Kolmogorovian Censorship Hypothesis

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As in religion and art for a long while, now in science, there is no fundamental principle that is not questioned; there is no nonsense that would not be believed by some people. (Planck 1930)

Kolmogorovian (Classical) Probability

vs.

Quantum Probability

Kolmogorovian (Classical) Probability

Event algebra: $\mathcal{A}$ Boolean algebra

Probability: $p : \mathcal{A} \to [0,1]$ such that

1. $p(\mathbb{1}) = 1$
2. $p(A \vee B) = p(A) + p(B) - p(A \wedge B)$

Quantum Probability

Event algebra: $L(H)$ subspace lattice of a Hilbert space

Probability: $p : L(H) \to [0,1]$ generated by a density operator W

$p(E) = tr(WE)$

Pitowsky theorem*

$$\mathbf{p} = (p_1, p_2, \ldots p_n, \ldots p_{ij}, \ldots)$$

Denote $R(n, S) \cong \mathbb{R}^{n+|S|}$ the linear space consisting of real vectors of this type. Let $\varepsilon \in \{0, 1\}^n$ be an arbitrary n-dimensional vector consisting of 0's and 1's. For each ε we construct the following $\mathbf{u}^\varepsilon \in R(n, S)$ vector:

$$u_i^\varepsilon = \varepsilon_i \qquad u_{ij}^\varepsilon = \varepsilon_i \varepsilon_j \qquad i = 1, 2, \ldots n \quad (i, j) \in S$$

The set of convex linear combinations of u^ε's is called a classical correlation polytope:

$$c(n, S) = \left\{ \mathbf{f} \in R(n, S) \,\middle|\, \mathbf{f} = \sum_\varepsilon \lambda_\varepsilon \mathbf{u}^\varepsilon \, ; \, \lambda_\varepsilon \geq 0; \sum_\varepsilon \lambda_\varepsilon = 1 \right\}$$

Theorem (Pitowsky 1989) *The correlation vector* $\mathbf{p}$ *admits a representation in a Kolmogorovian probability space if and only if* $\mathbf{p} \in c(n, S)$.

*Pitowsky, I. (1989): *Quantum Probability – Quantum Logic*, Lecture Notes in Physics **321**, Springer, Berlin.

“Bell-type” inequalities

The condition $\mathbf{p} \in c(2, \{(1,2)\})$ is equivalent with the following inequalities:

$$
\begin{gathered}
0 \leq p_{12} \leq p_1 \leq 1 \\
0 \leq p_{12} \leq p_2 \leq 1 \\
p_1 + p_2 - p_{12} \leq 1
\end{gathered}
$$

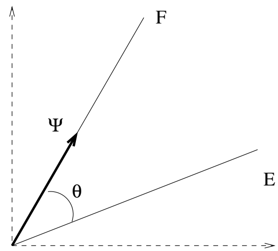
Similarly, for $\mathbf{p} \in c(4, \{(1,3),(1,4),(2,3),(2,4)\})$:

$$
\begin{gathered}
0 \leq p_{ij} \leq p_i \leq 1 \\
0 \leq p_{ij} \leq p_j \leq 1 \qquad\qquad i = 1,2 \;\; j = 3,4 \\
p_i + p_j - p_{ij} \leq 1
\end{gathered}
$$

$$
\begin{array}{rcccl}
-1 & \leq & p_{13} + p_{14} + p_{24} - p_{23} - p_1 - p_4 & \leq & 0 \\
-1 & \leq & p_{23} + p_{24} + p_{14} - p_{13} - p_2 - p_4 & \leq & 0 \\
-1 & \leq & p_{14} + p_{13} + p_{23} - p_{24} - p_1 - p_3 & \leq & 0 \\
-1 & \leq & p_{24} + p_{23} + p_{13} - p_{14} - p_2 - p_3 & \leq & 0
\end{array}
$$

Clauser–Horne inequalities.

Nonsensical probabilities for non-commuting elements



It can be shown* that for *any* two noncommuting elements $E_1, E_2 \in L(H)$ there always exists a pure state ψ such that

$$\underbrace{\langle \psi, E_1 \psi \rangle}_{1} + \underbrace{\langle \psi, E_2 \psi \rangle}_{>0} - \underbrace{\langle \psi, (E_1 \wedge E_2)\, \psi \rangle}_{0} > 1$$

It not simply violates the Kolmogorovian axiom 2, but it is a nonsense.

*Szabó, L. E. (2001): Critical reflections on quantum probability theory, in: *John von Neumann and the Foundations of Quantum Physics*, M. Rédei and M. Stoeltzner (eds.), Kluwer Academic Publishers, Dordrecht.

