Are tractor beams possible?

arXiv:1303.3237

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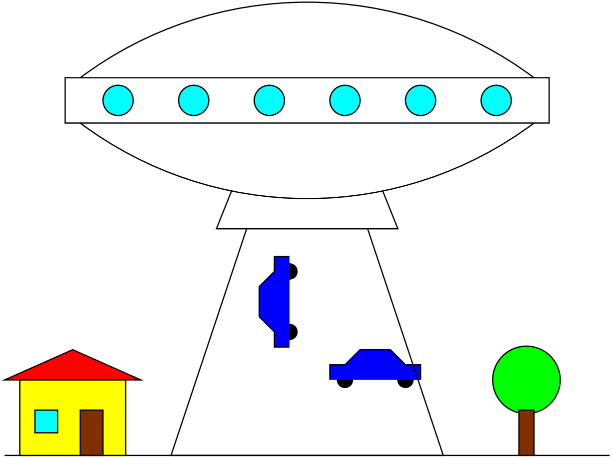
2013. május 24.

Outline

Tractor beam

Is this possible?



- A wave is scattered
- The scatterer is pulled towards the source

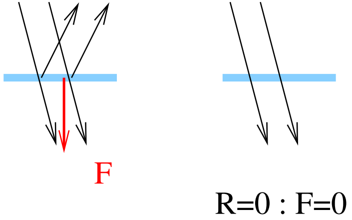
Radiation pressure

Scattering of light:

$$P = (1 + R^2 - T^2)W\,,$$

$$W = \epsilon_0 E_0^2/2$$

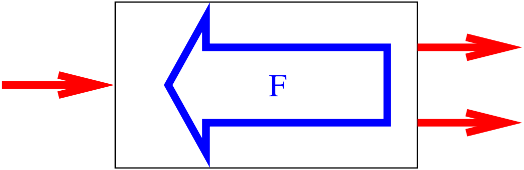
R: reflection, T: transmission



Can the pressure be negative?
What objects can be pulled towards the radiation source?
Unitarity (conservation of energy): $R^2 + T^2 = 1$, $P = 2WR^2 > 0$.
How can P be negative?

What is negative radiation pressure?

Mizraki, A. and Fainman, Y., *Opt. Lett.* **35** 3405 (2010)
“gain medium”: No unitarity!, $T > 1$



Is this really negative radiation pressure?

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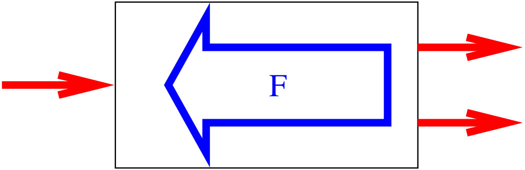

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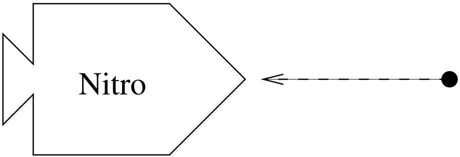
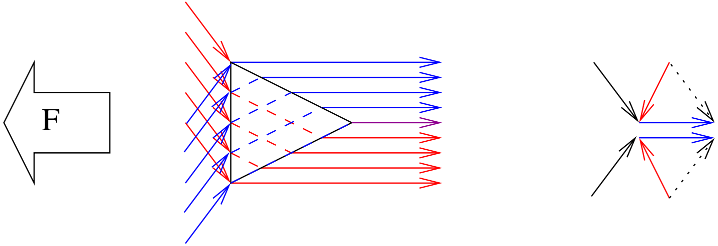


Does the bullet pull the rocket?

