Self-consistent viscous phase space distributions from kinetic theory

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an amalgamation of:

DM & Zack Wolff, PRC95 (2017), 024903

Zack Wolff & DM, PRC96 (2017), 044909

M. Damodaran et al, arxiv:1707.00793

... plus work in progress

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Outline

I. Hydro and δf problem

II. Self-consistent δf from kinetic theory

III. Shear viscous δf, effect on harmonic flow

IV. Bulk viscous δf, some results

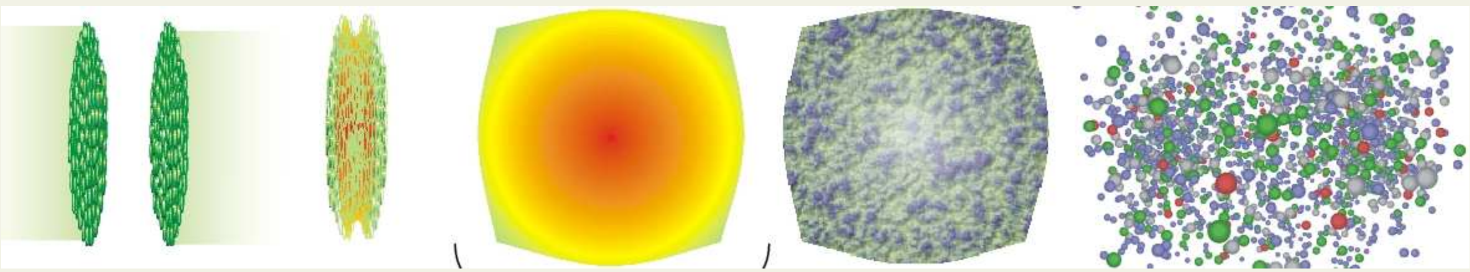
V. Summary

I. Heavy ion physics

bang two heavy nuclei together to study the quark-gluon plasma

e.g., at Large Hadron Collider (LHC) or Relativistic Heavy Ion Collider (RHIC)

initial nuclei **quark-gluon plasma** **hadron gas**



[image: S. Bass]

$\uparrow$ $\mathcal{O}(10\ \mathrm{fm})$ $\downarrow$

$<$——— $\mathcal{O}(10\ \mathrm{fm}/c)$ ———$>$

Initconds

Hydrodynamics

- **hydro fields, e.g,** $e(\vec{r}, t)$
- **equation of state,**
- **viscosities, relax. times**

Kinetic theory / flight to detectors

Fluid dynamics

assumption of local thermalization quite successful in explaining observables

conserved currents:

$$\partial_\mu T^{\mu\nu}(x) = 0 \quad , \quad \partial_\mu N_B^\mu(x) = 0$$

if dissipation - extra fields and equations of motion:

$$T^{\mu\nu}(x) = T^{\mu\nu}_{ideal}(x) + \delta T^{\mu\nu}(x) \qquad \delta T^{\mu\nu} \equiv \pi^{\mu\nu} - \Pi \Delta^{\mu\nu}$$

$$N^\mu(x) = N^\mu_{ideal}(x) + \delta N^\mu(x)$$

Navier-Stokes hydro: $\pi^{\mu\nu}_{NS} = \eta[\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}(\partial \cdot u)]$, $\Pi_{NS} = -\zeta(\partial \cdot u)$

causal 2nd-order dissipative hydrodynamics ($\neq$ Navier-Stokes):

$$\delta \dot{T}^{\mu\nu} = A^{\mu\nu}(\{T^{\alpha\beta}, N^\gamma\}) \quad , \quad \delta \dot{N}^\mu = B^\mu(\{T^{\alpha\beta}, N^\gamma\})$$

Israel & Stewart '79; Muronga '04; Baier, Romatschke et al '06; Heinz, Song et al; Teaney et al
DM, Huovinen et al; Koide, Kodama et al; Schenke, Jeon et al; Denicol, Rischke et al; Strickland et al...

utilizes equation of state ($p(e, n_B)$, $T(e, n_B)$), transport coeffs (η, ζ, κ_B), relaxation times (τ_η, τ_ζ, τ_κ)

The δf problem (hydro $\to$ particles)

hydro gives N^μ & $T^{\mu\nu}$, but experiments measure particles

$$N_B^\mu(\vec{r},t) \equiv \sum_i b_i \int \frac{d^3p}{E} p^\mu f_i(p,\vec{r},t)$$

$$T^{\mu\nu}(\vec{r},t) \equiv \sum_i \int \frac{d^3p}{E} p^\mu p^\nu f_i(\vec{p},\vec{r},t)$$

- **in local equilibrium (ideal hydro) - 1-to-1 map to thermal distributions**

$$T_{LR}^{\mu\nu}(x) = diag(e,p,p,p) \qquad \Leftrightarrow \qquad f_{eq,i}(x,p) = \frac{g_i}{(2\pi)^3} \frac{1}{e^{(p^\mu u_\mu - \mu_i)/T} + a}$$

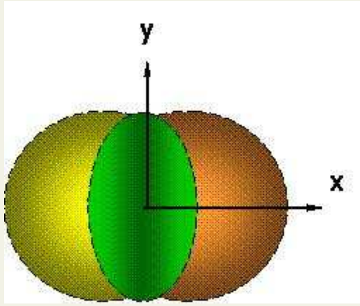
- **near local equilibrium (viscous hydro) - "few to many"**

$$\begin{array}{c} T^{\mu\nu}(x) = T^{\mu\nu}_{ideal}(x) + \delta T^{\mu\nu}(x) \\ N^\mu(x) = N^\mu_{ideal(x)} + \delta N^\mu(x) \end{array} \qquad \Leftarrow \qquad f_i(x,p) = f_{eq,i}(x,p) + \delta f_i(x,p)$$

$\Rightarrow$ **question of δf (even for single-species systems!)**

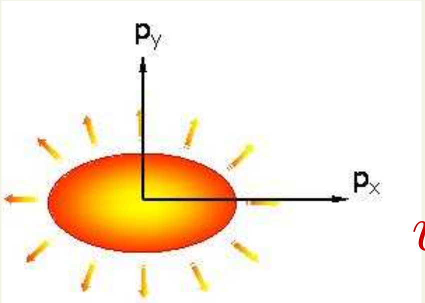
Elliptic flow (v_2) and viscosity

initial spatial anisotropy converts to final momentum space anisotropy



$$\longrightarrow$$

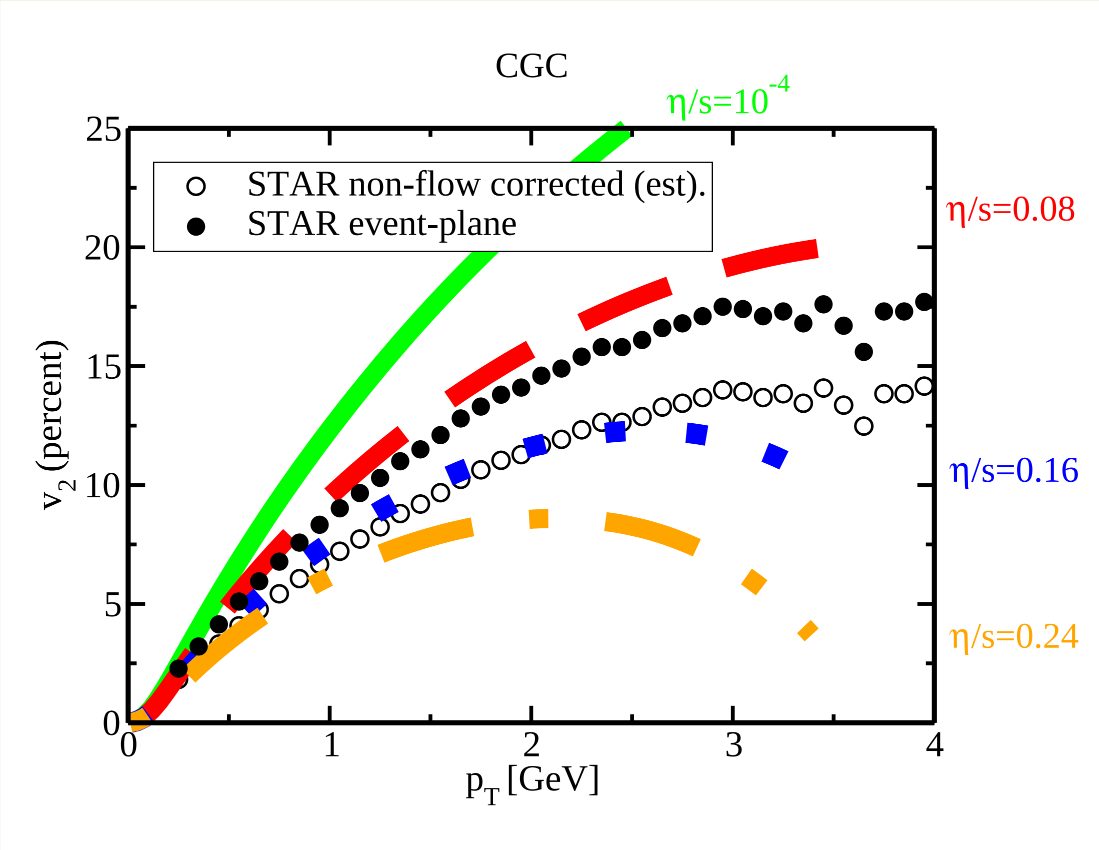
$$\varepsilon \equiv \frac{\langle x^2 - y^2 \rangle}{\langle x^2 + y^2 \rangle}$$



$$v_2 \equiv \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle} \equiv \langle \cos 2\phi_p \rangle$$

can be used to measure viscosity

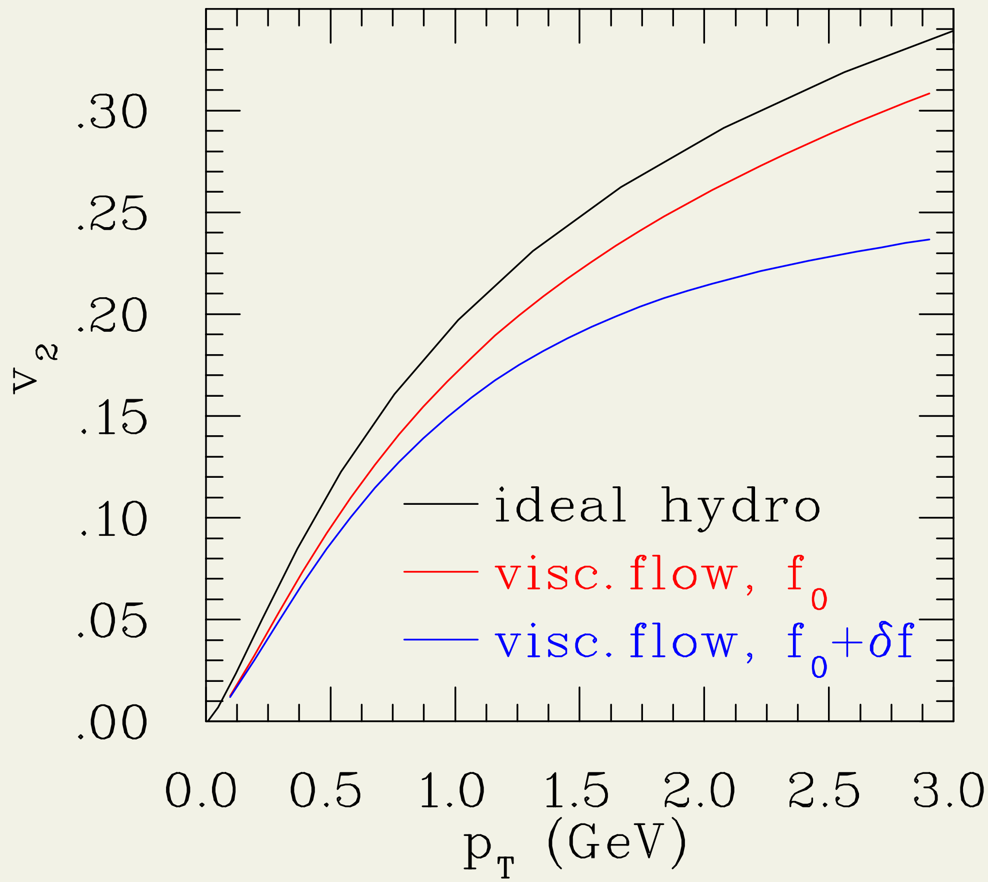
e.g., Romatschke & Luzum, PRC78 ('08):

