Particle interpretation in interacting systems

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- in Monte Carlo simulations particles are identified by $e^{-m\tau}$ imaginary time dependence $\Rightarrow$ E-level of the Hamiltonian
  How can it be temperature dependent? How can it melt?
- fields acquire wave fct. renormalization with $Z \leq 1$
  Are they "whole" degrees of freedom? And if $Z \to 0$?
- When are two particles indistinguishable?
- In a crossover everything changes continuously
  How can the number of dof change continuously?

Goal: answer to these questions & go beyond. . .

Outlines

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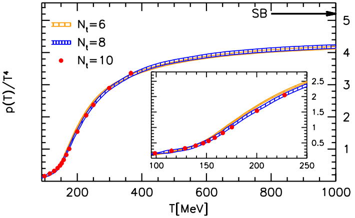
The QCD phase structure

Present day particle physics experiments:
CERN LHC, BNL RHIC, Fermilab Tevatron: hadron-hadron colliders

QCD at finite temperature/chemical potential?



(Sz. Borsanyi et al, JHEP 1011 (2010) 077)

- at low T: Hadron Resonance Gas (HRG)
- SB limit at high T:
  8 gluon + 2 relativistic quark dof
- continuous phase transition (crossover): what are the dof here?

Why is so difficult to treat QCD equation of state?

- strong interactions
- gauge symmetry
- zero mass particles
- bound states at low T
- crossover near $T_c = 157\,\mathrm{MeV}$
  hadrons at high T phase? – observable
  quarks at low T phase? – not observable
- fluid near phase transition region
  - fluidity measure $\eta/s \approx \hbar k/(4\pi)$ (viscosity at own scale): small $\Rightarrow$ very "good" liquid
  - no oscillating density-correlation $\Rightarrow$ not ordinary liquid (more like supercritical water, permanent fluid)

Is it a particle system?

- NO? – strong interactions may reorganize the system fully, no more and less important components: no hope of analytic treatment
- YES? with the usual perturbative dof – improved calculation of interactions (eg. HTL, high loop DR, Polyakov loop dynamics, Boltzmann eq. with 2-3 scattering, etc.)
- YES? but the elementary dof are not the usual particles (eg. $N = 4$ SYM, 5D AdS gravity duality; 2PI dressed quasiparticles)

  $\Rightarrow$ a perturbative approach is possible...

So the main question is:

What are the elementary degrees of freedom of an interacting particle system?

Outlines

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By subdividing matter we arrive at point masses $\Rightarrow$ particles

Experience: dynamics of matter can be understood from the dynamics of elementary parts

- One particle state: point in the phase space P
- Multiparticle state $\in P^N$, N is the particle number

evolution (dynamics):

- classical mechanics, Newton's law
- Boltzmann equation $\rightarrow$ equilibrium
- fails under a scale (quantum effects)
- other failure: yields non-extensive thermodynamics
  $\Rightarrow$ indistinguishability of particles, Gibbs paradox

Outlines

- State of a system: Hilbert space
- transformations, measurements: operators
- Identification/definition of particle is not always possible in general...

Possible methods:

- particle number operator, Fok space
- spectral defintion and time evolution
- dynamical definition: longest living exciations (linear response theory)
- OR linear response theory at $T > 0$
- statistical/thermodynamical definition

Experience: in certain (idealized, free) systems these definitions yield the same concept

$\Rightarrow$ particle

Particle number operator

Conserved quantities:

- generator of time evolution: $\hat{H}$ Hamiltonian
- Q mutually commuting conserved operator set (eg. $Q = \{\hat{\mathbf{p}}, J^2, J_z, \dots\}$ and $[Q_i, \hat{H}] = 0$)
  $\Rightarrow$ common eigensystem (quantum channel, SSC)

In free systems $\exists \hat{N} \in Q$ number operator

- Ground state (vacuum): $N = 0$ sector
- def.: Particle: state in $N = 1$ SSC
  $\exists$ 1-particle QM, wave function, Schrödinger equation.
- Multiparticle states: Fok space construction