Tricky beam

Sukhov, S. and Dogariu, A. *Phys. Rev. Lett.* **107** 203602 (2011)



- Geometrical optics: very simple
- They have also solved Maxwell's equations (FEL)
- Also possible for general (absorption free) scatterers
- Examples: sphere, a number of tiny spheres
- One has to know scattering data of scatterer

Two channel scattering

- Consider two channels with different momenta
  e.g., spin up and down particles in a magnetic field $B_z < 0$
- Incoming beam: smaller momentum channel, x direction
- Scattering between channels
  e.g., revert some spins, the scatterer can be a region with $B_x \neq 0$
- Outgoing ptcles carry more momentum
- Balanced by force acting on the scatterer in the direction opposite to that of the incoming beam
- Incoming plane wave acts as a tractor beam

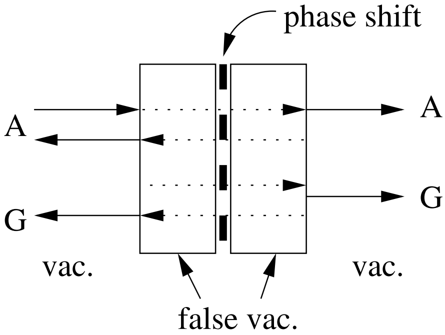
A simple model: $k_i^2 = \omega^2 - m_i^2$, $m_u > m_d$
Scatterer: constant potential for $-L \leq x \leq L$

$$F_u/A_u^2 = k_u(1 + |R_{uu}|^2 - |T_{uu}|^2) + k_d(|R_{du}|^2 - |T_{du}|^2)$$

Wide range of parameters: effects of reflection dominated by scattering between channels

Outline

A strip of false vacuum I



- Vacuum: two modes: $k_a^2 + 4 = \omega^2$, $k_g^2 = \omega^2$
- False vacuum: two modes, $k_i^2 = \omega^2$
- Phase shift screen: $\pm\delta$ for the two modes

A strip of false vacuum II

- Analytical force formula
  $\frac{F_{\mathrm{fvs}}}{|a|^2} = k_a^2(1 + |R_a|^2 - |T_a|^2) + k_g^2(|R_g|^2 - |T_g|^2)$
- very simple for $L \to 0$

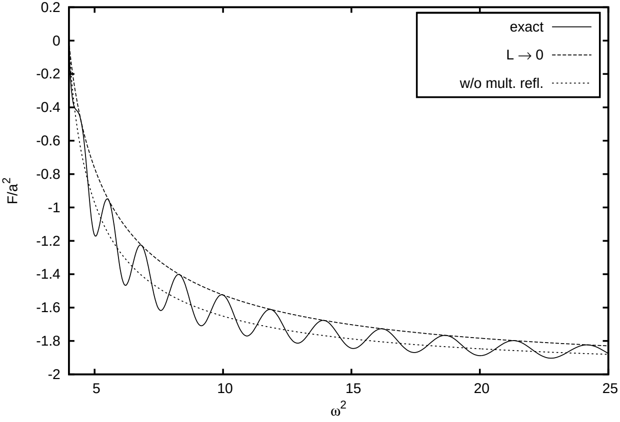
$$F_{L\to 0} = a^2 \frac{4k_a^2(k_a^2 - k_g^2)\sin^2\delta}{k_a^2 + 6k_a k_g + k_g^2 - (k_a - k_g)^2\cos 2\delta}$$

- and also neglecting multiple reflections

$$F_{\mathrm{fvs}} \approx |a|^2 \frac{2k_a^2(k_a^2(k_i + k_g)^2 + k_i^2(k_i^2 + 2k_i k_g - 7k_g^2))}{(k_a + k_i)^2(k_i + k_g)^2}$$

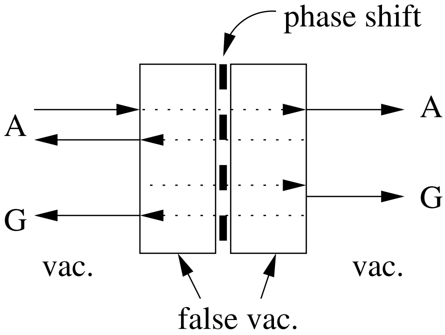
- Oscillations: Fabry–Perot (or: antireflex coating)

A strip of false vacuum III



Index of refraction I

Similar to the false vacuum strip:

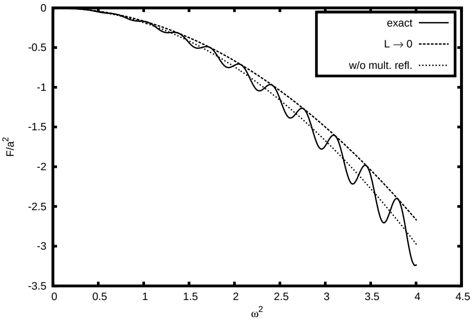


- Outside: two modes, “a” and “g”
- Inside: two modes, opposite phase shifts
- $\omega_i = c_i k_i$, $c_i = c/n_i$, $n_{a,g,b}$: indices of refraction

Index of refraction II

- analytical force formula for $L \to 0$:

$$F_{a,L=0} = \frac{2(n_g - n_a)}{n_a^2(n_a + n_g)}\omega^2 \,, \qquad F < 0 \text{ if } n_a > n_g$$



Outline

2D scattering

Assumptions: free propagation far from the scatterer
Scattering asymptotics: multichannel scattering

$$\psi_a \sim A_a e^{ik_a x} + \frac{e^{ik_a r}}{\sqrt{r}} \sum_b f_{ab}(\vartheta) A_b$$

Cross section from channel b to a

$$\frac{d\sigma_{ab}}{d\vartheta} = \frac{k_a}{k_b} |f_{ab}(\vartheta)|^2$$

defined as energy flux into angle ϑ for unit flux incoming wave
Scattering operator

$$S_{ab}(\vartheta) = \delta_{ab}\delta(\vartheta) + \sqrt{\frac{ik_a}{2\pi}} f_{ab}(\vartheta)$$

mapping: incoming wave $\mapsto$ outgoing wave

Partial waves

Easier to solve ODEs

$$\psi_a = \sum_{\ell=-\infty}^{\infty} \mathrm{e}^{i\ell\vartheta} R_{a,\ell}(r)$$

with radial functions $R_{a,\ell}$

S-matrix

Incoming plane wave $\mathrm{e}^{ikx} = \sum_\ell i^\ell \mathrm{e}^{i\ell\vartheta} J_\ell(kr)$; Asy for $r \to \infty$

$$R_\ell \sim (\dots)(S_{ab,\ell} A_b H^{(1)} + H^{(2)} A_a), \quad H^{(1,2)} = J_\ell \pm iY_\ell \sim \mathrm{e}^{\pm ikr}/\sqrt{r}$$

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- Multiple (n) channels S_ℓ an $n \times n$ matrix
- Unitarity (energy conservation) $SS^\dagger = 1$, optical thm
- Symmetry: time reversal (in many cases)
  Perturbative problems: can be violated by the background

Force acting on the scatterer I

Momentum balance Assuming free propagation for $r \to \infty$, substitute into

$$\mathbf{F} = -\lim_{R\to\infty} R \int_{-\pi}^{\pi} \bar{\mathbf{T}}\mathbf{e}_r R \mathrm{d}\vartheta \,,$$

$\mathbf{T}$: stress tensor, $\mathbf{e}_r$ radial unit vector

Using the partial wave expansion a master formula is obtained:

$$F = F_x + iF_y = -4 \sum_{\ell} \left\{ A^\dagger S^\dagger_{\ell+1} K S_\ell A - A^\dagger K A \right\} \,,$$

$A = (A_1, \ldots, A_n)^T$ amplitude, $K = \mathrm{diag}(k_1, \ldots, k_n)$ wave numbers
Consequence of unitarity:

$$\mathrm{Re}\, |A_a|^2 k_a (1 - S^*_{aa,\ell+1} S_{aa\ell}) > 0$$

one channel radiation pressure positive; NRP: $k_b > k_a$ necessary

Force acting on the scatterer II

Force expressed with cross sections

$$F_x = -2k^2\sigma_{\|}|A|^2\,,$$
$$F_y = -2k^2\sigma_{\perp}|A|^2\,,$$

with $\sigma_{\|} = \int_{-\pi}^{\pi} \mathrm{d}\sigma(\vartheta)(\cos\vartheta - 1)$ and $\sigma_{\perp} = \int_{-\pi}^{\pi} \mathrm{d}\sigma(\vartheta)\sin\vartheta$
Also in QM: average momentum transfer

$$F = F_x + iF_y = -4\sum_{\ell}\left\{A^\dagger S^\dagger_{\ell+1} K S_\ell A - A^\dagger K A\right\}\,,$$