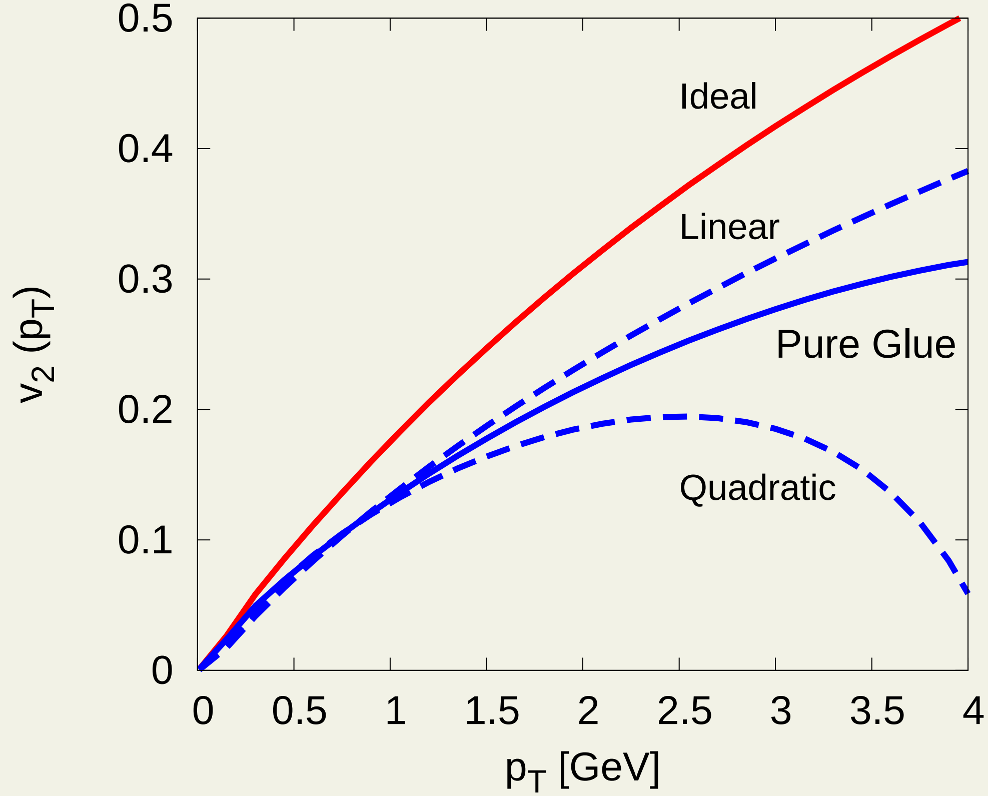


however, result sensitive to δf

DM & Wolff, PRC95 & PRC96 ('17)
Dusling, Moore, Teaney, PRC81 ('09)
...

Huovinen & DM ('08) **Grad** $\delta f \propto p^2$ Dusling, Moore et al ('09) - **linear response** $\delta f \sim p^{1.5}$





large effects at higher momenta (δf **increases)**

Parametrizations for shear $(\pi^{\mu\nu})$

Grad ansatz - from "14-moment" expansion in kinetic theory

$$\delta f^{Grad}(x,\vec{p}) = \frac{1}{2T(x)^2[e(x)+p(x)]} p^\mu p^\nu \pi_{\mu\nu}(x)\, f^{\rm eq}(x,\vec{p}) \quad \sim \; p^2 f^{\rm eq}$$

power law generalization of Grad e.g., DM & Wolff, PRC95 ('17)

$$\delta f = \frac{C(\alpha)}{2T^2(e+p)} \left(\frac{p\cdot u}{T}\right)^{\alpha-2} p^\mu p^\nu \pi_{\mu\nu}\, f^{\rm eq} \quad \sim \; p^\alpha f^{\rm eq}$$

relaxation time approx: $p\cdot\partial f(x,\vec{p}) = \frac{(p\cdot u)}{\tau_{REL}}[f^{\rm eq}(x,\vec{p}) - f(x,\vec{p})]$

$$\Rightarrow \delta f_{RTA} = \frac{\tau_{REL}}{2}\frac{p_\mu p_\nu}{(p\cdot u)}\frac{\pi^{\mu\nu}}{\eta T} f^{\rm eq} \quad \sim \; p^1 f^{\rm eq}$$

stretched thermal distribution: e.g., Strickland & Romatschke, PRD 71 ('05); Tinti et al

$$f^{\rm eq} + \delta f_{SR} = N e^{-\sqrt{E^2 + a\, p_z^2}/\Lambda} \quad \text{more generally} \quad f^{\rm eq}(\vec{p}) \to f(\sqrt{p^\mu \Xi_{\mu\nu} p^\nu})$$

instead: calculate from microcopic dynamics $\rightarrow$ kinetic theory

II. Covariant transport

(on-shell) phase-space density $f(x, \vec{p}) \equiv \frac{dN(\vec{x},\vec{p},t)}{d^3x d^3p}$

transport equation:

$$p^\mu \partial_\mu f_i(x,p) = C^i_{2\to 2}[\{f_j\}](x,p) + C^i_{2\leftrightarrow 3}[\{f_j\}](x,p) + \cdots$$

with, e.g.,

$$C^i_{2\to 2} = \frac{1}{2}\sum_{jkl}\int_{234} (f_3^k f_4^l - f_1^i f_2^j) W^{ij\to kl}_{12\to 34} \qquad \left(\int_j \equiv \int \frac{d^3p_j}{2E_j}\,, \qquad f_a^k \equiv f^k(x,p_a)\right)$$

thermalizes (in box), fully causal and stable

near hydro limit transport coeffs & relaxation times:

$\eta \approx 1.2 T/\sigma$, $\qquad \zeta \approx small \times T/\sigma$, $\qquad \tau_\pi \approx 1.2 \lambda_{tr}$

δf from linearized kinetic theory

Dusling, Moore, Teaney, PRC81 ('09); DM, JPG38 ('12); DM & Wolff, arXiv:0404.7850

crux: near local equilibrium, small gradients $\partial_\mu X$ $\to$ expect small $\delta f \propto \partial_\mu X$

late-time, near-equilibrium behavior universal, and can be studied systematically de Groot, et al ('70s)... Arnold, Moore, Yaffe, JHEP 0011... Denicol et al PRD85 ('12)...

operationally: linearize in $\delta f = f - f^{\rm eq}$, and put $\partial_t \to 0$

$$p \cdot \nabla f^{\rm eq} + p \cdot \nabla \delta f = C[f^{\rm eq}, \delta f] + \mathcal{O}(\delta f^2)$$

integral eqn. relates δf to gradients in the system (one way to derive transport coeffs)

—

local thermal equilibrium: $f^{\rm eq}(x, \vec{p}) = \frac{g}{(2\pi^3)} \exp\left[\frac{\mu(x) - p \cdot u(x)}{T(x)}\right]$

shear: $T = const$, $\mu = const$, $2\sigma^{\mu\nu} \equiv \nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}(\partial u) \neq 0$

bulk: $T = const$, $\mu = const$, $\partial u \neq 0$

For shear: $p \cdot \nabla f^{\rm eq} = -\frac{T^2}{2} f^{\rm eq} P^{\mu\nu}(p) X_{\mu\nu}(x)$

where

$$P^{\mu\nu} \equiv \frac{1}{T^2}\left[\Delta^\mu_\alpha \Delta^\nu_\beta p^\alpha p^\beta - \frac{1}{3}\Delta^{\mu\nu}(\Delta_{\alpha\beta}p^\alpha p^\beta)\right] \ , \quad X^{\mu\nu} \equiv \frac{\sigma^{\mu\nu}}{T}$$

With $\delta f \equiv \phi f^{\rm eq}$, decomposition into irreducible tensors in x and p gives

$$\phi_i = \chi_i(\tilde{p})\, P^{\mu\nu} X_{\mu\nu}, \qquad \tilde{p} \equiv \frac{p_{LR}}{T}$$

and the integral eqn. becomes

$$-\frac{1}{2} P_{1\cdot 1} f^{\rm eq}_{1i} \quad = \quad \frac{1}{T^2} \sum_{jk\ell} \iiint\limits_{2\,3\,4} f^{\rm eq}_{1i} f^{\rm eq}_{2j}\, \bar{W}^{ij\to k\ell}_{12\to 34}\, \delta^4(12-34)\,(\chi_{3k}P_{3\cdot 1} + \chi_{4\ell}P_{4\cdot 1}$$
$$-\chi_{1i}P_{1\cdot 1} - \chi_{2j}P_{2\cdot 1})$$

with

$$\chi_{ai} \equiv \chi_i(|\tilde{\vec{p}}_a|) \quad , \qquad P_{a\cdot b} \equiv P^{\mu\nu}_a P_{b,\mu\nu} = (\tilde{\vec{p}}_a \tilde{\vec{p}}_b)^2 - \frac{1}{3}|\tilde{\vec{p}}_a|^2 |\tilde{\vec{p}}_b|^2 \ . \qquad (1)$$

This is equivalent to maximizing

$$Q[\chi] \quad = \quad \frac{1}{2T^2}\sum_i \int\limits_1 P_{1\cdot 1} f^{\rm eq}_{1i}\chi_{1i} + \frac{1}{2T^4}\sum_{ijk\ell} \iiiint\limits_{1\,2\,3\,4} f^{\rm eq}_{1i} f^{\rm eq}_{2j}\, \bar{W}^{ij\to k\ell}_{12\to 34}\, \delta^4(12-34)$$
$$(\chi_{3k}P_{3\cdot 1} + \chi_{4\ell}P_{4\cdot 1} - \chi_{1i}P_{1\cdot 1} - \chi_{2j}P_{2\cdot 1})\chi_{1i}$$

The maximum gives the shear viscosity: $\eta = 8 Q_{max} T^3/5$.

Variational solution

This is analogous to how a linear algebra problem

$$A|\chi\rangle = |b\rangle \qquad \Rightarrow \qquad |\chi\rangle = A^{-1}|b\rangle$$

is equivalent to a maximization/minimization problem

$$Q[\chi] \equiv \frac{1}{2}\langle\chi|A|\chi\rangle - \langle\chi|b\rangle \ , \qquad \delta Q = 0$$

(if A is symmetric and negative/positive definite)

In our case, $|\chi\rangle$ is infinite dimensional. In practice we truncate

$$\chi_i(p) \approx \sum_{k=0}^{n} c_{i,k}\phi_k(p)$$

and solve the finite-dimensional maximization problem to get $c_{i,k} \to \chi_i(p)$

