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# Contributions to Nonlinear Dynamics, Chaos and Complex Systems

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# Nonlinear Dynamics, Chaos and Complex Systems

## Group of the URJC, Madrid, Spain



## Dynamics of partial control

Juan Sabuco,<sup>1</sup> Miguel A. F. Sanjuán,<sup>1</sup> and James A. Yorke<sup>2</sup>

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Q compact (i.e., closed and bounded)

$$q_{n+1} = f(q_n)$$

### Admissible trajectories

$$q_{n+1} = f(q_n) + \xi_n + u_n$$

for  $n = 1, 2, 3, \dots$

$$\xi_0 > u_0 > 0$$

$$|\xi_n| \leq \xi_0$$

$$|u_n| \leq u_0.$$

We call such  $\xi_n$  and  $u_n$  admissible.

3/10

## PHILOSOPHICAL TRANSACTIONS A

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Research



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## Partially controlling transient chaos in the Lorenz equations

Rubén Capeáns<sup>1</sup>, Juan Sabuco<sup>1</sup>, Miguel A. F. Sanjuán<sup>1</sup>  
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### PARTIAL CONTROL OF CHAOS: HOW TO AVOID UNDESIRABLE BEHAVIORS WITH SMALL CONTROLS IN PRESENCE OF NOISE

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Recent Progress in Controlling Chaos

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Miguel A F Sanjuán • Celso Grebogi



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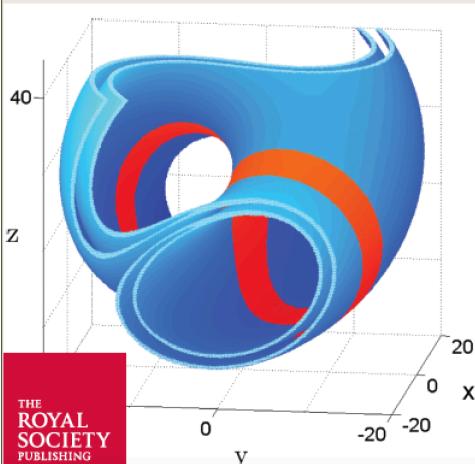
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## PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A

MATHEMATICAL, PHYSICAL AND ENGINEERING SCIENCES

### Horizons of cybernetical physics

Theme Issue compiled and edited by Alexander Fradkov



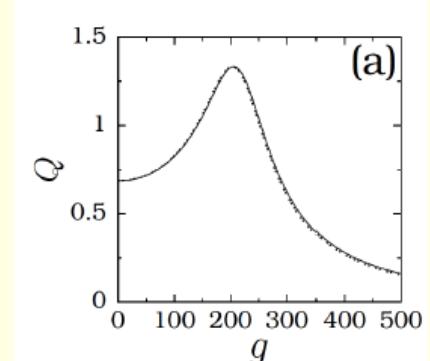
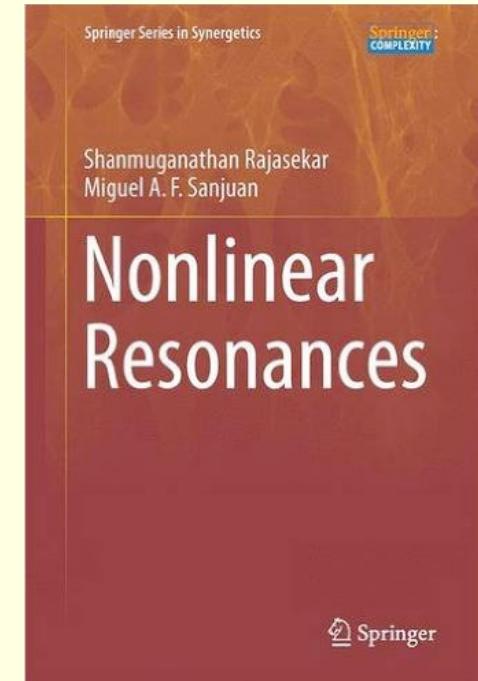
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# Vibrational Resonance and Nonlinear Resonances

$$\dot{x} = f(x) + \varepsilon \cos \omega t + \xi(t)$$

- A.  $\xi(t)$  is a noise SR
- B.  $\xi(t) = g \cos \Omega t$      $\Omega \gg \omega$  VR
- C.  $\xi(t)$  is a chaotic signal CR

$$\ddot{x} + d\dot{x} + \omega_0^2 x + \beta x^3 = f \cos \omega t + g \cos \Omega t, \quad \Omega \gg \omega$$