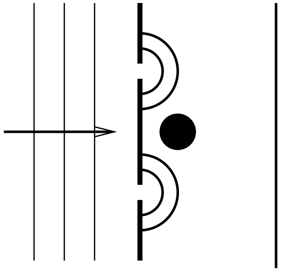
- Neighbouring partial waves $(\ell, \ell+1)$
- Positive contribution of diagonal elements (unitarity)
- More general than expression with cross-section (asymptotics of partial waves, not sum)

Outline

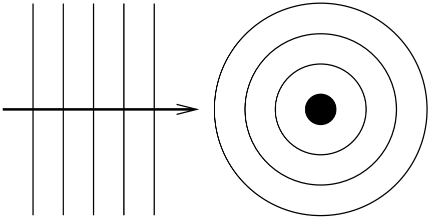
Aharonov–Bohm scattering I

The Aharonov–Bohm effect:
Motion of a charged particle in a region with $\mathbf{B} = 0$

Double slit expreriment:



Scattering:



- Both experiments show flux dependence
- Holonomy is also physical not just field strength $\mathbb{P}e^{i\int \mathbf{A}d\mathbf{r}}$
- Reaction force (deflected beam): Force acting on the scatterer

Aharonov–Bohm scattering II

Schrödinger-equation

$$-i\dot{\psi} = (\nabla - i\mathbf{A})^2\psi\,,$$

with electromagnetic vector potential

$$\mathbf{A}(r, \vartheta, z) = \frac{A_0}{r}\mathbf{e}_\vartheta\,.$$

$2\pi A_0$ flux; outside $\mathbf{B} = 0$.
Fixed energy: $\psi(\mathbf{r}, t) = \mathrm{e}^{-i\omega t}\psi(\mathbf{r})$.
Scattering asymptotics (?)

$$\psi \sim \mathrm{e}^{ikx} + \frac{f(\vartheta)}{\sqrt{r}}\mathrm{e}^{ikr}$$

Cross sections: $\mathrm{d}\sigma/\mathrm{d}\vartheta = |f(\vartheta)|^2$
Scattering amplitude: $f(\vartheta) \sim \frac{\sin^2 \pi A_0}{2\pi}\frac{1}{\sin^2(\vartheta/2)}$.

Aharonov–Bohm scattering III

Partial waves: $s_\ell \propto J_{|\ell - A_0|}$ (note index shift wrt plane wave)
Effect of scattering in the outgoing wave: phase shift δ_ℓ,
$S_\ell = \exp(2i\delta_\ell)$
$(J_\nu(z) \sim \cos(z - \nu\pi/2 - \pi/4)/\sqrt{2\pi z})$

$$\ell \geq 0 \;:\; \delta_\ell = \frac{A_0 \pi}{2}\,, \quad \ell \leq -1 \;:\; \delta_\ell = \pi\ell - \frac{A_0\pi}{2}\,.$$

thus

$$F_x = -|\phi_0|^2 4k \sum_{\ell=-\infty}^{\infty} \{\cos\left[2(\delta_\ell - \delta_{\ell-1})\right] - 1\}$$
$$= -|\phi_0|^2 4k\,(\cos(2\pi A_0) - 1) \approx 16|\phi_0|^2\pi^2 A_0^2 k\,,$$
$$F_y = -|\phi_0|^2 4k \sum_{\ell=-\infty}^{\infty} \sin\left[2(\delta_\ell - \delta_{\ell-1})\right] = |\phi_0|^2 4k\sin(2\pi A_0) \approx 8|\phi_0|^2\pi A_0 k\,.$$

Analog problem: force acting on a superfluid vortex (GPe)