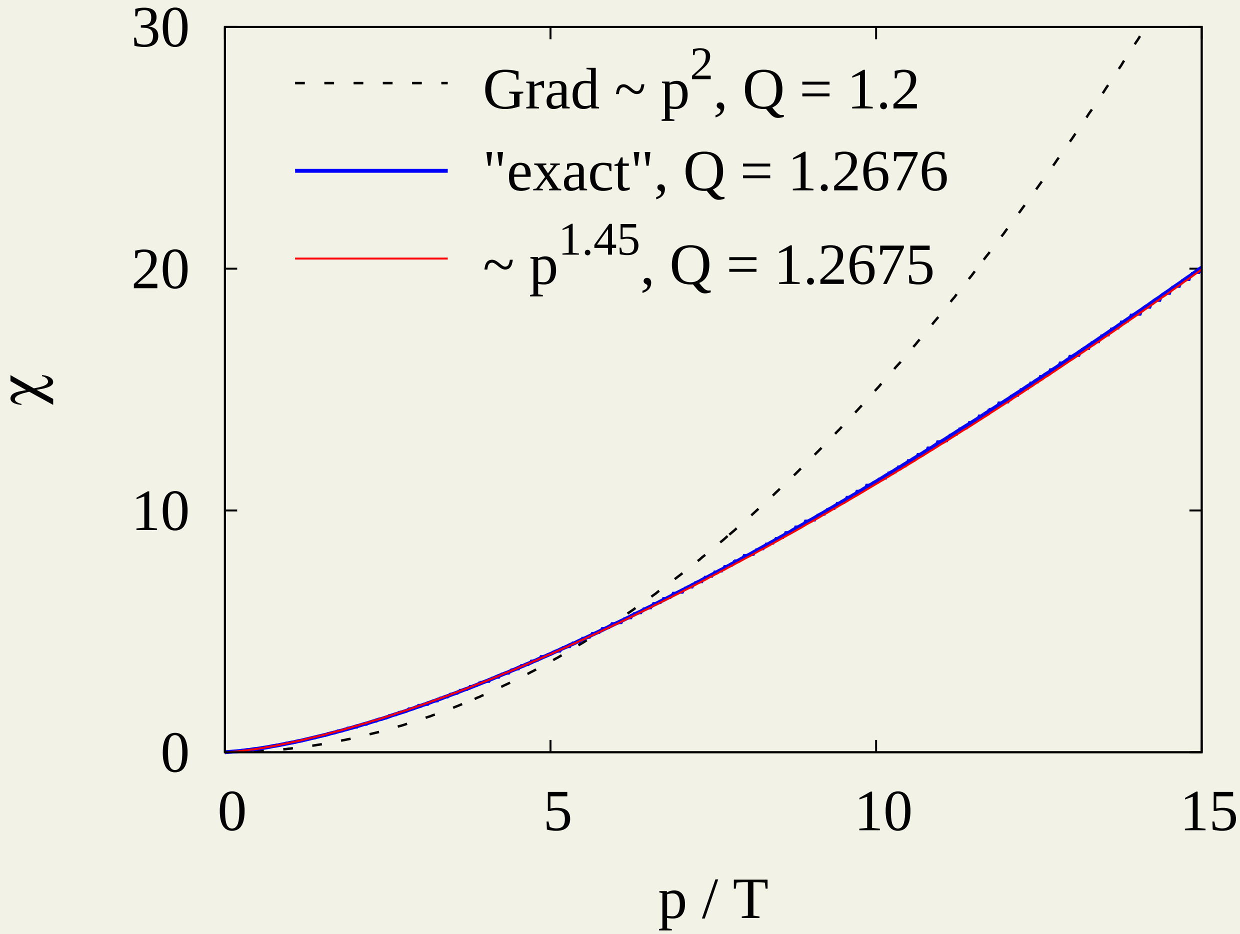
—

For isotropic σ_{ij}, need integration in 4D for each A_{ij} $\to$ suitable for GPUs

III. Shear δf - one-component case

Grad ($\propto p^2$) vs linear response - massless gas with isotropic $\sigma = const$:

DM ('11)

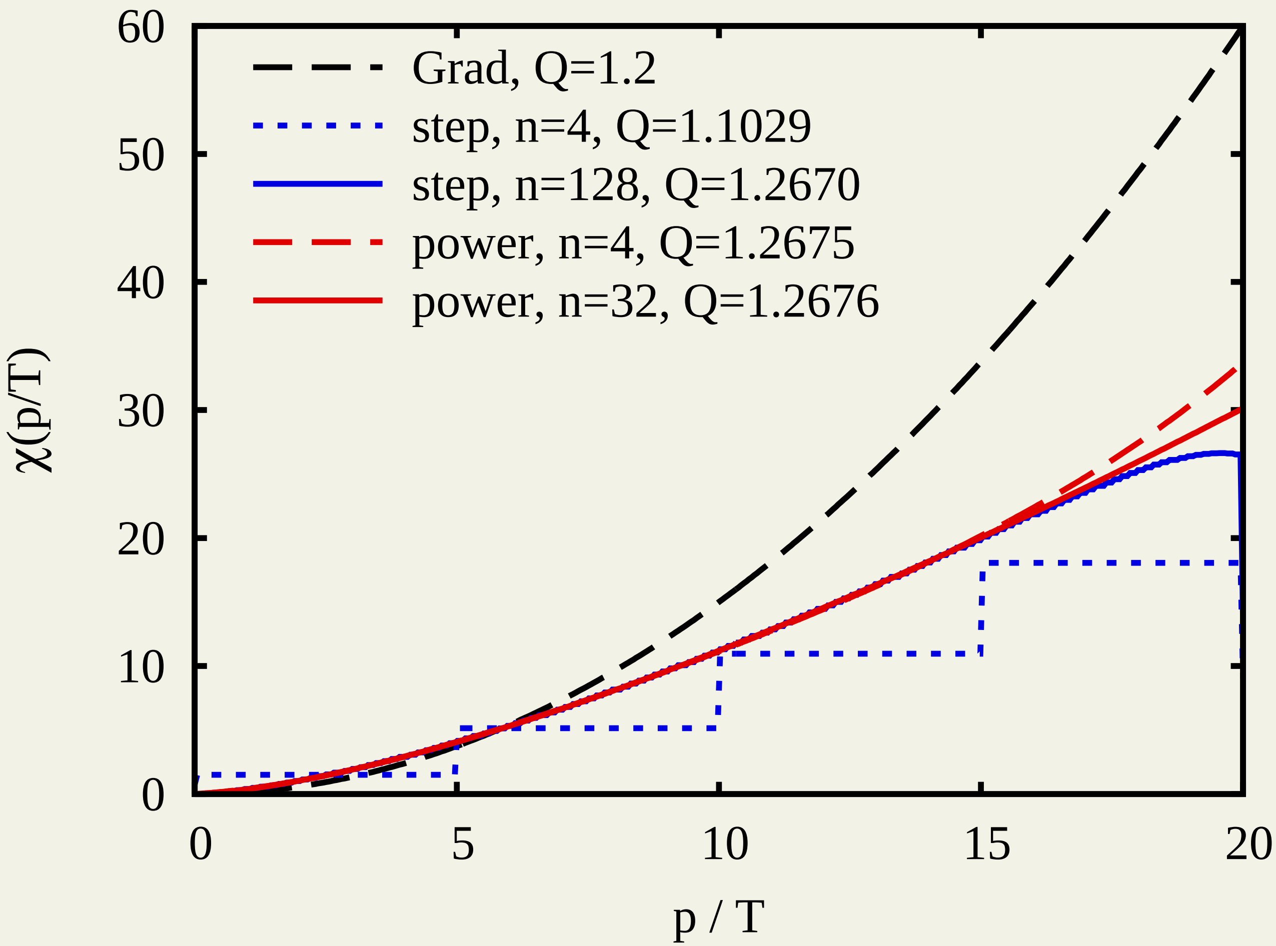


$$\delta f = \chi(p/T) \frac{T}{n\sigma} \frac{\pi_{\mu\nu}\hat{p}^\mu \hat{p}^\nu}{\eta T} f_{eq}$$

$$\boldsymbol{\eta \simeq \frac{1.2676 T}{\sigma}}$$

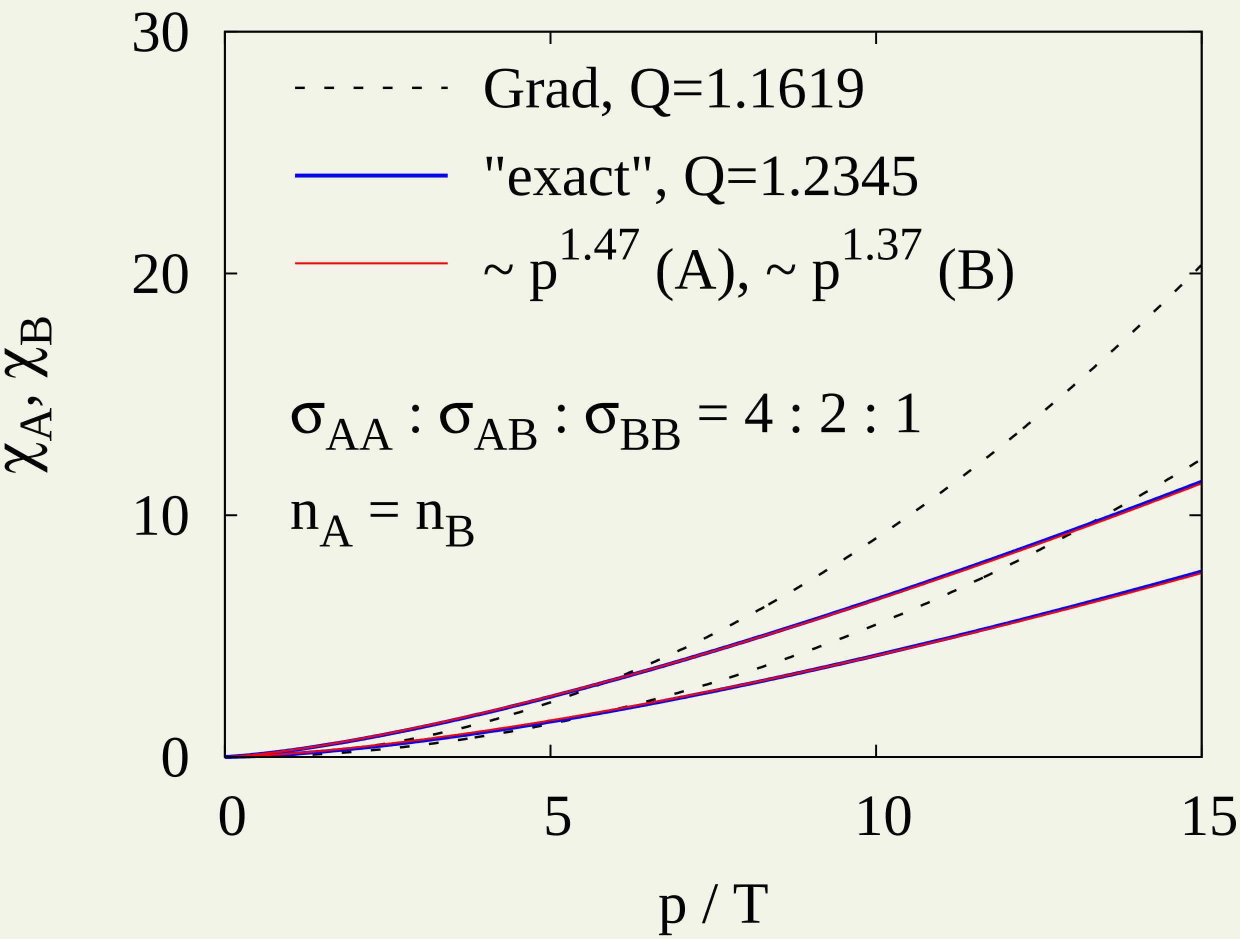
Dusling, Moore et al: **similar** $\sim p^{1.5}$ **for forward-peaked** $2 \to 2$ **and** $1 \leftrightarrow 2$

indeed converged



two-component, massless gas - weaker than quadratic δf_i too

DM ('11)



Hadron gas shear δf

studied a variety of systems: with elastic $ij \to ij$ processes DM & Wolff, PRC95

i) π **gas with** $\sigma = const$

ii) π **gas with** $\sigma_{\pi\pi}(s)$ **fit to PDG tables**

iii) $\pi - N$ **gas with** $\sigma^{eff}_{\pi\pi} = 30$ **mb,** $\sigma^{eff}_{\pi N} = 50$ **mb,** $\sigma^{eff}_{NN} = 20$ **mb**

[fit to Prakash et al, Phys. Rep. 227 ('93)]

iv) hadron gas with 49 effective species (states up to $m = 1.650$ **GeV)**

[total 144 species but lumped isospin states together]

—

we generally find smaller δf **for larger mass, at fixed** σ

one also expects larger cross sections for baryons (additive quark model)

$\Rightarrow$ **generally smaller viscous corrections for baryons than for mesons**