# Fractal Structures in Nonlinear Dynamics

REVIEWS OF MODERN PHYSICS, VOLUME 81, JANUARY–MARCH 2009

## Fractal structures in nonlinear dynamics

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Departamento de Física, Universidad Rey Juan Carlos, Tulipán s/n, 28933 Móstoles, Madrid, Spain and Centro de Astrobiología, CSIC-INTA, Ctra. de Ajalvir km. 4, 28850 Torrejón de Ardoz, Madrid, Spain

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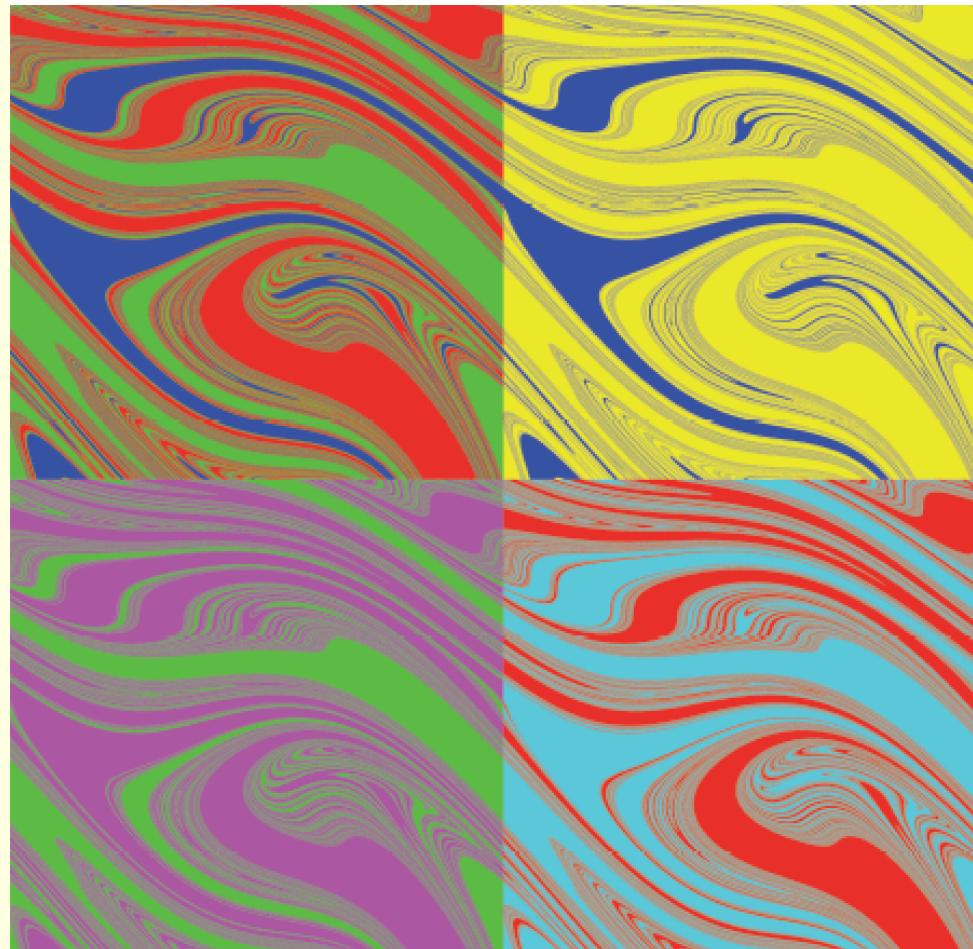
REPORTS ON PROGRESS IN PHYSICS

doi:10.1088/0034-4885/76/1/016001

## New developments in classical chaotic scattering

Jesús M Seoane and Miguel A F Sanjuán

Nonlinear Dynamics, Chaos and Complex Systems Group, Departamento de Física, Universidad Rey Juan Carlos, Tulipán s/n, 28933 Móstoles, Madrid, Spain



# SCIENTIFIC REPORTS

OPEN

## Ascertaining when a basin is Wada: the merging method

Alvar Daza<sup>1</sup>, Alexandre Wagemakers<sup>1</sup> & Miguel A. F. Sanjuán<sup>1,2,3</sup>

Trying to imagine three regions separated by a unique boundary seems a difficult task. However, this is exactly what happens in many dynamical systems showing Wada basins. Here, we present a new perspective on the Wada property: *A Wada boundary is the only one that remains unaltered under the action of merging the basins*. This observation allows to develop a new method to test the Wada property, which is much faster than the previous ones. Furthermore, another major advantage of the merging method is that a detailed knowledge of the dynamical system is not required.

