPhydBBMG model [11] agree There is Current Tension between the degenerate Solutions of RAA-v2 puzzle with

Ebe CIBJET using sQGMP rates

And ebe-vUSPH with BBMG wQGP rates

We will Need further observables to Break this theory degeneracy. Dijet acoplanarity may discriminate very different internal rates of the two soft+hard frameworks

$$\Gamma_{ab}(q_{\perp},T) = \rho_b(T) d^2 \sigma_{ab}(T) / d^2 q_{\perp}$$

Shuzhe Shi, J.Liao, MG: arXiv:1804.01915

Compare sQGMP to wQGP and Zakharov's mQGP JETP Lett.(2015) where q+g are not suppresed but a monopole component added.

The suppressed semi-QGP components of sQGMP require larger monopole density than in mQGP to compensate the loss q+q component entropy to fit the lattice EOS P/T or S(T)



The mQGP monopole component is not big enough to reduce η /s close the SYM limit And it underpredicts jet v2 data , in contrast to the CUJET 3 component semi-QGMP model

Extra Slides Below

Lin Chen, Guang-You Qin *, Shu-Yi Wei, Bo-Wen Xiao, Han-Zhong Zhang PLB 773 (2017) 672 "Probing transverse momentum broadening...

distribution in the Sudakov resummation formalism as follows

$$\frac{d\sigma}{d\Delta\phi} = \sum_{a,b,c,d} \int p_{\perp\gamma} dp_{\perp\gamma} \int p_{\perp J} dp_{\perp J} \int dy_{\gamma} \int dy_{J} \int db \\
\times x_{a} f_{a}(x_{a},\mu_{b}) x_{b} f_{b}(x_{b},\mu_{b}) \frac{1}{\pi} \frac{d\sigma_{ab\to cd}}{d\hat{t}} b J_{0}(|\vec{q}_{\perp}|b) e^{-S(Q,b)},$$
(1)

where J_0 is the Bessel function of the first kind, q_{\perp} is the transverse momentum imbalance between the photon and the jet $\vec{q}_{\perp} \equiv \vec{p}_{\perp\gamma} + \vec{p}_{\perp J}$, which takes into account both initial and final transverse momentum kicks from vacuum Sudakov radiations and medium gluon radiations. Here we define $x_{a,b} = max(p_{\perp\gamma}, p_{\perp J})(e^{\pm y_{\gamma}} + e^{\pm y_{J}})/\sqrt{s_{NN}}$ as

The vacuum Sudakov factor $S_{pp}(Q, b)$ is defined as

$$S_{pp}(Q,b) = S_P(Q,b) + S_{NP}(Q,b)$$
 (2)

where the perturbative S_P Sudakov factor depends on the incoming parton flavour and outgoing jet cone size. The perturbative Sudakov factors can be written as [35–37]

pQCD Vacuum Shower
$$S_P(Q,b) = \sum_{q,g} \int_{\mu_b^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left[A \ln \frac{Q^2}{\mu^2} + B + D \ln \frac{1}{R^2} \right]$$
 (3)

At the next-to-leading-log (NLL) accuracy, the coefficients can be expressed as $A = A_1 \frac{\alpha_s}{2\pi} + A_2 (\frac{\alpha_s}{2\pi})^2$, $B = B_1 \frac{\alpha_s}{2\pi}$ and $D = D_1 \frac{\alpha_s}{2\pi}$, with the value of individual terms given by the following table, where both A and B terms are summed over the corresponding incoming parton flavours.

$$A_1$$
 A_2 B_1 D_1 quark C_F $K \cdot C_F$ $-\frac{3}{2}C_F$ C_F gluon C_A $K \cdot C_A$ $-2\beta C_A$ C_A

Here C_A and C_F are the gluon and quark Casimir factor, respectively. $\beta = \frac{11}{12} - \frac{N_f}{18}$, and $K = (\frac{67}{18} - \frac{\pi^2}{6})C_A - \frac{10}{9}N_fT_R$. $R^2 = \Delta \eta^2 + \Delta \phi^2$ represents the jet cone-size, which is set to match the experimental setup. The implementation of the non-perturbative Sudakov factor $S_{NP}(Q, b)$ follows the prescription given in Refs [61, 62]. In the Sudakov resummation formalism, following the usual b^* prescription, the factorization scale is set to be $\mu_b \equiv \frac{c_0}{b_\perp} \sqrt{1 + b_\perp^2/b_{max}^2}$,

Logarithmic approximations, quark form factors, and quantum chromodynamics





FIG. 4. Theoretical approximations to the cross section defined in the text. The long-dashed line is the soft logarithmic approximation [LA, (1), (2), (3)]. The solid line is the DLLA Eq. (2.12). The dashed line is the corresponding one-gluon contribution.

A. Kulesza, W.J. Stirling/Nuclear Physics B 555 (1999) 279-305

WIGYUIASSY WIGNER 5/25/18

It is convenient to return to the general notation
of the Introduction and define
$$\eta = Q_T^2 / s$$
 and $\lambda = \alpha_s C_F / \pi$. Thus Eq. (2.11) can be written as
 $\frac{1}{\sigma_0} \frac{d\sigma}{d\eta} \Big|_{\text{DLLA}} = \frac{\lambda}{\eta} \ln \frac{1}{\eta} \exp\left(-\frac{\lambda}{2} \ln^2 \eta\right) \theta(1-\eta)$
 $= \frac{d}{d\eta} F_{\text{DLLA}}(\eta) \theta(1-\eta)$ (2.12)
with $F_{\text{DLLA}}(\eta)$ identified from Eq. (1.1).

with $C_F = \frac{4}{3}$, $T_R = \frac{1}{2}$ and N = 3. It is instructive to see how the logarithms in *b*-space generate logarithms in q_T -space. For illustration, we take only the leading coefficient $A^{(1)} = 2C_F$ to be non-zero in $e^{S(b,Q^2)}$, and assume a fixed coupling α_S . This corresponds to

$$\frac{d\sigma}{dq_T^2} = \frac{\sigma_0}{2} \int_0^\infty b \, db \, J_0(q_T b) \exp\left[-\frac{\alpha_s C_F}{2\pi} \ln^2\left(\frac{Q^2 b^2}{b_0^2}\right)\right]. \tag{6}$$

The expressions are made more compact by defining new variables $\eta = q_T^2/Q^2$, $z = b^2Q^2$, $\lambda = \alpha_S C_F/\pi$, $z_0 = 4 \exp(-2\gamma_E) = b_0^2$. Then

$$\frac{1}{\sigma_0} \frac{d\sigma}{d\eta} = \frac{1}{4} \int_0^\infty dz J_0(\sqrt{z\eta}) e^{-\frac{\lambda}{2} \ln^2(z/z_0)}$$
(7)

and we encounter the same expression as in [6], which describes the emission of soft and collinear gluons with transverse momentum conservation taken into account. The result

The conclusion is then that the subleading logarithms which arise from a correct treatment of transverse-momentum conservation can play a major role in filling in the zero at $\eta = 0$ and obscuring the maximum which was present near $\ln 1/\eta \sim 1/\lambda$ in the DLLA. It is informative to di-