χ_i^{grad}

for $\sigma_{ij} = 30$ **mb**

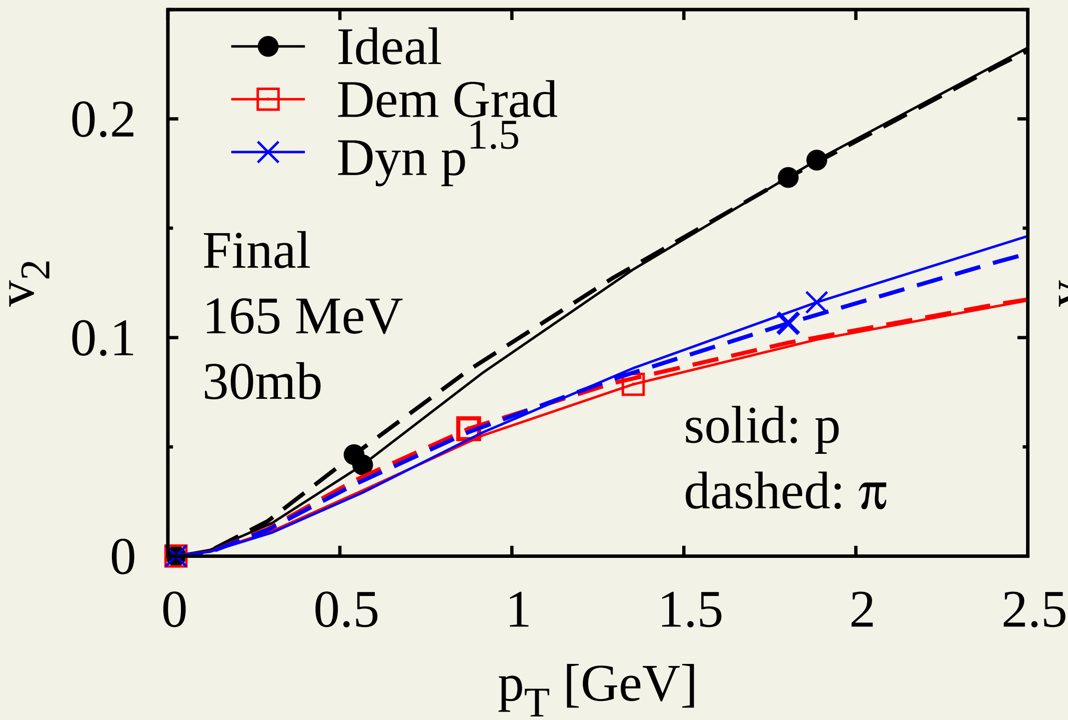
Species	T = 100	120	140	165 MeV
π	1.08	1.13	1.17	1.21
K	0.89	0.96	1.02	1.08
η	0.87	0.94	1.00	1.06
f_0	0.85	0.92	0.98	1.04
ρ	0.80	0.87	0.93	0.99
ω	0.80	0.86	0.93	0.99
K^*(892)	0.77	0.83	0.90	0.96
N	0.76	0.82	0.88	0.94
η'(958)	0.75	0.82	0.88	0.94
f_0(980)	0.75	0.81	0.87	0.93
a_0(980)	0.75	0.81	0.87	0.93
ϕ(1020)	0.74	0.81	0.86	0.92
Λ	0.72	0.79	0.84	0.90
h_1(1170)	0.72	0.78	0.83	0.89
Σ	0.71	0.77	0.83	0.89
b_1(1235)	0.71	0.76	0.82	0.88
Δ(1232)	0.71	0.76	0.82	0.88
a_1(1260)	0.71	0.77	0.82	0.88
K_1(1270)	0.70	0.76	0.81	0.87
f_2(1270)	0.70	0.76	0.81	0.87
f_1(1285)	0.70	0.76	0.81	0.87
η(1295)	0.70	0.75	0.81	0.87
π(1300)	0.70	0.75	0.81	0.87

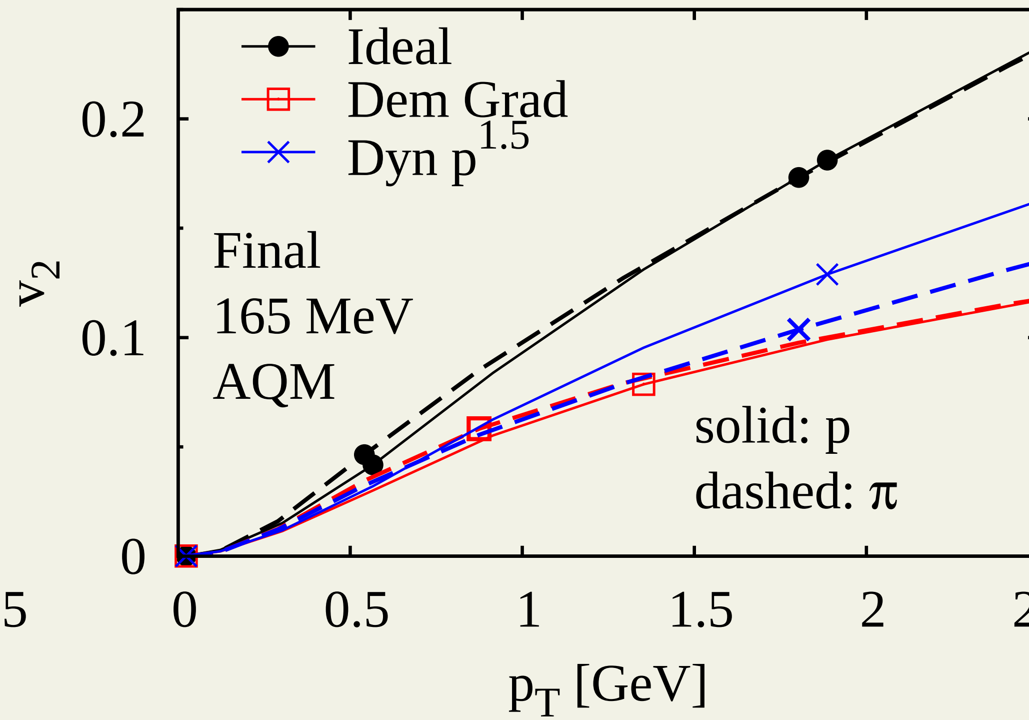
Effect on elliptic flow of π & p

DM & Wolff, PRC95: $\boldsymbol{v_2(p_T)}$ **in Au+Au at RHIC,** $b = 8$ **fm,** $\eta/s = 0.1$

$\sigma_{ij} = 30$ **mb** $\sigma_{MM} : \sigma_{MB} : \sigma_{BB} = 4 : 6 : 9$

(additive quark model)

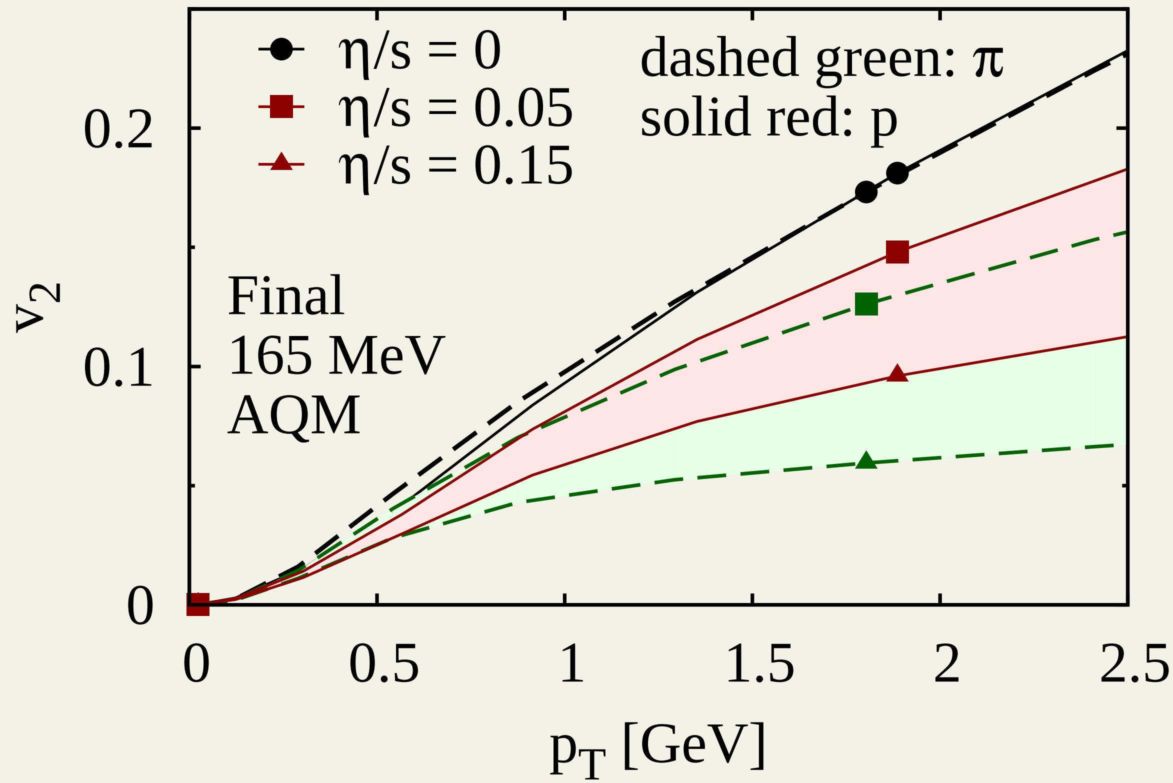




$\pi - p$ **splitting grows larger with self-consistent** δf

Sensitivity to shear viscosity

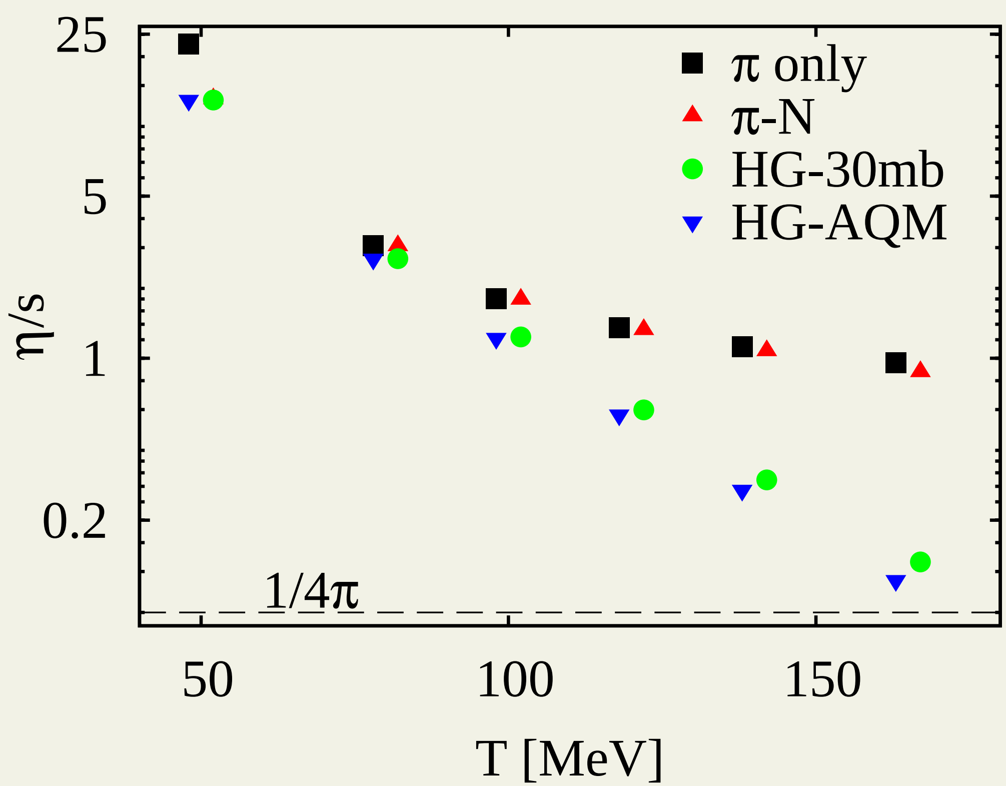
DM & Wolff, PRC95



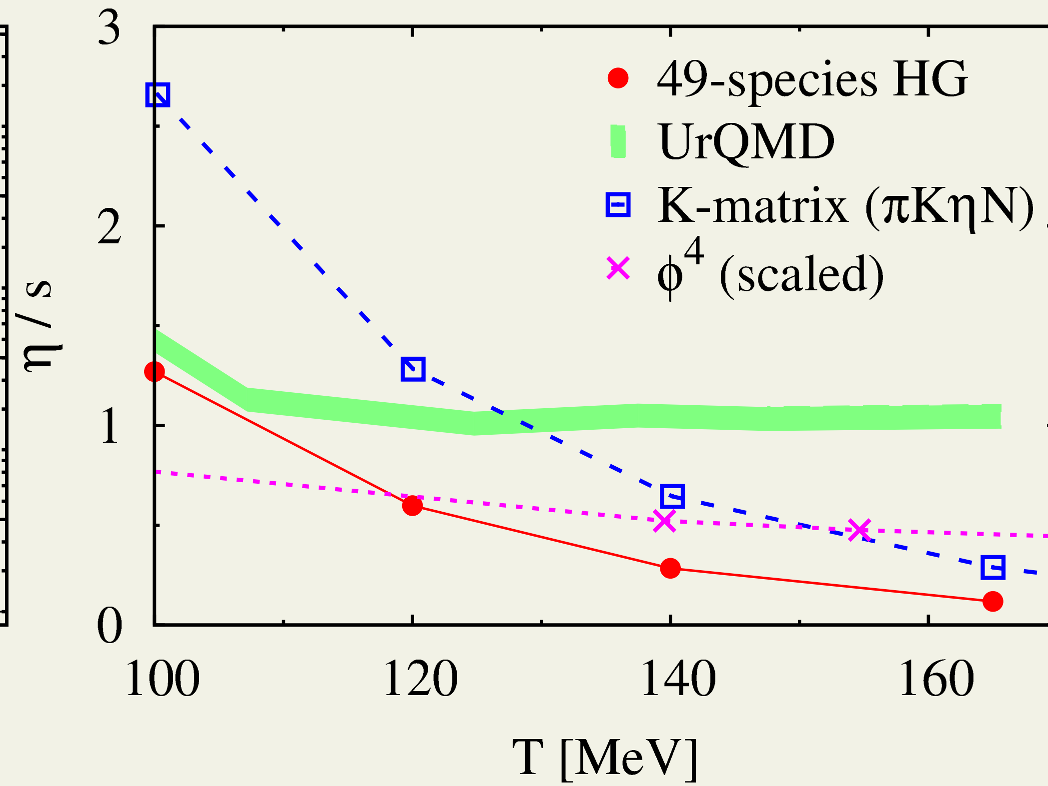
with self-consistent δf, need up to 50% larger η/s to get same v_2 suppression