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# SCIENTIFIC REPORTS

OPEN

## Basin entropy: a new tool to analyze uncertainty in dynamical systems

Alvar Daza<sup>1</sup>, Alexandre Wagemakers<sup>1</sup>, Bertrand Georgeot<sup>2</sup>, David Guéry-Odelin<sup>3</sup> & Miguel A. F. Sanjuán<sup>1</sup>

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# Modeling Biological Systems

Physics Reports 501 (2011) 1–74

Contents lists available at ScienceDirect

Physics Reports

journal homepage: [www.elsevier.com/locate/physrep](http://www.elsevier.com/locate/physrep)



## Map-based models in neuronal dynamics

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Journal of Theoretical Biology 349 (2014) 74–81

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Journal of Theoretical Biology

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## Avoiding healthy cells extinction in a cancer model

Álvaro G. López<sup>a</sup>, Juan Sabuco<sup>a</sup>, Jesús M. Seoane<sup>a</sup>, Jorge Duarte<sup>b</sup>, Cristina Januário<sup>b</sup>, Miguel A.F. Sanjuán<sup>a,\*</sup>

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journal homepage: [www.elsevier.com/locate/cnsns](http://www.elsevier.com/locate/cnsns)



Research paper

## Nonlinear cancer chemotherapy: Modelling the Norton-Simon hypothesis

Álvaro G. López<sup>a</sup>, Kelly C. Jarosz<sup>b,c</sup>, Antonio M. Batista<sup>d</sup>, Jesús M. Seoane<sup>a,\*</sup>, Ricardo L. Viana<sup>e</sup>, Miguel A.F. Sanjuán<sup>a,f</sup>

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Ecological Complexity

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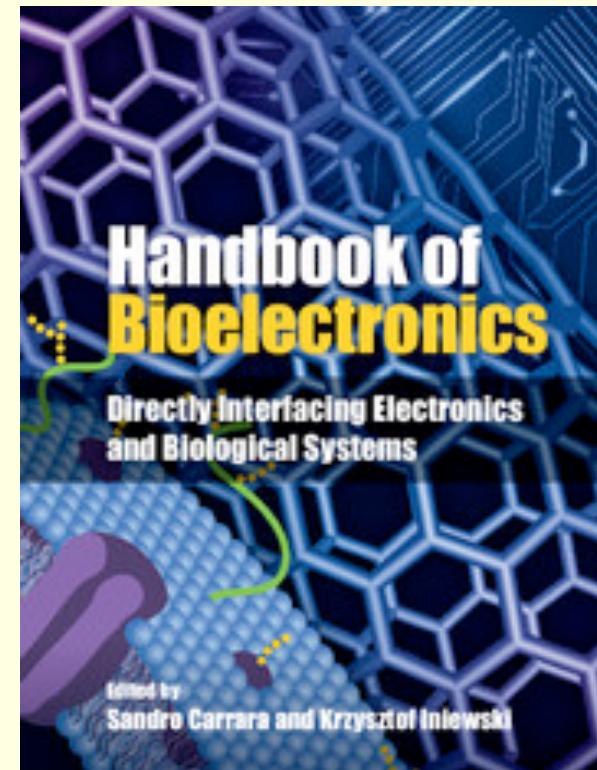


Original Research Article

## When less is more: Partial control to avoid extinction of predators in an ecological model

Rubén Capeáns, Juan Sabuco, Miguel A.F. Sanjuán<sup>\*</sup>

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# Applications in Physics: Galactic Dynamics, Chaos & Entanglement, Cold Atoms, Black Holes

A&A 595, A68 (2016)  
DOI: 10.1051/0004-6361/201629206  
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Astronomy  
& Astrophysics

## Role of dark matter haloes on the predictability of computed orbits

Juan C. Vallejo<sup>1,2</sup> and Miguel A. F. Sanjuán<sup>2</sup>

Monthly Notices  
of the  
ROYAL ASTRONOMICAL SOCIETY  
MNRAS 447, 3797–3811 (2015)



doi:10.1093/mnras/stu2733

## The forecast of predictability for computed orbits in galactic models

J. C. Vallejo<sup>1,2\*</sup> and M. A. F. Sanjuán<sup>2</sup>

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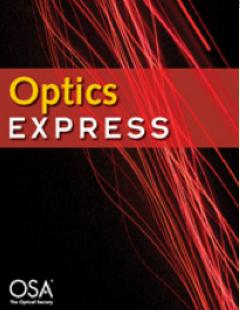
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Impact of quantum–classical correspondence on entanglement enhancement by single-mode squeezing