Field operators, spectral function

We can introduce some concepts

- annihilation operator $\hat{a}_p : N+1\,\mathrm{part} \to N\,\mathrm{part}$
- field operator: $\hat{\Psi}(\mathbf{p},t) \sim \mathcal{N}_p a_p$
  $\hat{\Psi}(x)\left|0\right\rangle \sim$ particle state at position x
- spectral function: $\varrho(t) = \langle 0|[\hat{\Psi}(t), \hat{\Psi}^\dagger(0)]_\pm|0\rangle$ ($\pm$ fermionic/bosonic).
  calculate it in Fourier space (include complete system):

$$\varrho(\omega > 0) = \sum_n 2\pi\delta(\omega - E_n)|\left\langle 0|\Psi|n\right\rangle|^2$$

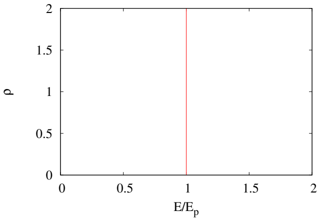
$\Rightarrow$ useful tool to obtain spectral density

the same can be repeated for SSC Q:

- $\hat{A}_Q \quad \Rightarrow \quad \Psi_Q \quad \Rightarrow \quad \varrho_Q(t) = \langle 0|[\hat{\Psi}_Q(t), \hat{\Psi}_Q^\dagger(0)]_\pm|0\rangle$
- $\Psi_Q \sim \Psi^N$, but there can be multiple choices
- $\varrho_Q(t)$ yields the energy spectrum at quantum numbers Q

Spectrum and time dependence at $N = 1$

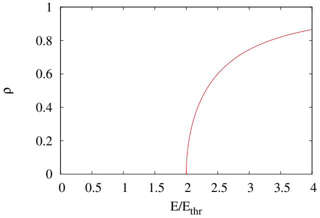
$N = 1$ sector, fixed $\hat{\mathbf{p}}$, Q (bound states are internal dof, see later)



- one single energy level at E_p (dispersion relation)
- time dependence of a 1-particle is unique $\sim e^{-iE_p t}$

Spectrum and time dependence at $N = 2$

$N = 2$ sector, for example with $\mathbf{p} = 0$, fixed other Q



- lots of energy levels (continuum at infinite volume) $|\mathbf{q}, -\mathbf{q}\rangle$ states have zero complete momentum
- in relativistic systems $\varrho(\omega) \sim \Theta(\omega - 2m)\sqrt{1 - \frac{4m^2}{\omega^2}}$
- time dependence is not unique: $\sum_n c_n e^{-iE_n t}$ $\Rightarrow$ need infinite initial conditions, or history

$\Rightarrow$ not a particle-like spectrum!

Linear response theory

Create a state at $t = 0$ and observe field operator:

$$iG_{ret}^{(Q)}(t) = \Theta(t)\langle[\hat{\Psi}_Q(t), \hat{\Psi}_Q^\dagger(0)]_\pm\rangle = \Theta(t)\varrho_Q(t)$$

- for $t > 0$ equivalent to the spectral function
- can be defined at finite temperature ($\langle.\rangle \to \frac{1}{Z}\,\mathrm{Tr}\, e^{-\beta\hat{H}}$)

For a 1-particle state: $iG_{ret}(\mathbf{p}, t) \sim e^{-iE_p t}$ for all temperatures
$\Rightarrow$ the same unique time dependence

(2-particle state: for large times $\varrho_A(t) \sim t^{-3/2}$; different at finite T)

Thermodynamics

Partition function $Z = e^{-\beta V f} = \mathrm{Tr}\, e^{-\beta \hat{H}} = \sum_n e^{-\beta E_n}$.
In free systems $N = 1$ sector determines the complete thermodynamics

$$f = \sum_Q (\mp) T \int \frac{d^3\mathbf{p}}{(2\pi)^3} \ln(1 \pm e^{-\beta(E_{Q,p} - \mu_Q)})$$

- all particles yield equal weight contribution
  $\Rightarrow$ particles $\equiv$ thermodynamical dof
- only the energy levels count (not the way we measure them)
- in relativistic systems at $T \to \infty$ Steffan-Boltzmann limit

$$P_{SB} = \frac{\pi^2}{90} \left(N_b + \frac{7}{8} N_f \right).$$

$N_{b/f}$ are the number of bosonic/fermionic particle species.

Outlines

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- particle number
- spectral defintion and time evolution
- dynamical definition: longest living exciations (linear response theory)
- OR linear response theory at $T > 0$
- statistical/thermodynamical definition

Gave the same particle concept for free systems

They yield different concepts for interacting systems!