Aharonov–Bohm scattering IV

Iordanskii force controversy C. Wexler, D.J. Thouless, *Phys. Rev.* **B58** R8897–R8900 (1998):

$$f(-\vartheta) = f(\vartheta) \quad \Rightarrow \quad F_y = 0$$

A.L. Shelankov *Europhys. Lett.* **43** (1998) 623
M.V. Berry *J. Phys. A: Math. Gen.* **32** (1999) 5627:
scattering asymptotics does not hold in forward direction

$$\psi(\mathbf{r}, t) = \mathrm{e}^{-i(\omega t - kx)}\phi(\mathbf{r})$$

and for $y \ll \sqrt{x}$

$$\phi(x > 0, y) \sim \cos(A_0\pi) - \frac{2i^{1/2}}{\sqrt{\pi}} \sin(A_0\pi)\sqrt{\frac{k}{2}} \frac{y}{\sqrt{x}}$$

transversal force F_y from this region (although not F_x)

Outline

Scattering on a vortex I

Consider two scalar fields, ϕ and ψ

- A vortex in ϕ: $\phi \sim v\exp(i\vartheta)$ (broken U(1) symmetry) e.g. ϕ^4 potential $V = \lambda(\phi^*\phi - v^2)^2/2$
- Scattering of ψ
- Coupling $\Delta\mathcal{L} = g\phi\psi^2 + c.c.$

Field equation

$$(\omega^2 - m^2 + \nabla^2)\psi = 2g\phi^*\psi^*\,,$$

Coupling for $r \to \infty$: transformation to mass eigenstates

$$\begin{pmatrix}\psi\\ \psi^*\end{pmatrix} = \begin{pmatrix}\mathrm{e}^{-i\vartheta/2} & i\mathrm{e}^{-i\vartheta/2}\\ \mathrm{e}^{i\vartheta/2} & -i\mathrm{e}^{i\vartheta/2}\end{pmatrix}\begin{pmatrix}u\\ d\end{pmatrix}$$

- heavy mode u ($m_u^2 = m^2 + 2gv$)
- light mode d ($m_d^2 = m^2 - 2gv$)
- transformation matrix depends on ccordinate ϑ

Scattering on a vortex II

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\vartheta/2} & ie^{-i\vartheta/2} \\ e^{i\vartheta/2} & -ie^{i\vartheta/2} \end{pmatrix}, \quad \psi = U\rho\,, \quad \rho = \begin{pmatrix} u \\ d \end{pmatrix}$$

- trf matrix U depends on angular coordinate ϑ
- Commuting U and ∇: artificial gauge potential induced

$$i\mathbf{A}\frac{\sigma_2}{2} = \frac{1}{r} U^\dagger \frac{\partial U}{\partial \vartheta} \mathbf{e}_\vartheta$$

  where $\mathbf{A} = \mathbf{e}_\vartheta / r$ (Kazan 1985; March-Russell, Preskill, Wilczek 1992)
- Field equations

$$\left(\nabla + i\mathbf{A}\frac{\sigma_2}{2}\right)^2 \rho - K^2\rho = 0\,, \quad \sigma_2 = \begin{pmatrix} & -i \\ i & \end{pmatrix},$$

- two channels, different masses
- coupling due to Aharonov–Bohm gauge potential

Solution of the scattering problem I

Partial waves

$$(u,d) = \sum_{\ell=-\infty}^{\infty} \mathrm{e}^{i(\ell+\gamma)\vartheta}(u_\ell(r), d_\ell(r))$$

with $\gamma = 1/2$. The radial functions satisfy

$$u_\ell'' + \frac{u_\ell'}{r} - \frac{\eta_u^2}{r^2}u_\ell + \frac{c}{r^2}d_\ell + k_u^2 u_\ell = 0\,,$$
$$d_\ell'' + \frac{d_\ell'}{r} - \frac{\eta_d^2}{r^2}d_\ell + \frac{c^*}{r^2}u_\ell + k_d^2 d_\ell = 0\,,$$

where $\eta_u^2 = \eta_d^2 = (\ell+1/2)^2 + 1/4$ and $c = i(\ell+1/2)$. We use rescaled variables such that $v = 1$, $m_u = 2$ (threshold at $\omega = 2$), and for simplicity sake, present numerical data for $m_d = 1$.