Sijo K. Joseph<sup>a</sup>, Lock Yue Chew<sup>b</sup>, Miguel A.F. Sanjuán<sup>a,\*</sup>

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The effect of geometry on the classical entanglement in a chaotic optical fiber

Sijo K. Joseph<sup>1\*</sup> and Juan Sabuco<sup>1</sup> and Lock Yue Chew<sup>2</sup> and Miguel A. F. Sanjuán<sup>1</sup>

<sup>1</sup>Nonlinear Dynamics, Chaos and Complex Systems Group, Departamento de Física, Universidad Rey Juan Carlos, Tulipán s/n, 28933 Móstoles, Madrid, Spain

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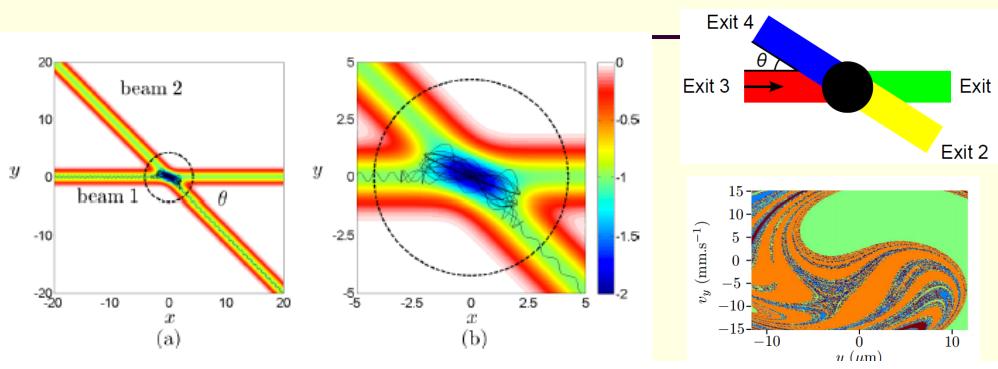
Juan C. Vallejo  
Miguel A.F. Sanjuán

# Predictability of Chaotic Dynamics

A Finite-time Lyapunov Exponents  
Approach

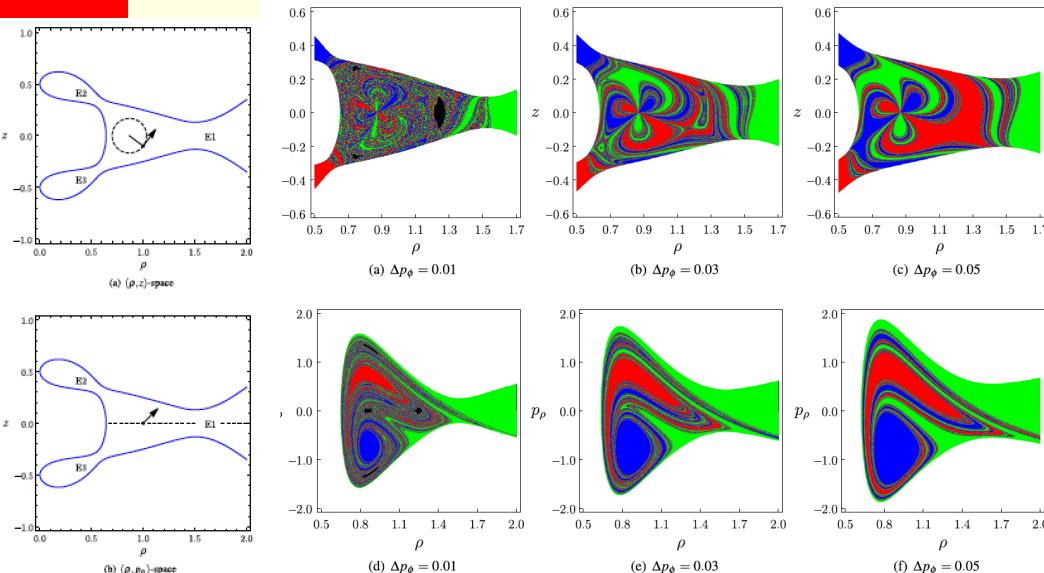
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## Chaotic dynamics and fractal structures in experiments with cold atoms