- particle number – $\hat{N}$ is not conserved ✗
- spectral defintion and time evolution
- dynamical definition: longest living exciations (linear response theory)
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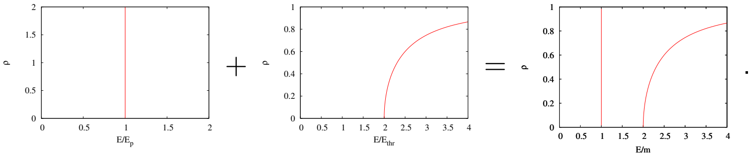
Gave the same particle concept for free systems

They yield different concepts for interacting systems!

Spectral defintion and asymptotic states

spectra of different N sectors mix together



- multiple energy levels
- time dependence is not unique, depends on the history ✗

Solution I: asymptotic particle state

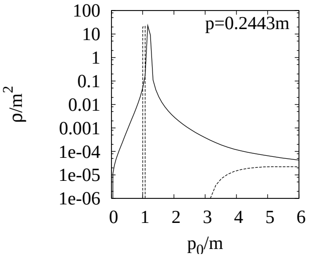
- at $T = 0$: may $\exists$ discrete E-level
- linear response for long times: $Ze^{-iEt} + Ct^{-3/2}e^{-iE_{thr}t}$
  long time behaviour unique ✓
  Z is wave function renormalization (sum rule)

- particle number – $\hat{N}$ is not conserved ✗
- spectral defintion, longest living exciations:at zero temperature – asymptotic states
- linear response theory at massless case or at $T > 0$
- statistical/thermodynamical definition

Usually there is no clear distinction between particle and continuum states, if

- zero mass excitation (no gap)
- unstable particle particle and decay products mix
- $T > 0$ environment: scattering on thermal bath particles

Mixing particle and continuum states: quasiparticles



- no unique time dependence ✗
- no asymptotic states ✗

(AJ, PRD76 (2007) 125004 [hep-ph/0612268])

- linear response: $\varrho(t) = Ze^{-iEt-\gamma t} + f_{bckg}(t) = \mathrm{pole} + \mathrm{cut}$
- for large Z and small γ: complex pole dominates long time dependence ✓ $\Rightarrow$ quasiparticle
  - not a single energy level: collective, multiparticle state!
  - $T = 0$ and $T \neq 0$ time dependence are different!
    $\Rightarrow$ environment-dependent quasiparticle definition

Mathematical treatment of quasiparticles

Can quasiparticles be standalone degrees of freedom? From several point of view they are particle-like:

- quasiparticles dominate long time dependence ✓
- particle-like contribution to free energy (Beth, Uhlenbeck) ✓
  $\delta Z \sim \int_0^\infty \frac{d\omega}{\pi} \frac{\partial \delta}{\delta \omega} e^{-\beta\omega} \sim e^{-\beta E}$:$\delta_\ell(\varepsilon)$ phase shift jumps π-t at poles

  (Landau, Lifsitz V.; R.F Dashen, R. Rajaraman, PRD10 (1974), 694.)

We should write up a Lagrangian:

$$\mathcal{L} = \sum_Q \Psi_Q^\dagger \mathcal{K}_Q(i\partial)\Psi_Q + \mathcal{L}_{int}$$

BUT: exponential damping with local kernel
$\hat{H} \to \hat{H} - i\gamma \quad \Rightarrow$ loss of unitarity! ✗

Solution

We have to take into account the complete spectrum!

(Ward, Luttinger, Phys.Rev. 118 (1960) 1417; G. Baym, Phys. Rev. 127 (1962) 1391; Cornwall Jackiw, Tomboulis, Phys.Rev. D10 (1974) 2428-2445;J. Berges and J. Cox, Phys. Lett. B 517 (2001) 369)

In Φ-derivable (or 2PI) approach we solve self-consistently the SD equations: $G^{-1} = G_0^{-1} - \Sigma(G)$.

Corollary

- quasiparticles are collective excitations
- no local representation of quasiparticles

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The Bloch-Nordsieck model

Folklore: particle-like states are always quasiparticles...
Real spectrum in case of zero mass excitations?

By chance, $\exists$ 3+1D solvable model: Bloch-Nordsieck model

(F. Bloch and A. Nordsieck, Phys. Rev. 52 (1937) 54.)

