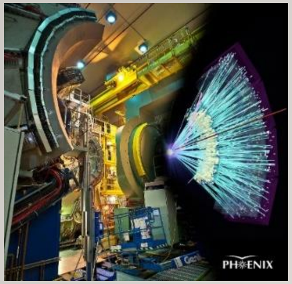
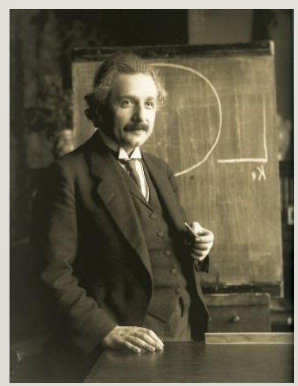
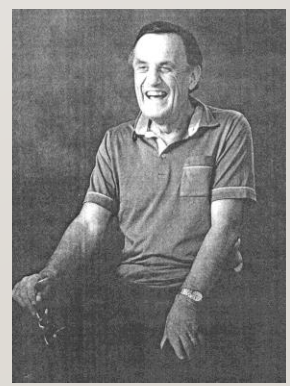
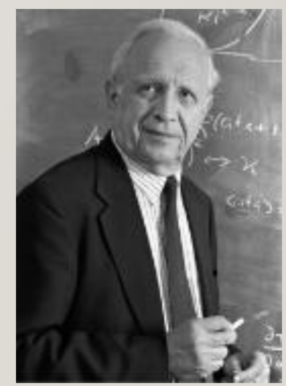