Hadron gas η/s

DM & Wolff ('17)



DM & Wolff, PRC96



η/s **drops quite low as number of species increases - mostly driven by** s

agreement within factor of 2 with Prakash et al K**-matrix result**

do <u>not</u> find flat η/s **vs** T **like Bass et al did (UrQMD)**

one-component massive gas w/ isotropic $\sigma = const$ also reproduced

de Groot et al:

$$\eta^{Grad} = \frac{15z^2 K_2^2(z) h^2(z)}{16[(15z^2+2)K_2(2z) + (3z^3+49z)K_3(2z)]} \cdot \frac{T}{\sigma}$$

where

$$h(z) \equiv \frac{zK_3(z)}{K_2(z)} \,, \qquad\qquad z \equiv m/T$$

[fixed typo in *de Groot et al* kinetic theory book]

IV. Bulk δf models (Π)

Grad ansatz: Monnai & Hirano, PRC 80 ('09)

$$\delta f_{Grad} = -f_{eq}\Pi[A + B(p\cdot u) + C(p\cdot u)^2] \equiv -f_{eq}\Pi[A + BE_{LR} + CE_{LR}^2]$$

relaxation time approx: Dusling & Schäfer, PRC85 ('12)

$$\delta f_{DS} = -f_{eq}\frac{\Pi}{\zeta T^2}\left(\frac{p_{LR}^2}{3E_{LR}} - c_s^2 E_{LR}\right)$$

postulated linear deformations Pratt & Torrieri, PRC82 ('10)

$$f_{eq} + \delta f_{PT} \simeq f_{eq}(p_i \to \lambda_{ij} p_j,\ T \to T',\ \mu \to \mu')$$

$$[f_{eq} \sim e^{\mu/T - E(p)/T},\ E(p) = \sqrt{m^2 + p^2}]$$

like for shear, linearize in $\delta f = f - f_{eq}$, and put $\partial_t \to 0$

$$p \cdot \nabla f_{eq} = C[f_{eq}, \delta f]$$

but now keep bulk $(\nabla \cdot u)$ deformation in flow fields

$$p \cdot \nabla f_{eq} = -f_{eq} \frac{p_\mu p_\nu}{2T} \left[\tfrac{2}{3}\Delta^{\mu\nu}(\nabla \cdot u) + \nabla^\mu u^\nu + \nabla^\nu u^\mu - \tfrac{2}{3}\Delta^{\mu\nu}(\nabla \cdot u)\right]$$

$$\Rightarrow \quad f_{eq} \frac{p_{LR}^2}{3} \frac{(\nabla \cdot u)}{T} = C[f_{eq}, \delta f]$$

solution of integral eqn. must be of form

$$\delta f(x, \vec{p}) = \chi\left(\frac{p \cdot u}{T}\right) \frac{(\nabla \cdot u)}{T} f_{eq}$$

subject to two constraints that δf adds no energy or particle number:

$$\int \frac{d^3p}{E} (p \cdot u)^2 \delta f = 0 \ , \qquad \int \frac{d^3p}{E} (p \cdot u) \delta f = 0$$

$\Rightarrow$ **bulk δf varies in sign(!)**

Variational solutions

the integral eqn for the bulk δf can be turned into a constrained variational (maximization) problem $\delta Q[\chi] = 0$:

$$Q[\chi] = \frac{1}{2T^8} \iiiint_{1234} f_{1,eq} f_{2,eq}\, \chi_1 \left(\chi_3 + \chi_4 - \chi_1 - \chi_2\right) W_{12\to 34} \delta^4(12-34) - \frac{1}{3T^4}\int_1 f_{1,eq}\chi_1 p_1^2 + \alpha_E \int_1 f_{1,eq}\chi_1 E_1^2 + \alpha_N \int_1 f_{1,eq}\chi_1 E_1 \qquad (2)$$

where $\alpha_{E,N}$ are Lagrange multipliers, $\delta^4(12-34) \equiv \delta^4(p_1 + p_2 - p_3 - p_4)$, and

$$\int_i \equiv \int \frac{d^3 p_i}{2E_i}\ , \qquad f_{i,eq} \equiv f_{eq}(p_i)\ , \qquad \chi_i \equiv \chi(p_i)\ , \qquad W_{12\to 34} = \frac{2}{\pi} s \sigma_{TOT}(s)$$

Maximum gives the bulk viscosity: $\zeta = 4 Q_{max} T^3$.

Bulk viscosity vs mass

Grad ansatz: $\delta f_{Grad} = f_{eq}[A + B(p \cdot u) + C(p \cdot u)^2]$

only 1 free parameter because must have $\delta e = 0$**,** $\delta n = 0$

For $\sigma = const$**, text book Grad result** de Groot et al (80s)

$$\zeta_{Grad} = \frac{z^2 K_2^2(z)[(5 - 3\gamma)\hat{h} - 3\gamma]^2}{16[2K_2(2z) + zK_3(2z)]} \frac{T}{\sigma} \qquad \text{with} \qquad z \equiv \frac{m}{T}$$

where $\gamma \equiv c_p / c_v$

$$\hat{h} \equiv \frac{h}{n} = \frac{zK_3(z)}{K_2(z)} \; , \qquad \qquad \frac{\gamma}{\gamma - 1} = 5 + \hat{h} - \hat{h}^2$$

ultrarelativistic limit $(z \to 0)$**:** $\zeta_{Grad} \sim z^4 T/\sigma \to \; 0$

nonrelativistic limit $(z \to \infty)$**:** $\zeta_{Grad} \sim z^{-3/2} T/\sigma \to \; 0$

Why bulk viscosity vanishes:

In UR limit

$$T^\mu_{\ \mu} = \int \frac{d^3p}{E} p^\mu p_\mu f \ \propto \ m^2 \ \to \ 0$$

In other words, $T^i_i \equiv 3(p_{eq} + \Pi) = T^{00} \equiv e$, and so since Π vanishes in equilibrium $(e = 3p_{eq})$, then it must vanish always.

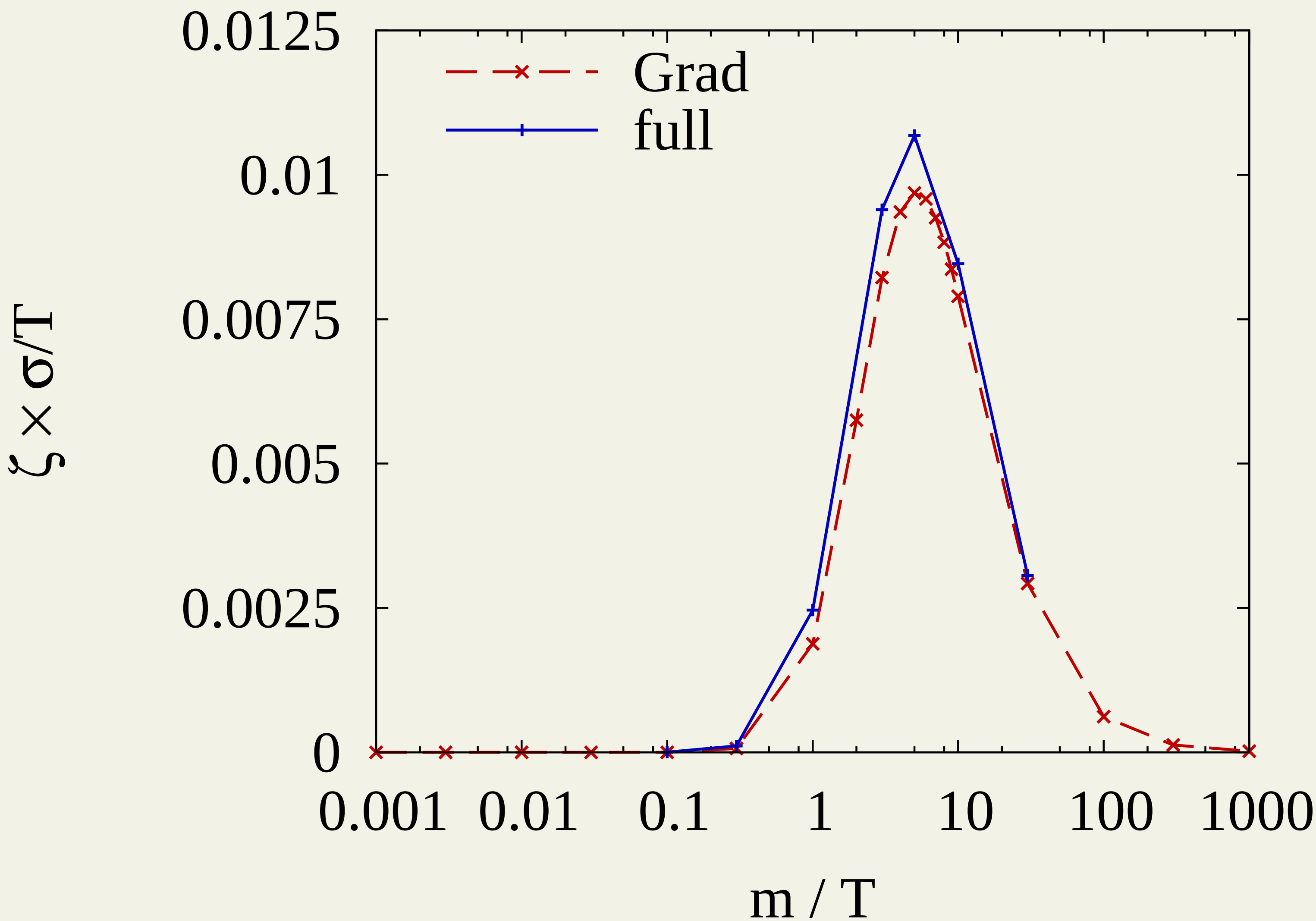
Whereas in NR limit

$$\Pi \equiv \frac{1}{3}\delta T^i_i = \frac{1}{3}\int \frac{d^3p}{E}|\vec{p}|^2 \delta f \ \to \frac{1}{3m}\int d^3p\,|p|^2\,\delta f$$