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \Psi^\dagger (i u_\mu D^\mu - m)\Psi, \qquad iD_\mu = i\partial_\mu - eA_\mu,\ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

(e fermion charge, $\alpha = e^2/(4\pi)$ fine structure constant, m fermion mass)

- 1-component QED ($\gamma^\mu \to u^\mu$)
- spin-statistics theorem $\Rightarrow$ fermion is an incoming (hard) test charge
- deep IR regime of real QED (H. A. Weldon, Phys. Rev. D 44, 3955 (1991).)

Solution

Solution at $T = 0$

- functional methods (F. Bloch and A. Nordsieck, Phys. Rev. 52 (1937) 54.; N.N. Bogoliubov and D.V. Shirkov)
- Fradkin representation (H.M. Fried, Greens Functions and Ordered Exponentials (Cambridge University Press, 2002))
- Schwinger-Dyson equations & Ward-identities (A. I. Alekseev, V. A. Baikov and E. E. Boos, Theor. Math. Phys. 54, 253 (1983) [Teor. Mat. Fiz. 54, 388 (1983)]; AJ and P. Mati, Phys. Rev. D 85 (2012) 085006.)

At $T > 0$

- long time evolution (J. -P. Blaizot and E. Iancu, Phys. Rev. D 55 (1997) 973.)
- complete spectrum (AJ and P. Mati, arXiv:1301.1803)

Solution at $T = 0$

1-loop perturbation theory for fermion propagator $(u\cdot p = u_\mu p^\mu)$

$$\mathcal{G}(p) = \frac{1}{u\cdot p - m - \Sigma(p)} = \frac{1}{u\cdot p - m}\,\frac{1}{1+\frac{\alpha}{\pi}\ln\frac{m-u\cdot p}{\mu}}.$$

$\Rightarrow$ divergent near $u\cdot p\to m$ $\Rightarrow$ resummation needed

- exact SD equation (operator EoM) in Feynman gauge:

$$\Sigma(p) = -ie^2\int\frac{d^4k}{(2\pi)^4}\,G(k)\mathcal{G}(p-k)u_\mu\Gamma^\mu(k;p-k,p).$$

- Ward identities (consequence of current conservation)

$$k_\mu\Gamma^\mu(k;p-k,p) = \mathcal{G}^{-1}(p) - \mathcal{G}^{-1}(p-k).$$

- In this model the WI can be solved, because $\Gamma^\mu = u^\mu\Gamma$! The equations form a closed set, analytic solution is possible.

The exact solution reads: $\varrho(p) = \dfrac{Z\Theta(u\cdot p - m)}{(u\cdot p - m)^{1+\alpha/\pi}}$.

$\Rightarrow$ exponentiation of perturbative result.

Not fully satisfying solution...

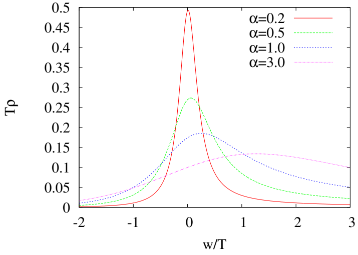
- gauge dependent
- not normalizable: $\int d\omega \varrho(\omega)$ divergent
  must be compensated with $Z = 0$ $\Rightarrow$ $0\cdot\infty$ type problem!
- real time dependence?
  dimensional analysis: $\int d\omega e^{-i\omega t}\varrho(\omega) \to e^{-imt} t^{\alpha/\pi}$
  growing correlation in time?? unitarity??

for physical answer: regularization $\Rightarrow$ finite temperature

Finite temperature results

- SD & WI written up in real time formalism
- Analytic solution exists for $u_\mu = (u, 0, 0, 0)$ (standing fermion), in real time: $\varrho(t) \sim (\sinh \pi T t)^{\alpha/\pi}$
- inverse Fourier transform exists for pure imaginary α
  $\Rightarrow$ perform FT, then analytic continuation!