Approximate solution of radial equations I

Decoupled approximation: ignore coupling

$$d_\ell(r) = A_\ell J_{\eta d}(k_d r)$$

and thus

$$S_\ell = \begin{pmatrix} e^{2i\xi_u} & 0 \\ 0 & e^{2i\xi_d} \end{pmatrix} \qquad \xi_{u,d} = \frac{\pi}{2}(\ell - \eta_{u,d})$$

works well for $\omega \ll m_d$ (closed channel u), $F_y = 0$
Numerical solution (above the threshold)

- u channel: $F_{x,u} < 0$ tractor beam
- Phases depend weakly on frequency
- Scattering between channels important
- $F_y \neq 0$

Approximate solution of radial equations II

Simplified perturbation theory

- calculate phase for d at the threshold (2nd order pert th)

$$\delta_d = \xi_d + \Delta\,, \quad \text{with } \Delta = \arctan\left[\frac{|c|^2\pi/4}{\eta_u^2\eta_d + \eta_u\eta_d^2}\right]$$

- calculate matrix elements with Bessel functions with the right phase

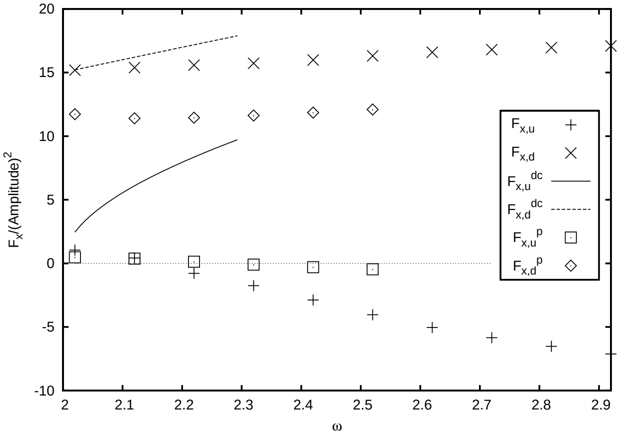
$$S_\ell \approx \frac{1}{N}\begin{pmatrix} \mathrm{e}^{2i\delta_u} & \mathrm{e}^{i(\delta_u+\delta_d)} i\pi c I \\ \mathrm{e}^{i(\delta_u+\delta_d)} i\pi c^* I & \mathrm{e}^{2i\delta_d} \end{pmatrix}$$

with

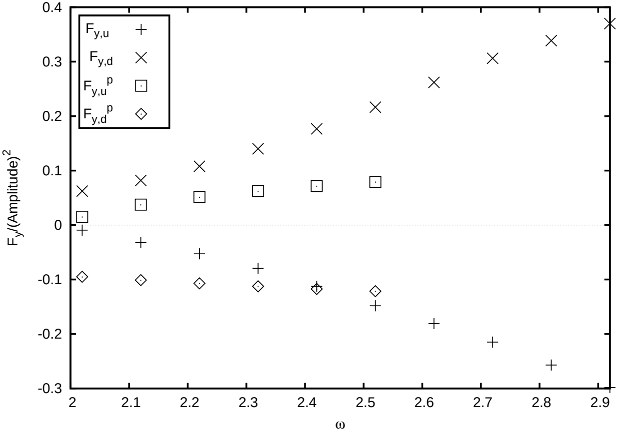
$$I = \left(\frac{k_u}{k_d}\right)^{\nu_u} \Gamma(\bar\nu) \frac{{}_2F_1\left(\nu', \bar\nu, \nu_u + 1, k_u^2/k_d^2\right)}{2\Gamma\left(1-\nu'\right)\Gamma(\nu_u+1)}\,,$$

where $\nu_u = \eta_u$, $\delta_u = \pi(\ell - \nu_u)/2)$, $\bar\nu = (\nu_u + \nu_d)/2$, $\nu' = (\nu_u - \nu_d)/2$, and N is a normalization