with the general constraints $\delta e = 0$, $\delta n = 0$, so

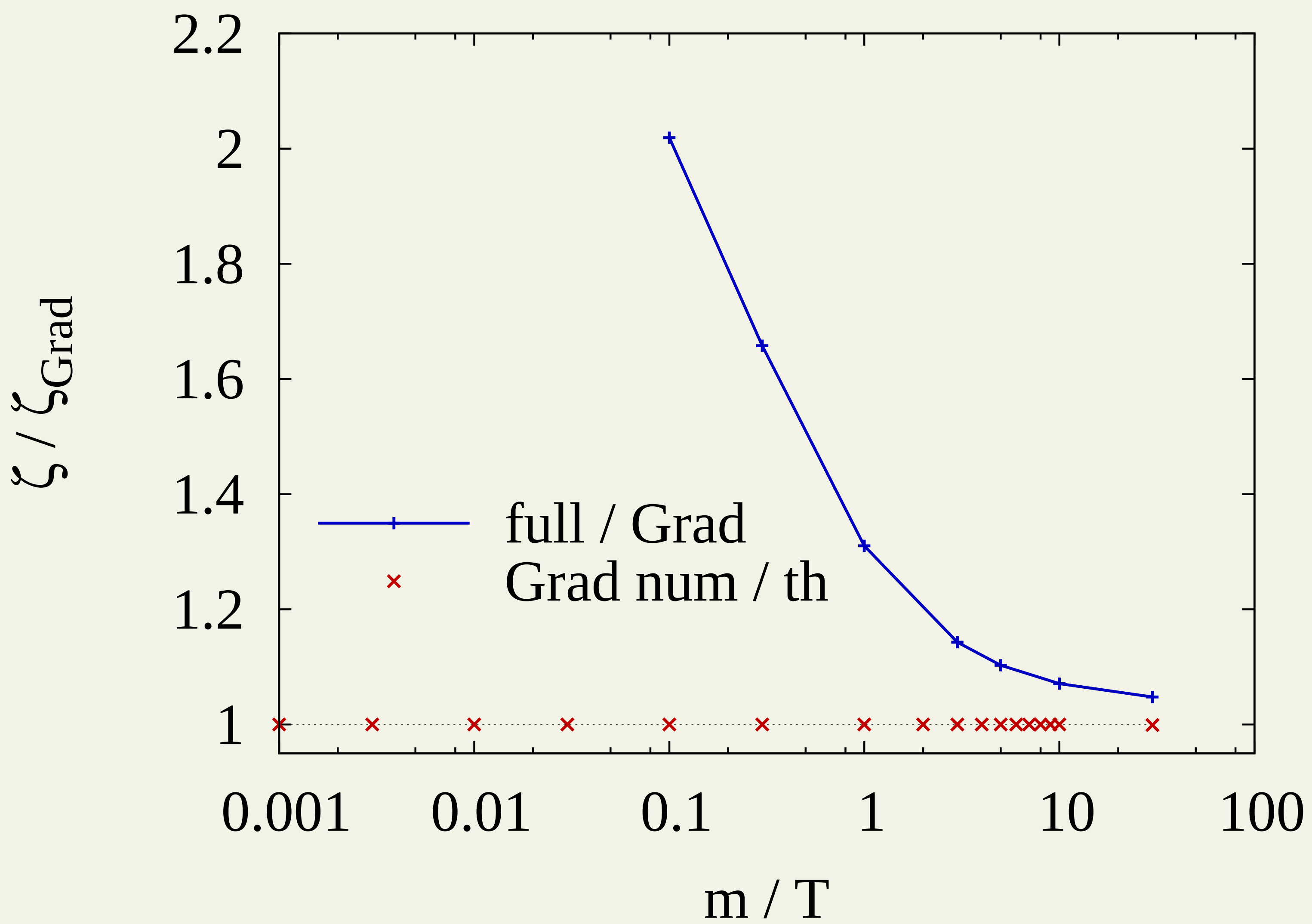
$$\delta e - m\delta n \equiv \delta e_{kin} = 0 = \int \frac{d^3p}{E}(E^2 - mE)\delta f \ \to \ \frac{1}{2m}\int d^3p\,|p|^2\,\delta f \ = \ 0$$

DM ('17)



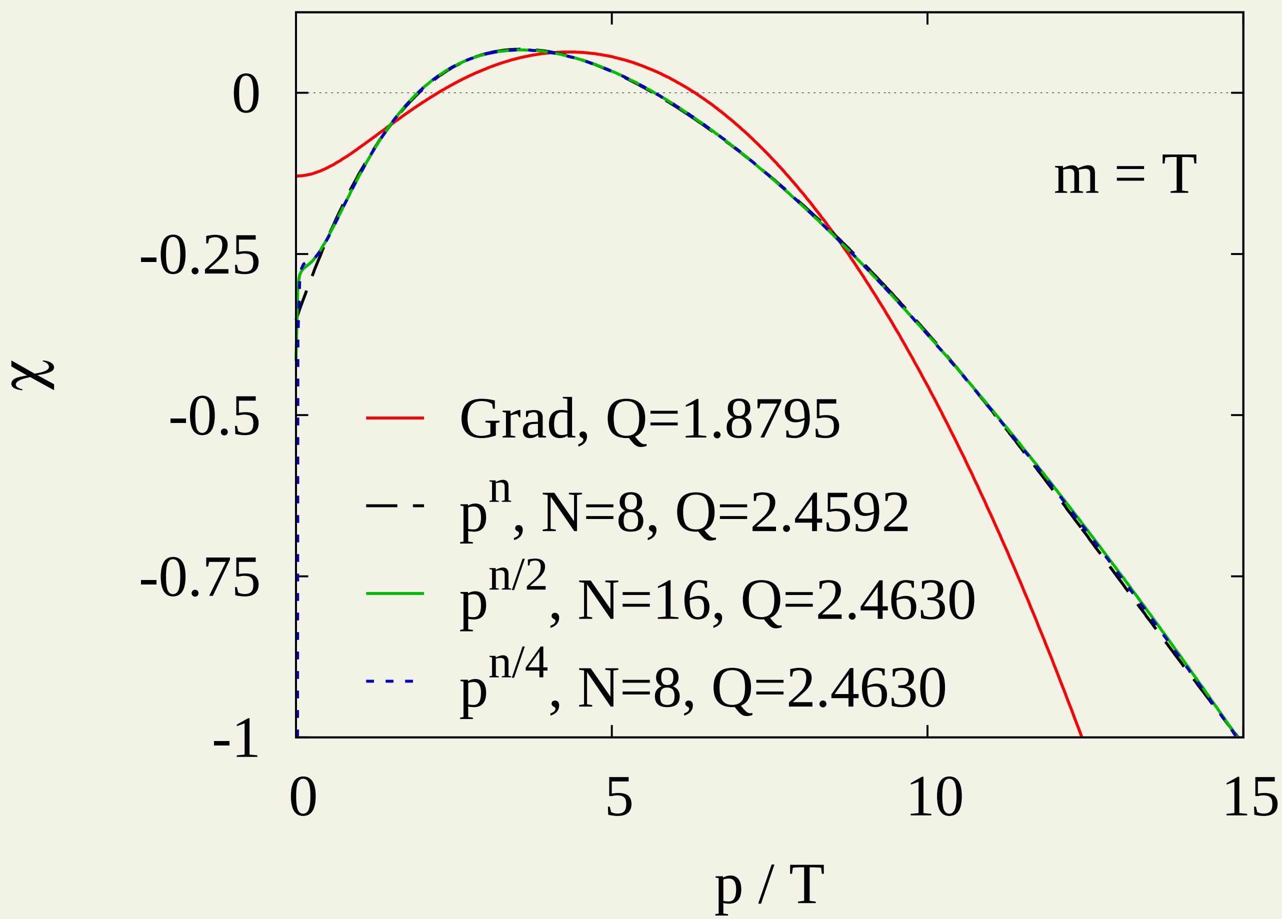
the Grad bulk viscosity result looks reasonably accurate

DM ('17)



except for $m/T \lesssim 1$

Full solution for χ deviates from Grad ansatz $\chi^{bulk} = Az^2 + B(\frac{p \cdot u}{T}) + C(\frac{p \cdot u}{T})^2$



e.g., less steep at high momenta

also differs from rel. time result

$$\delta f_{RTA} \propto \left(\frac{p^2}{3} - c_s^2 E^2\right)$$

because at low momenta

$$\delta f_{RTA} \propto -c_s^2 m^2 \quad < \; 0$$

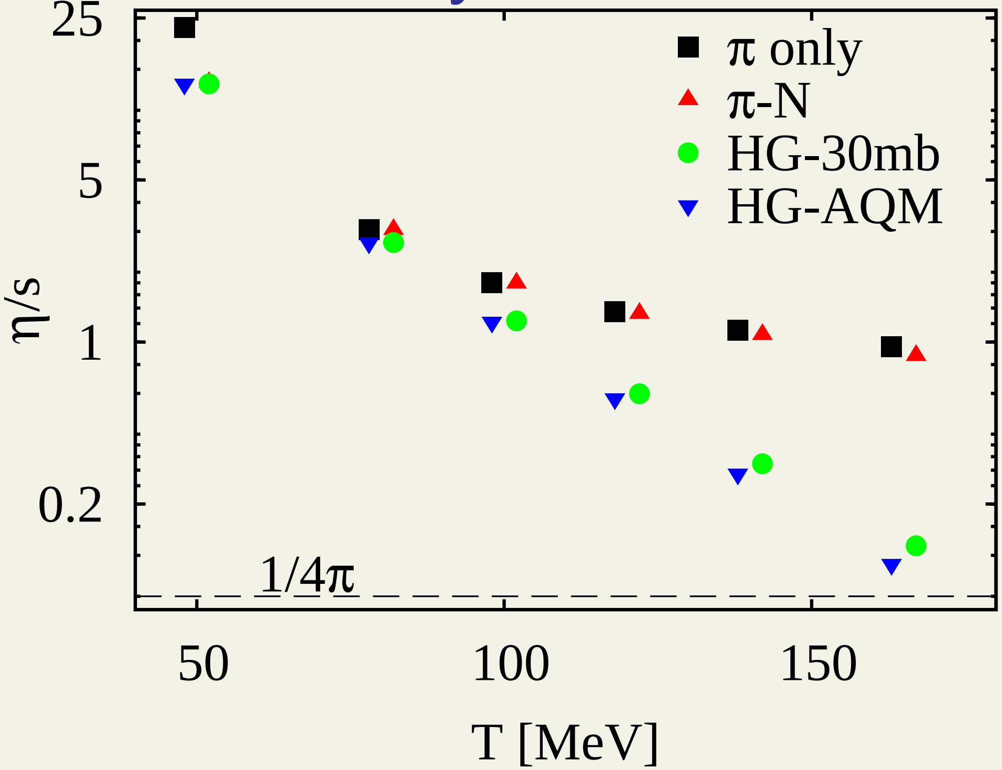
while at high momenta

$$\delta f_{RTA} \propto \left(\frac{1}{3} - c_s^2\right) p^2 \quad > \; 0$$

Hadron gas viscosity

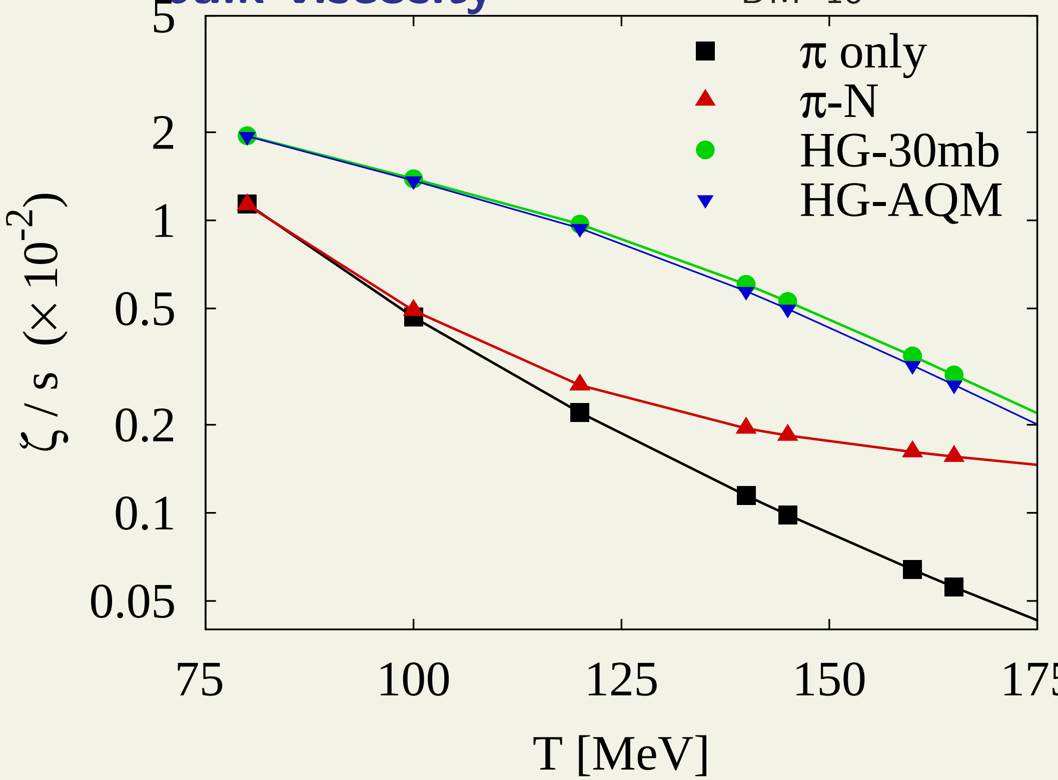
49-species, elastic $ij \to ij$ interactions with constant, isotropic cross sections that follow additive quark model (AQM) meson-baryon scaling: $\sigma_{MM} : \sigma_{MB} : \sigma_{BB} = 30 : 45 : 67.5$ mb.

shear viscosity



bulk viscosity

DM '18



With more species ζ/s is higher, especially at higher T.