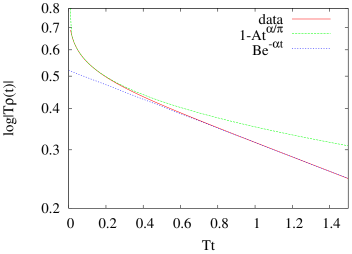
$$\varrho(x) = \frac{N_\alpha \beta \sin \alpha\, e^{x/2}}{\cosh x - \cos \alpha} \left| \Gamma\left(1 + \frac{\alpha}{2\pi} + i\frac{x}{2\pi}\right)\right|^{-2},$$

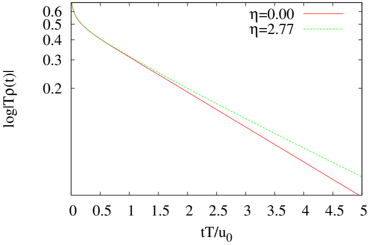


- function of $x = \beta(u \cdot p - m)$ $(x = \beta w)$
- $\alpha \to 0$ and $T \to 0$ limits OK.
- normalizable, $Z \sim T^{\alpha/\pi}$
- for other u: numerical convolution

Real time dependence

Fourier transform of the physically sensible result: $\varrho(t) = e^{-imt}\bar{\varrho}(t)$





- for long times $Tt \gg 1$: $\sim e^{-\alpha_{eff}(u)Tt}$ quasiparticle behaviour
- for short times $Tt \ll 1$: $\sim 1 - Zt^{\alpha/\pi}$ not quasiparticle-like!
- at $T = 0$ $\varrho(t) \sim e^{-imt}$ $\Rightarrow$ no zero temperature dephasing!
- in real QED probably $\varrho(t) \sim (C_1 + C_2 t^{-3/2})e^{-imt}$

Misleading quasiparticle picture

As $T \to 0$ the damping becomes non-quasiparticle-like!
If we (uncorrectly) assume quasiparticle behaviour, we can have false conclusions! eg.:

$$-\frac{\varrho'(t)}{\varrho(t)} = \begin{cases} \gamma, & \text{if } \varrho \sim e^{-\gamma t} \\ Zt^{\beta-1}, & \text{if } \varrho \sim 1 - Zt^{\beta} \end{cases}$$

$\Rightarrow$ one may identify $\gamma = Zt^{\beta-1}$ dephasing time!

- in fact dephasing time is meaningless at $T = 0$!
- dephasing in solid state physics $\Rightarrow$ same phenomenon?

(P. Mohanty, E.M.Q. Jariwala, R.A. Webb, Phys. Rev. Lett. 78, 3366 (1997), [arXiv: cond-mat/9710095])

lesson: only the complete spectrum yields reliable time dependence!

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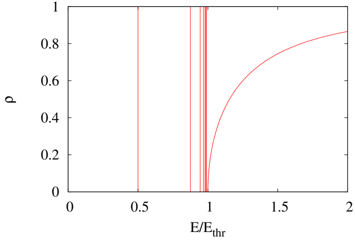
Bound states

Folklore: quasiparticles represent (thermodynamical) dof...
What is the case with bound states?
In case of attractive interactions, there can appear states *below* the free 2-particle thresholds $\Rightarrow$ bound states
For example: $e^- + p^+$, $\mathbf{p} = 0$, $J^2 = 0$ $\Rightarrow$ s-states of H-atom



- in Coulomb approximation energy levels $\Rightarrow$ particle, $E_n = \frac{E_0}{n^2}$
- in QED: ns states decay for $n > 1$ $\Rightarrow$ finite width $\gamma \sim \frac{\gamma_0}{n^3}$

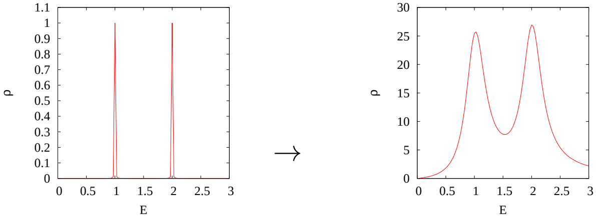
- quasiparticles $\Rightarrow$ experimentally observable
- thermodynamics? $\sum_n e^{-\beta E_0/n^2}$ is divergent!

Overlapping quasiparticle states

In reality at $T > 0$ or finite density: increased width

- finite collisional lifetime
- finite density $\Rightarrow$ maximal orbital size

$\Rightarrow$ *ns* states for large *n* overlap



But a quasiparticle is collective multiparticle state:
how to count common energy levels?

Thermodynamics from the complete spectrum

Build thermodynamics on the s-channel spectrum!