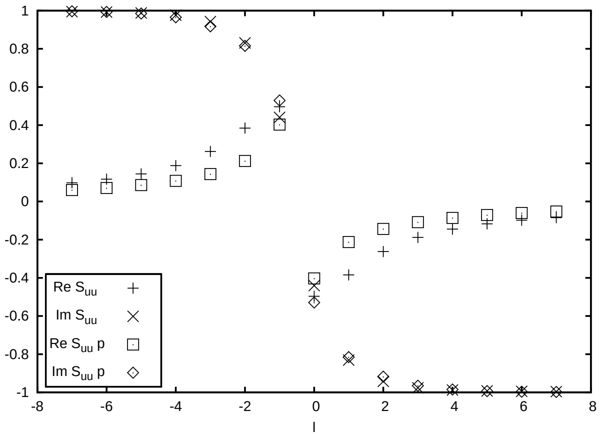
Force acting on the scatterer I



Force acting on the scatterer II



How good is the solution

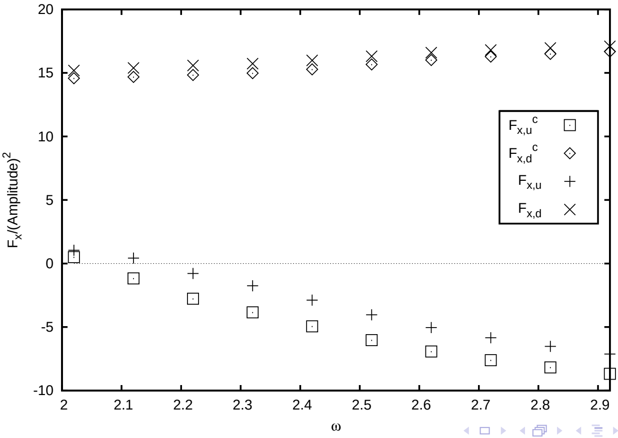


Errors for small ℓ: origin

Effect of a vortex core

More realistic vortex configuration:

$$\phi(r) = f(r)e^{i\vartheta}\,, \quad f(r \le R_c) = vr/R_c, f(r \ge R_c) = v$$



Cosmic strings

- Classical field theoretical solution
- Internal structure
- Thin, elongated object
- One cross section of a string: a vortex
- Localised energy density in the string

Important parameter: string tension

$$\mu = E/L$$

- Elevtroweak string: $G\mu \approx 10^{-32}$ (μ: 10 mg/Solar diam)
- GUT string: $G\mu \approx 10^{-6}$ (μ: Solar mass/Solar diam)

Cosmic strings are high energy localised objects that provide a link between astrophysics and particle physics.

Physics of cosmic strings I

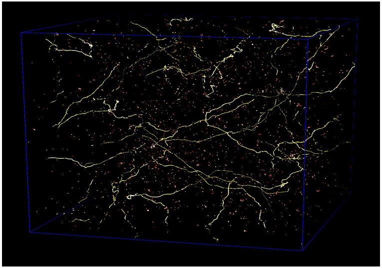
Evolution of cosmic strings

- Formation: during phase transitions
- Evolution of a string network
  - expansion: the network becomes more diuted
  - collisions, interlinking
  - radiation (e.g. at cusps formed in collisions)
  - string tension contracts loops
- Signatures of cosmic strings
  - Scattering of material off strings: structure formation: galaxies, voids, filamets (fractal dimension: Murdzek 2007)
  - Contribution to CMB anisotropy: best fit with GUT strings $G\mu = (2.04 \pm 0.13) \times 10^{-6}$, Contribution to multipole $\ell = 10$:$f_{10} = 0.11 \pm 0.05$ (Hindmarsh et al., 2007, 2008)
  - Gravitational lensing
  - Gravitational radiation

Physics of cosmic strings II

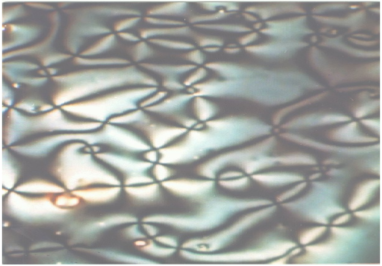
Analogies with condensed matter systems

- vortex filaments in superfluids
- Abrikosov vortices in superconductors
- defects in liquid crystals
- defect formation: Kibble–Zurek mechanism