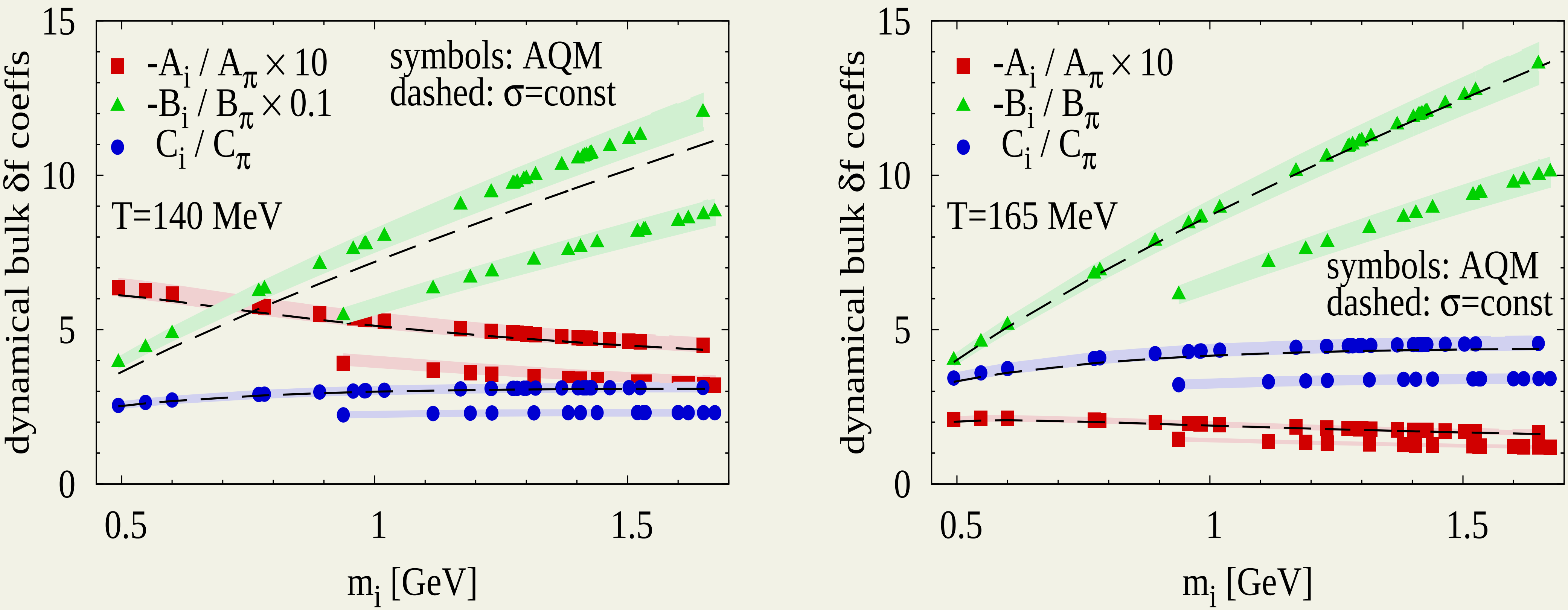
Still, $\zeta/\eta \sim 10^{-2}$ only. Same holds for species-independent $\sigma_{ij} = 30$ mb.

Species-dependent bulk δf

Now analyze the *species-dependent* **Grad basis used in the calculation**

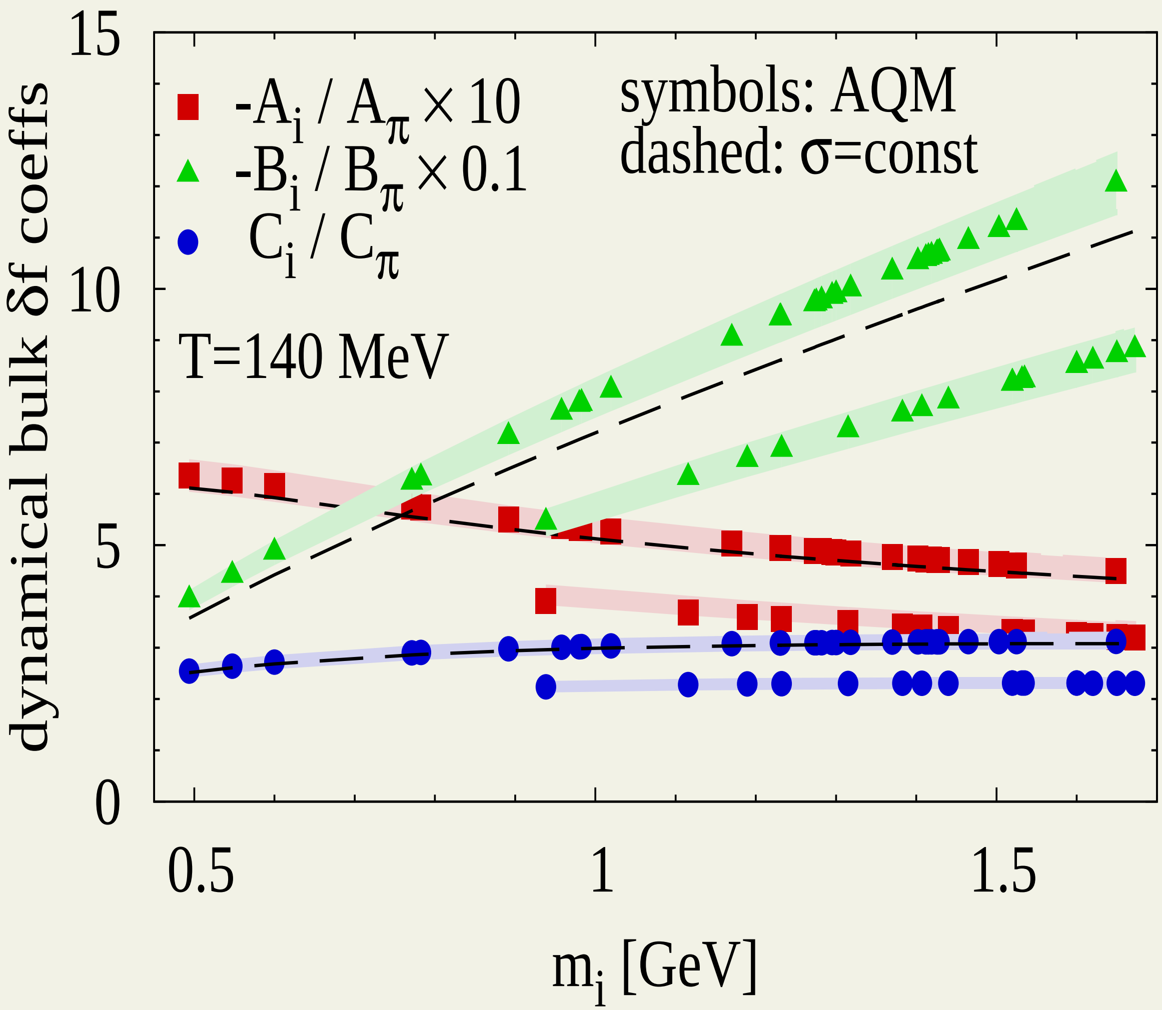
$$\chi_i = A_i z_i^2 + B_i(E/T) + C_i(E/T)^2$$

At $T = 140$ and 165 MeV, plot A_i, B_i, C_i relative to those for pions:

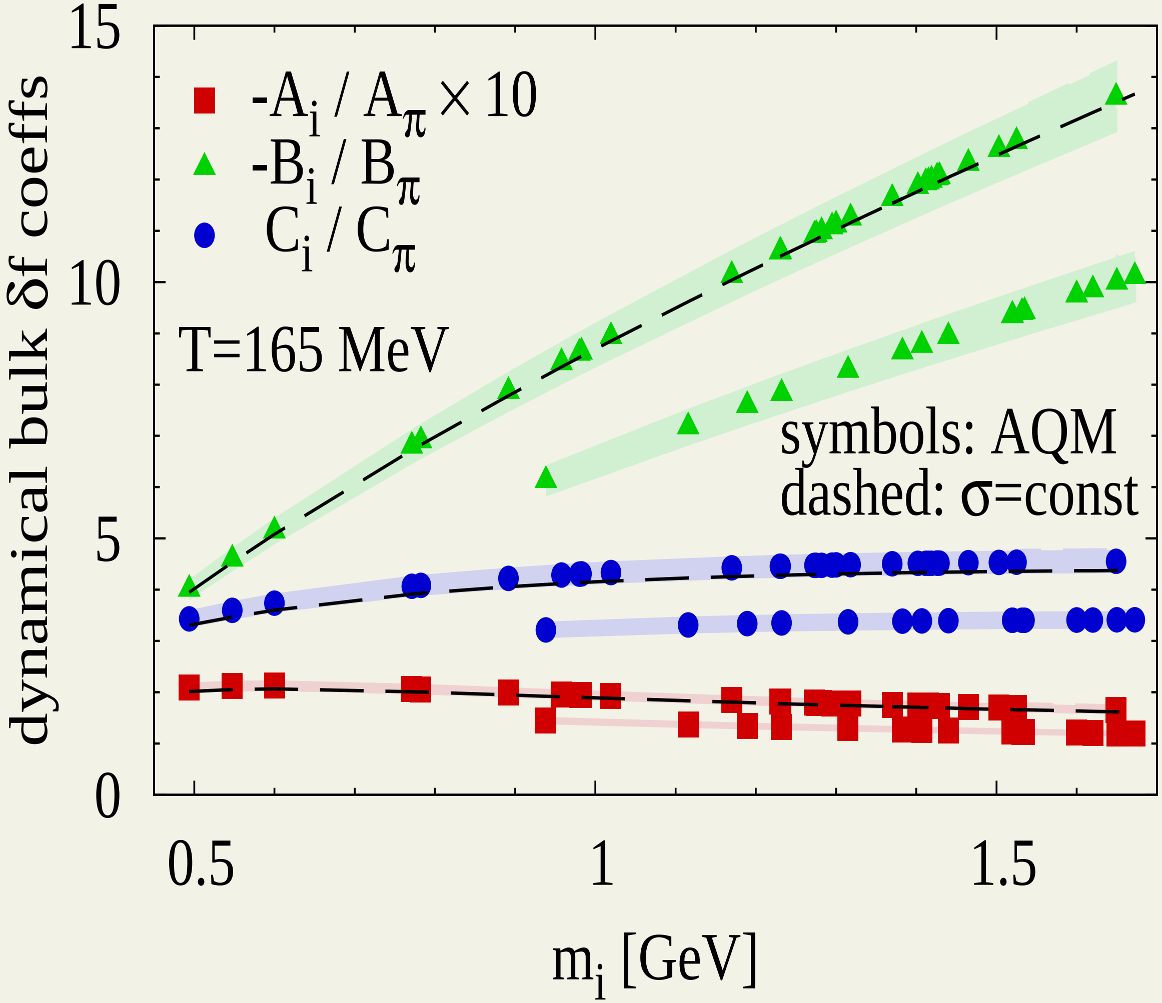


$\sim$25% smaller coeffs for baryons due to larger σ (AQM), but smooth mass dependence within both meson and baryon bands. Significant T dependence.