Technically:

$\rightarrow$ quadratic theory $\mathcal{L} \sim \Psi\mathcal{K}\Psi \quad \Rightarrow \quad$ energy-momentum tensor

$\rightarrow$ energy density $\varepsilon = \frac{1}{Z}\,\mathrm{Tr}\, e^{-\beta\hat{H}}\,\hat{T}_{00}$

$\rightarrow$ free energy, pressure from thermodynamical relations

Result:

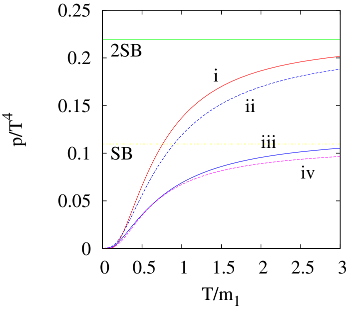
$$\varepsilon = \int \frac{d^4p}{(2\pi)^4}\,\Theta(p_0)\mathcal{H}(p)n(p_0)\varrho(p)$$

where

$$\mathcal{H}(p) = p_0\frac{\partial\mathcal{K}}{\partial p_0} - \mathcal{K}, \qquad \mathcal{K}^{-1}(p) = G(p) = \mathcal{P}\int\frac{d\omega}{2\pi}\,\frac{\varrho(\omega,\mathbf{p})}{p_0-\omega+i\varepsilon}.$$

- classical mechanical analogy: $\mathcal{K}$ quadratic kernel "Lagrangian" with $p_0 \sim \dot{q} \quad \Rightarrow \quad \mathcal{H}$ energy.
- ε does not depend on the normalization of ϱ.

Thermodynamics



$m_1 = 1,\ m_2 = 2$

- i.) $\gamma = 0$: 2 Dirac-deltas
- ii.) two independent $\gamma = 0.2$ peaks
- iii.) overlapping equal $\gamma = 0.2$ width peaks
- iv.) one $m = 1.2$ and $\gamma = 0.2$ peak

(AJ. Phys.Rev. D86 (2012) 085007)

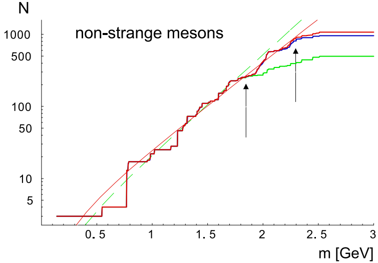
thermodynamics of overlapping peaks: if we had only one particle!

$\Rightarrow$ reduction of thermodynamical dof

Gibbs-paradox is resolved: continuous, analytic reduction of number of dof!

Coulomb spectrum of QCD

QCD bound state dynamics cannot be solved...
experimental evidence: exponentially rising energy level density



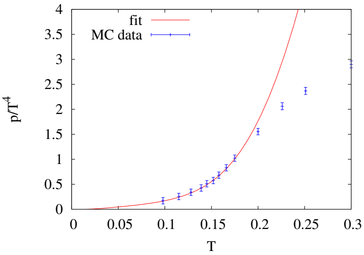
(W. Broniowski, W. Florkowski and L. Y. .Glozman, Phys. Rev. D **70**, 117503 (2004) [hep-ph/0407290].)

Hagedorn-spectrum:

$$\varrho_{hadr}(m) \sim (m^2 + m_0^2)^a e^{-m/T_H}$$

several fits (also $a = 0$) possible

Hagedorn thermodynamics



- MC data from BMW collaboration

  (Sz. Borsanyi et al, JHEP 1011 (2010) 077)
- fit: 1500 hadronic resonances, $m_0 = 120\,\mathrm{MeV}$, $T_H = 241\,\mathrm{MeV}$ and $a = 0$.

- very good fit to MC data
- for infinitely many resonances: divergent at $T > T_H$
- overestimation of pressure above $\approx 200\,\mathrm{MeV}$.

Reduction of thermodynamical dof

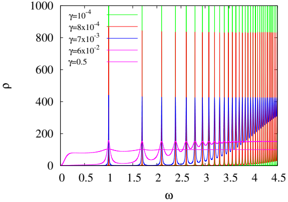
The reason is similar than in the previous case: full spectral function $\Rightarrow$ overlapping quasiparticle peaks

We consider three possible mechanisms

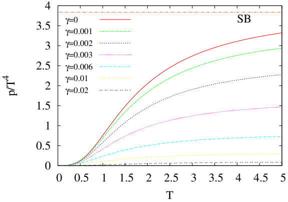
- quasiparticle peaks overlap with each other
- a quasiparticle peak overlap with the continuum
- a quasiparticle peak has vanishing wave function renormalization constant.