(Kép forrása: Martins and Shellard,

http://www.damtp.cam.ac.uk/research/gr/public/cs_evol.html)



(Srivastava, Testing Cosmic Defect Formation Theories in Liquid Crystal Experiments, COSLAB 2005, Smolenice)

Spontaneous symmetry breaking

- More symmetries at higher energies

$$\cdots \to \text{GUT} \to SU(3)_{\text{color}} \times SU(2)_{\mathrm{L}} \times U(1)_{\mathrm{Y}} \to SU(3)_{\text{color}} \times U(1)_{\text{em}}$$

transition from a symmetric state to one with broken symmetry: phase transition

- GUT $\sim 10^{15}$ GeV (at 10^{-36} s), electroweak: ~ 250 GeV (at 10^{-12} s)
- Mass terms for gauge potentials: coupling to scalar (Higgs) field

$$m^2 A_\mu A^\mu \to \Phi^\dagger \Phi A_\mu A^\mu$$

- Higgs effect: $\langle \Phi \rangle \neq 0$ in the broken symm phase
- Scalar field self interaction

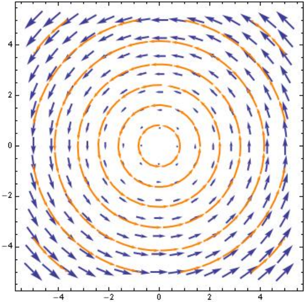
$$V(\Phi) = \beta(\Phi^\dagger \Phi - \eta^2)^2$$

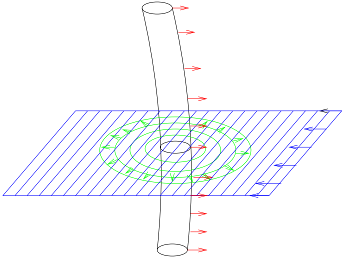
"Mexican hat", minima: $\Phi^\dagger \Phi = \eta^2$

- ground state degenerate: "phase" of $\langle \Phi \rangle$ undetermined

Scattering on a cosmic string I

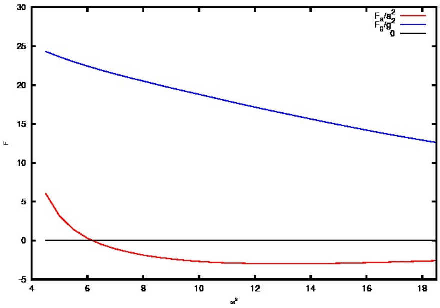
Field equations: $\ddot{\phi} - \nabla^2\phi + 2(\phi^*\phi - 1)\phi$
Similar to Gross–Pitaevskii, but dynamics is of second order





- Perturbations of the scalar field scattered
- Fits into the previous framework with $e_1 = e_2 = 0$, $q = -1$, $k_u^2 = \omega^2 - 4$, $k_d = \omega$.
- Two modes: amplitude/phase perturbed
- Vortex core is needed for NRP

Force acting on the cosmic string



- F_x vs ω for two channels:
  NRP: scattering of massive mode into massless one
- Preliminary time-evolution data (Romańczukiewicz): qualitative agreement
- Iordanskii force vanishes $F_y = 0$

Outline

Summary

Negative radiation pressure

- "Tractor beam" fairly generic in multichannel scattering
- many models in optics (beam prepared for the scattered)
- field theoretical models: surplus momentum in forward scattered wave
  - kink: nonlinearities
  - vortex: two channels
- models motivated by the case of scattering on the vortex: two channels
  - strip of false vacuum
  - different index of refraction
- effect expected in multi-channel scattering

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- “Tractor beam” fairly generic in multichannel scattering
- many models in optics (beam prepared for the scattered)
- field theoretical models: surplus momentum in forward scattered wave
  - kink: nonlinearities
  - vortex: two channels
- models motivated by the case of scattering on the vortex: two channels
  - strip of false vacuum
  - different index of refraction
- effect expected in multi-channel scattering

THANK YOU FOR YOUR ATTENTION!