$T = 140$ **MeV** $\chi_i = A_i z_i^2 + B_i(E/T) + C_i(E/T)^2$ [DM @ QM2018]



$T = 165$ Me $\chi_i = A_i z_i^2 + B_i(E/T) + C_i(E/T)^2$ [DM @ QM2018]



Effect on elliptic flow

Calculate v_2 in Au+Au at RHIC as for shear δf

- use 2+1D AZHYDRO

- estimate $\pi^{\mu\nu}$ and Π from flow gradients (Navier-Stokes)

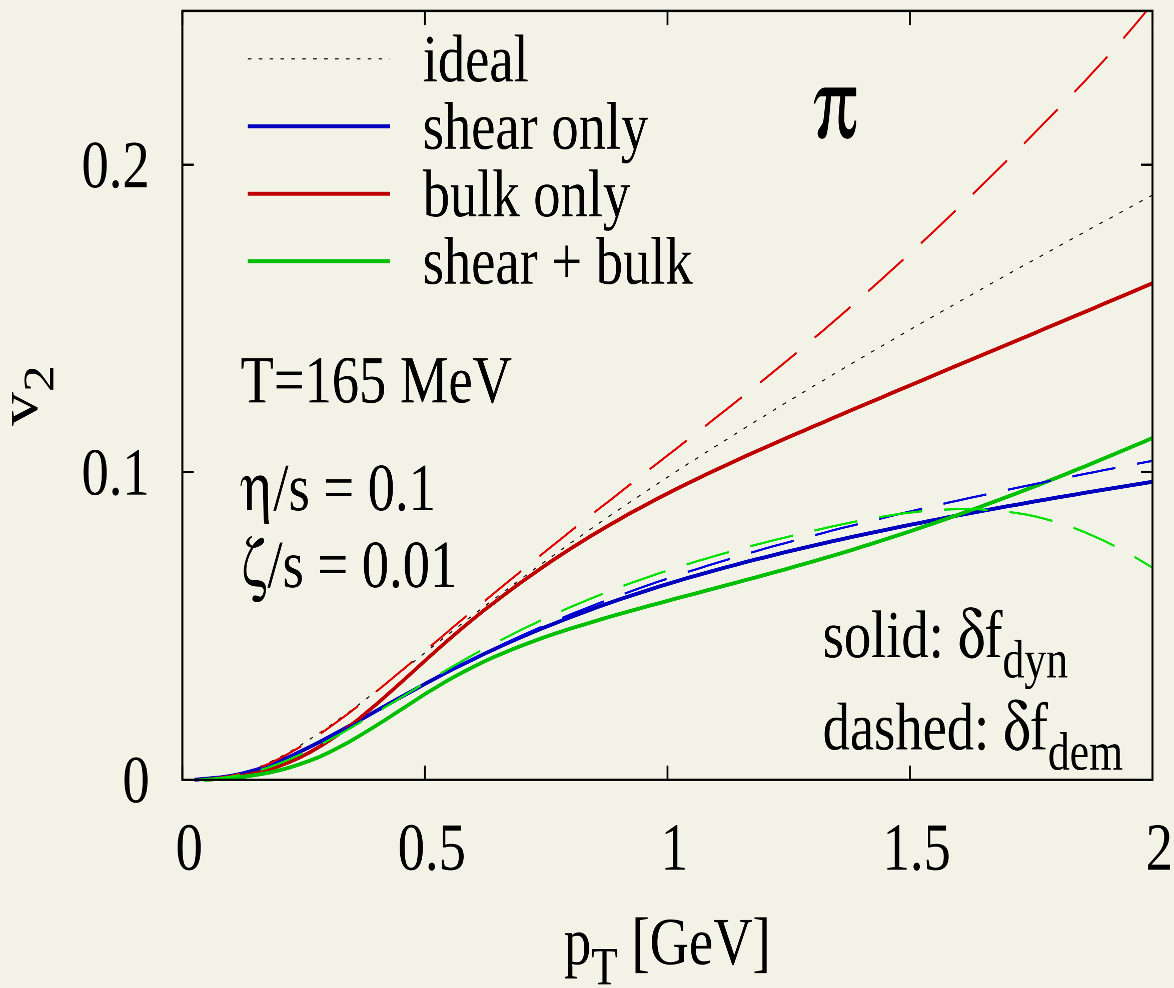
Contrast self-consistent bulk and shear δf_i (solid) with

- "democratic" species-independent Hirano-Monnai A, B, C

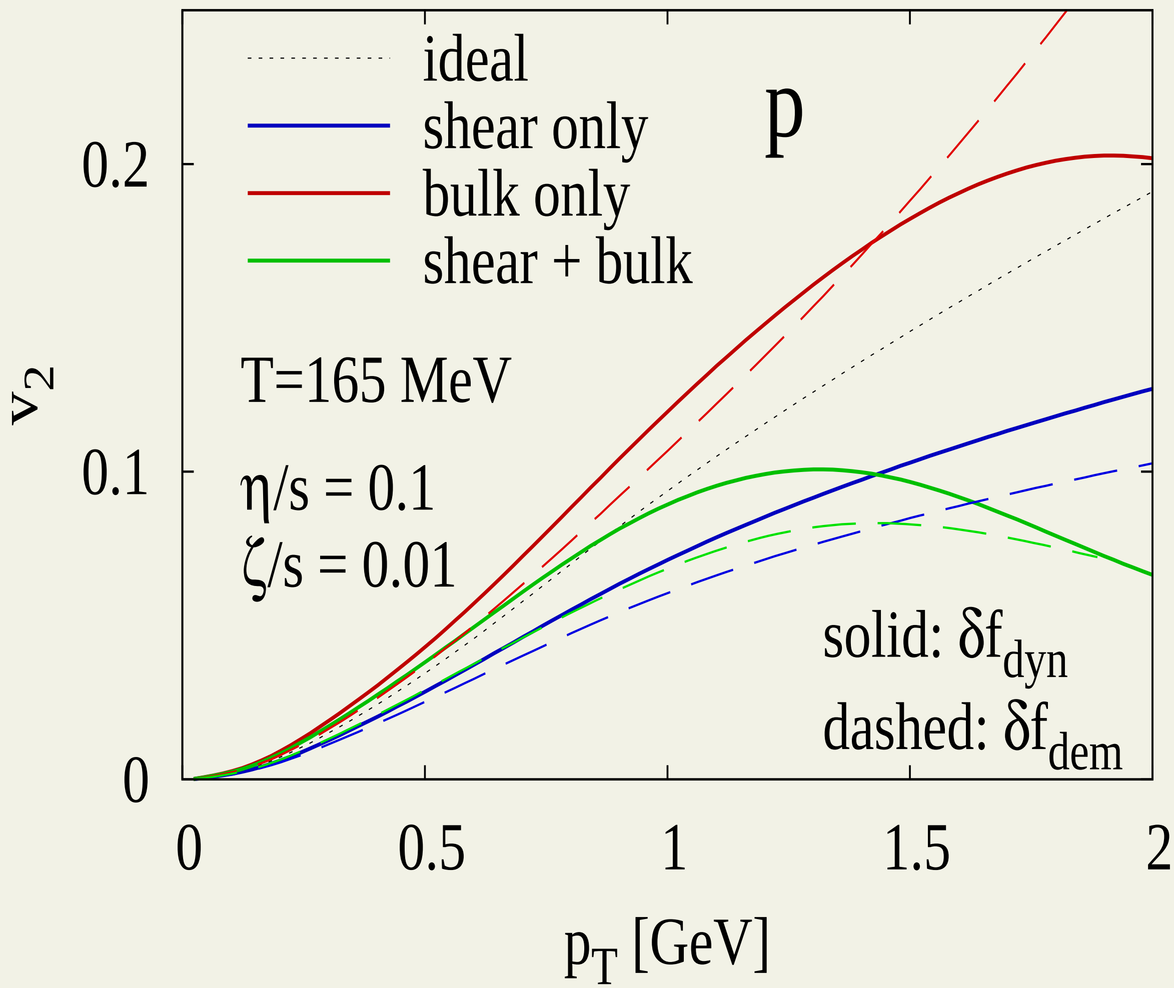
- and constant $\chi_i^\pi = \eta/2s$ (dashed)

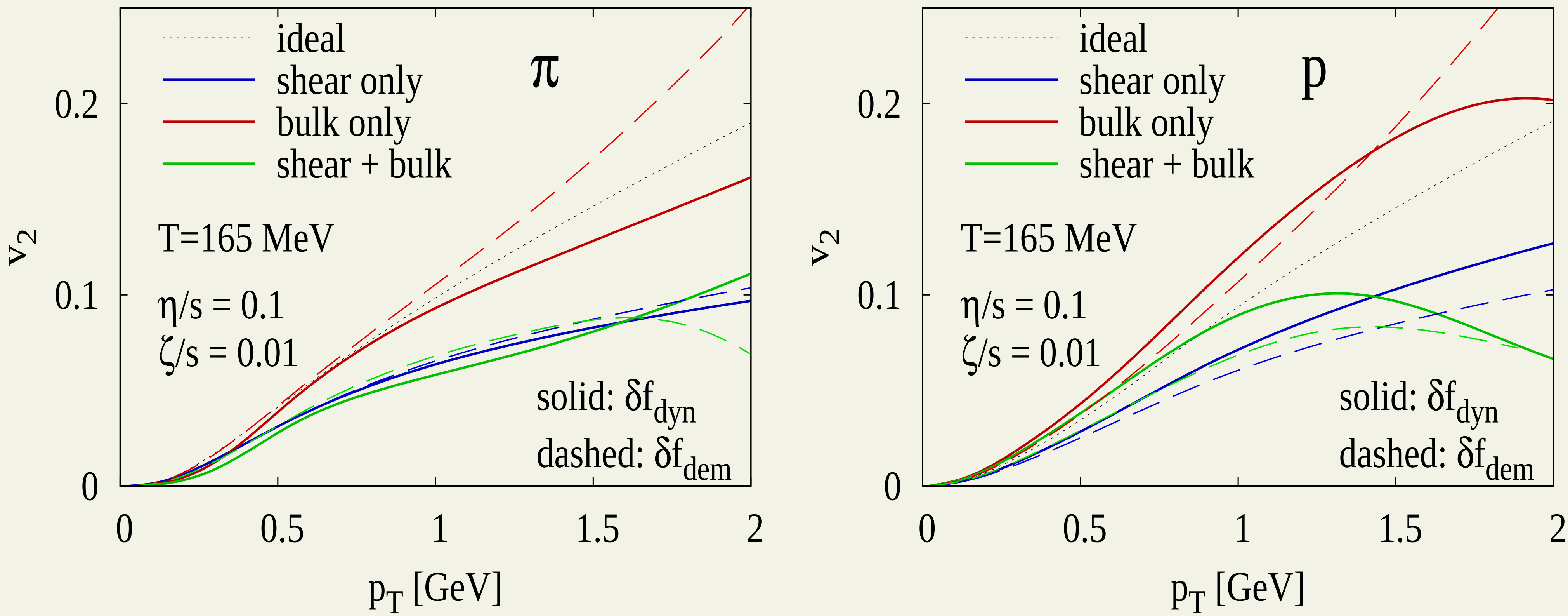
Set η/s=0.1 and large ζ/η=0.1.

pion $v_2(p_T)$


ideal
shear only
bulk only
shear + bulk
π
T=165 MeV
η/s = 0.1
ζ/s = 0.01
solid: δf_dyn
dashed: δf_dem
v_2
0.2
0.1
0
0
0.5
1
1.5
2
p_T [GeV]

proton $v_2(p_T)$


ideal
shear only
bulk only
shear + bulk
p
T=165 MeV
η/s = 0.1
ζ/s = 0.01
solid: δf_dyn
dashed: δf_dem
v_2
0.2
0.1
0
0
0.5
1
1.5
2
p_T [GeV]



-~10-20% variation in v_2 with δf model.

- generally more proton v_2 with self-consistent δf

- for π, correction even flips sign in pure bulk viscous case.

Summary

Comparison of hydrodynamic calculations to data inevitably requires a particlization model. Instead of *ad hoc* parameterizations, one can obtain self-consistent viscous phase space corrections from linearized kinetic theory.

Self-consistent shear and bulk viscous corrections affect identified particle $v_2(p_T)$ in A+A at RHIC, and very likely at the LHC too, even for $\zeta/\eta \sim 0.1$. Corrections for p+A reactions should be significant as well (higher gradients).

We find δf corrections that vary smoothly with mass, which lends itself to convenient parametrizations. Try our δf_i in your calculations, if you can.

Next steps:

- inelastic channels $\sigma_{ij\to k\ell}$
- full solution, go beyond Grad form for χ_{bulk}
- compare in detail to other bulk δf models
- implant into realistic hydro simulation
- heat flow ($\delta f \propto \nabla T$) and diffusion ($\delta f \propto \nabla n_i$)