Overlapping peaks

Hagedorn-distributed energy levels (35 peaks)



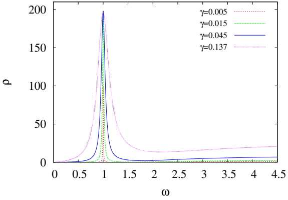
$\Rightarrow$



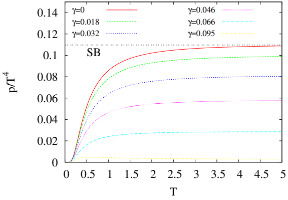
- spectra are shifted for better visibility
- already at small width the upper peaks melt into a continuum
  $\Rightarrow$ reduce pressure

Broadening peak at continuum

A bound state $m = 1$ quasiparticle & 2-particle threshold $m_{thr} = 2$



$\Rightarrow$

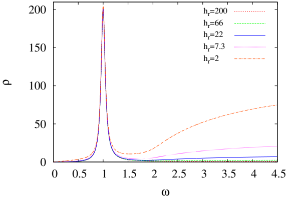


Dynamical vs. thermodynamical dof

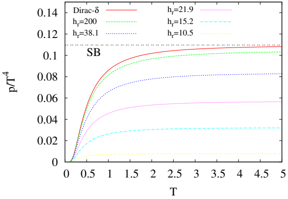
- at $\gamma = 0.14$: quasiparticle peak is clearly detectable in the spectrum
- it does not contribute to the pressure
- in MC: hadronic states are dynamically observable even at $T \gg T_c$! (AJ., P. Petreczky, K. Petrov, A. Velytsky, PRD75 (2007) 014506)
- at large γ: no quasiparticle peak!

Shrinking quasiparticle wave function renormalization

A bound state quasiparticle below a 2-particle threshold

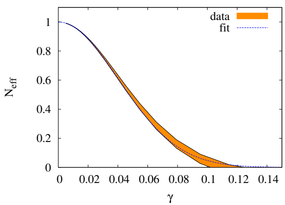


$\Rightarrow$

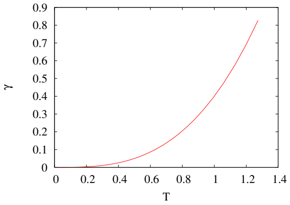


- quasiparticle is observable in dynamics (linear response), but does not contribute to thermodynamics
- for all ζ we find a peak in the spectrum $\Rightarrow$ chemical reaction

Consequences to Hagedorn spectrum

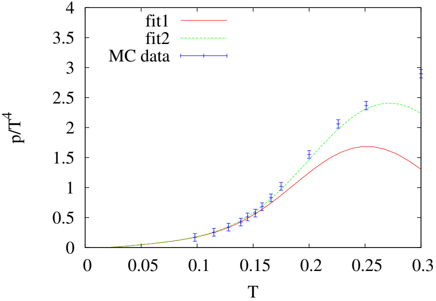


and



- reduction factor of thermodynamical dof: $N_{eff}(T) = \frac{p(T,\gamma)}{p(T,\gamma=0)}$
  $\Rightarrow$ slightly temperature dependent
- fit: Gaussian $e^{-\frac{\gamma^2}{2\gamma_0^2}}$, $\gamma_0 = 0.04$
- realistic $\gamma(T)$ not known for all hadrons; usually strongly nonlinear T-dependence (C.A. Dominguez, *et.al.*, JHEP 0708 (2007) 040)
  e.g. $\gamma(T) \sim T^3$

Pressure of the hadronic matter



Complete pressure: $P_{tot} = P_{hadr} + P_{QGP}$
With increasing temperature:

- partial pressure of hadrons decreases, $P_{hadr} < P_{tot}$.
- QGP pressure increases
- hadronic thermodynamics up to 1.5-2 T_c?

Outlines

Particle definition becomes dubious in interacting systems

- naive particle definitions are different in case of interaction
- asymptotic states only at zero temperature
- quasiparticles only for well separated, large peaks

There are cases, when the naive particle-like interpretation is not correct

- zero mass excitations $\Rightarrow$ no quasiparticles at $T \to 0$
- quasiparticles are not standalone degrees of freedom, they can disappear (melt)

Facit

The real generalization of particle concept is the treatment of the complete spectrum.