The Laboratory Record Argument*

> *There can not exist things—(quantum) events, properties, elements of reality etc.—whose relative frequency equals the quantum probability.*

Experiment	X_1	X_2	X_3	X_4	$X_1 \wedge X_3$	$X_1 \wedge X_4$	$X_2 \wedge X_3$	$X_2 \wedge X_4$
1	0	0	1	0	0	0	0	0
2	0	1	0	0	0	0	0	0
3	1	0	1	0	1	0	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99998	1	0	0	0	0	0	0	0
99999	0	0	1	0	0	0	0	0
N=100000	0	1	0	1	0	0	0	1
	N_1	N_2	N_3	N_4	N_{13}	N_{14}	N_{23}	N_{24}

The relative frequencies are

$$\nu_1 = \frac{N_1}{N}, \nu_1 = \frac{N_2}{N}, \ldots, \nu_{24} = \frac{N_{24}}{N}$$

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2	0	1	0	0	0	0	0	0
3	1	0	1	0	1	0	0	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
99998	1	0	0	0	0	0	0	0
99999	0	0	1	0	0	0	0	0
N=100000	0	1	0	1	0	0	0	1
	N_1	N_2	N_3	N_4	N_{13}	N_{14}	N_{23}	N_{24}

all rows are instances of the 2^4 classical truth functions $\boldsymbol{u}^\varepsilon$, $\varepsilon \in \{0,1\}^4$

N_ε = number of type-$\boldsymbol{u}^\varepsilon$ rows

The relative frequencies are

$$\nu_i = \sum_{\varepsilon\in\{0,1\}^4} \lambda_\varepsilon u_i^\varepsilon \qquad \nu_{ij} = \sum_{\varepsilon\in\{0,1\}^4} \lambda_\varepsilon u_{ij}^\varepsilon$$
$$\text{where } \lambda_\varepsilon = \frac{N_\varepsilon}{N}$$

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The relative frequencies are

$$\left.\begin{array}{c} \nu_i = \sum_{\varepsilon\in\{0,1\}^4} \lambda_\varepsilon u_i^\varepsilon \qquad \nu_{ij} = \sum_{\varepsilon\in\{0,1\}^4} \lambda_\varepsilon u_{ij}^\varepsilon \\ \text{where } \lambda_\varepsilon = \frac{N_\varepsilon}{N} \end{array}\right\} \Rightarrow$$

$$\boldsymbol{\nu} = (\nu_1, \nu_2, \ldots \nu_{24}) \in c(4, \{(1,3), (1,4), (2,3), (2,4)\}$$

The Laboratory Record Argument

There can not exist things—(quantum) events, properties, elements of reality etc.—whose relative frequency equals the quantum probability.

Consequently, $\nu_1, \nu_2, \ldots \nu_{24}$ must satisfy the Clauser–Horne inequalities.

The Laboratory Record Argument

There can not exist things—(quantum) events, properties, elements of reality etc.—whose relative frequency equals the quantum probability.

Consequently, $\nu_1, \nu_2, \ldots \nu_{24}$ must satisfy the Clauser–Horne inequalities. But, consider the typical numbers ascertained in the EPR experiment:

$$\begin{aligned} \nu_1 = \nu_2 = \nu_3 = \nu_4 &= \frac{1}{2} \\ \nu_{13} = \nu_{14} = \nu_{24} &= \frac{3}{8} \\ \nu_{23} &= 0 \end{aligned}$$

$$p_{13} + p_{14} + p_{24} - p_{23} - p_1 - p_4 = \frac{3}{8} + \frac{3}{8} + \frac{3}{8} - 0 - \frac{1}{2} - \frac{1}{2} \not< 0$$

It is a fact, however, that many probabilistic statements of quantum theory are tested experimentally by counting frequencies. How is this compatible with the difficulties outlined in the Laboratory Record Argument?

Kolmogorovian Censorship Hypothesis*

We never encounter "naked" quantum probabilities in reality.

$$p(A) = tr(WP_A) \cdot p(a)$$

What we observe is $p(A)$ and $p(a)$. They are real (Kolmogorovian) relative frequencies.

*Szabó, L. E. (1995): Is quantum mechanics compatible with a deterministic universe? Two interpretations of quantum probabilities, *Foundations of Physics Letters* **8**, 421.

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Quantum probabilities are nothing but classical conditional probabilities of outcomes of measurements of quantum observables, where the conditioning events are the events of choosing to set up a measuring device to measure a certain observable.

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Kolmogorovian Censorship Theorem

1. Let $(L(H), W)$ be a quantum probability space.

2. Let Γ be a countable set of observables, such that

$$[A, B] \neq 0 \quad \text{if } A \neq B \text{ for all } 0 \neq A, B \in \Gamma$$

3. Let a map $p_0 : \Gamma \to [0, 1]$ be such that $\sum_{A \in \Gamma} p_0(A) = 1$ and $p_0(A) > 0$ if $A \neq 0$.

Then there exists a classical probability space $(\mathcal{A}, p)$ with the following properties: For every spectral projection A_i for any observable $A \in \Gamma$ there exist events A_i^{cl} and a^{cl} in $\mathcal{A}$ such that

$$A_i^{cl} < a^{cl} \tag{1}$$

$$a^{cl} \wedge b^{cl} = 0 \quad \text{if } A \neq B \tag{2}$$

$$p(a^{cl}) = p_0(A) \tag{3}$$

$$p\left(A_i^{cl} | a^{cl}\right) = tr\left(WA_i\right) \tag{4}$$

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$$\begin{aligned} A_i^{cl} &< a^{cl} && (1) \\ a^{cl} \wedge b^{cl} = 0 & \quad \text{if } A \neq B && (2) \\ p(a^{cl}) &= p_0(A) && (3) \\ p\left(A_i^{cl} | a^{cl}\right) &= tr\,(WA_i) && (4) \end{aligned}$$

Proofs: Bana and Durt (1997); Szabó (2001); Rédei (2010)