Álvar Daza,<sup>1</sup> Bertrand Georgeot,<sup>2</sup> David Guéry-Odelin,<sup>3</sup> Alexandre Wagemakers,<sup>1</sup> and Miguel A. F. Sanjuán<sup>1,\*</sup>

PHYSICAL REVIEW D 98, 084050 (2018)

## Wada structures in a binary black hole system

Álvar Daza,<sup>1,\*</sup> Jake O. Shipley,<sup>2,†</sup> Sam R. Dolan,<sup>2,‡</sup> and Miguel A. F. Sanjuán<sup>1,3,4,§</sup>

## Understanding Complex Systems

Springer:  
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Miguel A. F. Sanjuán *Editors*Chaotic, Fractional,  
and Complex  
Dynamics: New  
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# Foundations of Nonlinear Dynamics & Chaos Theory

CHAOS 28, 103110 (2018)



## Low-dimensional paradigms for high-dimensional hetero-chaos

Yoshitaka Saiki,<sup>1,2,3</sup> Miguel A. F. Sanjuán,<sup>3,4</sup> and James A. Yorke<sup>3,5</sup>

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The dynamics on a chaotic attractor can be quite heterogeneous, being much more unstable in some regions than others. Some regions of a chaotic attractor can be expanding in more dimensions than other regions. Imagine a situation where two such regions and each contains trajectories that stay in the region for all time—while typical trajectories wander throughout the attractor. Furthermore, if arbitrarily close to each point of the attractor there are points on periodic orbits that have different unstable dimensions, then we say such an attractor is “hetero-chaotic” (i.e., it has heterogeneous chaos). This is hard to picture but we believe that most physical systems possessing a high-dimensional attractor are of this type. We have created simplified models with that behavior to give insight into real high-dimensional phenomena. Published by AIP Publishing. <https://doi.org/10.1063/1.5045693>

## Feature Articles

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## Computing Complex Horseshoes by Means of Piecewise Maps

Álvaro G. López\*,†, Álvar Daza\*, Jesús M. Seoane\*  
and Miguel A. F. Sanjuán\*,†

\**Nonlinear Dynamics, Chaos and Complex Systems Group,  
Departamento de Física, Universidad Rey Juan Carlos,  
Tulipán s/n, 28933 Móstoles, Madrid, Spain*

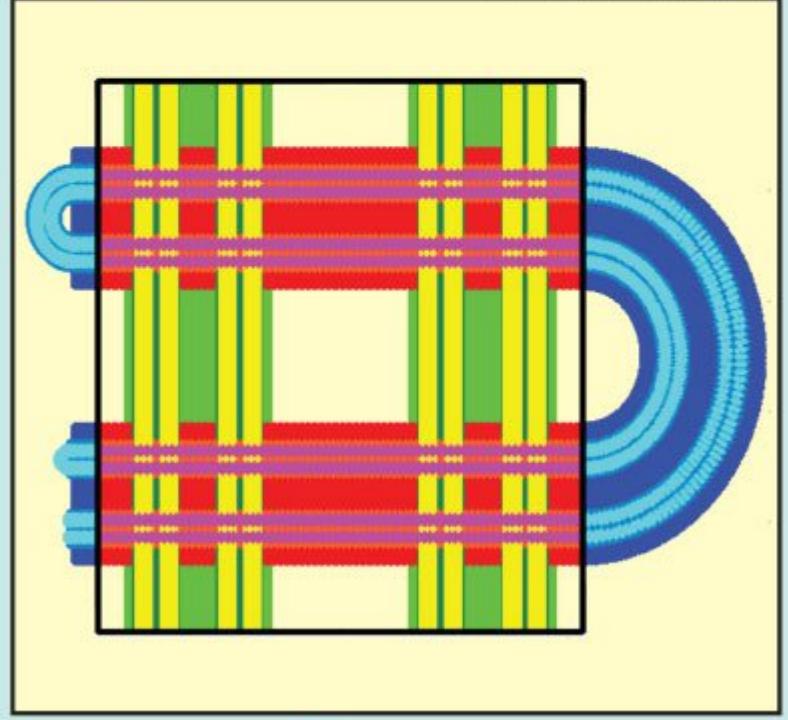
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# Las matemáticas y la física del caos

Manuel de León  
y Miguel Á. F. Sanjuán



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