QUANTUM STATISTICAL CORRELATIONS

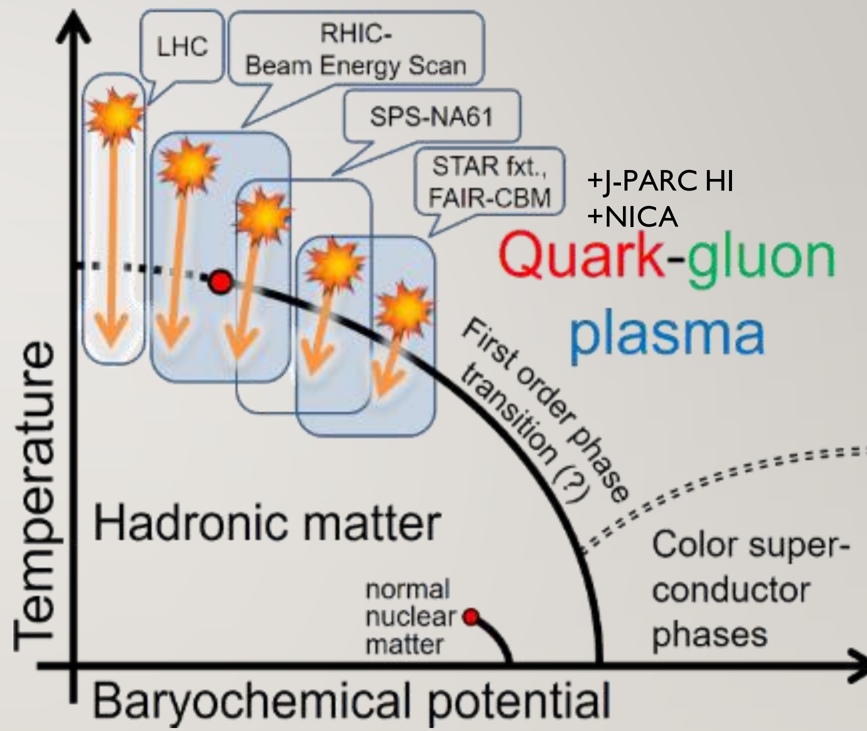
AND THE SEARCH FOR THE QCD CEP

MÁTÉ CSANÁD @ BALATON WORKSHOP 2019

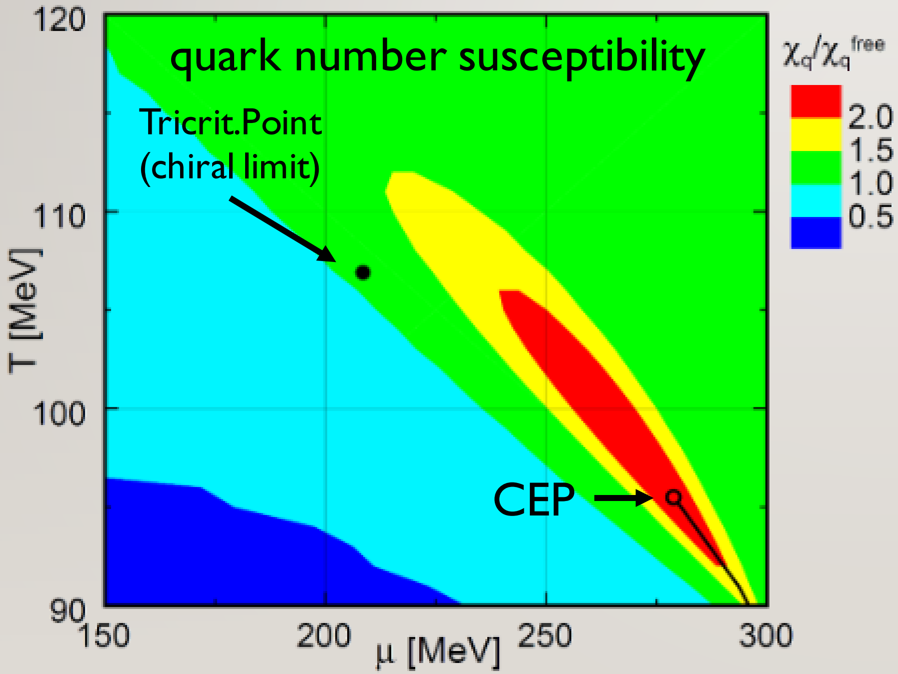
2/28 EXPLORING THE PHASE MAP OF QCD

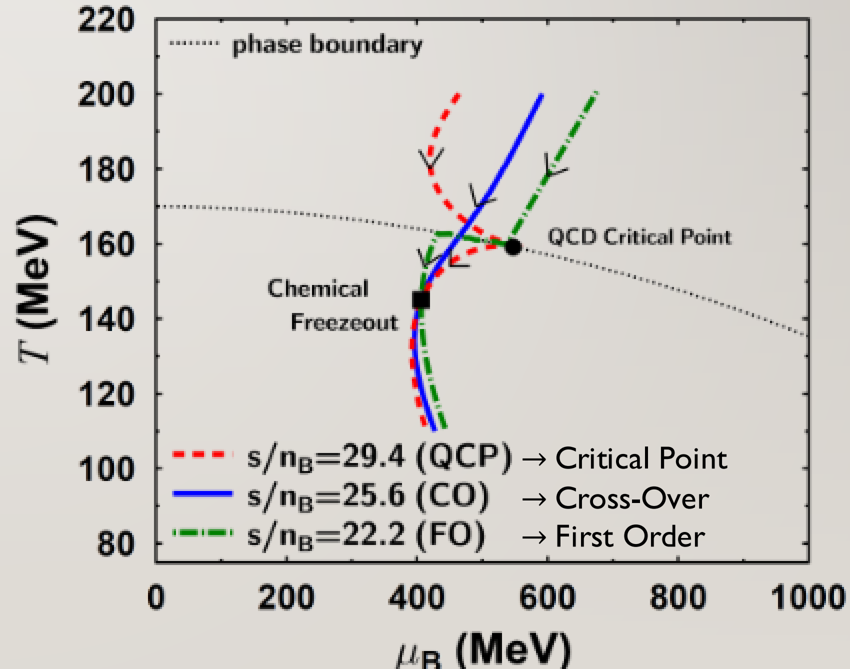
- Phase map: temperature versus matter excess (baryochem. pot. μ_B)
- Control parameters:
  - Collision energy, system
  - Collision geometry
- Crossover at low μ_B and $T \cong 170$ MeV
- Probably 1st order quark-hadron p.t. at high μ_B (NJL, bag model, etc)
- Critical End Point (CEP) in between?
- High μ_B: nuclear matter, neutron stars, color superconductors…
- Phase transition importance: even in core-collapse supernovae!



3/28 SEARCH FOR THE CRITICAL POINT POSSIBLE?

- Effects of the CEP in a broad region (via an effective potential ~ N_f=2 QCD)
  - Y. Hatta and T. Ikeda, PRD67,014028(2003) [hep-ph/0210284]
- Hydro evolution attracted to the critical point
  - M. Asakawa et al., PRL101,122302(2008) [arXiv:0803.2449]





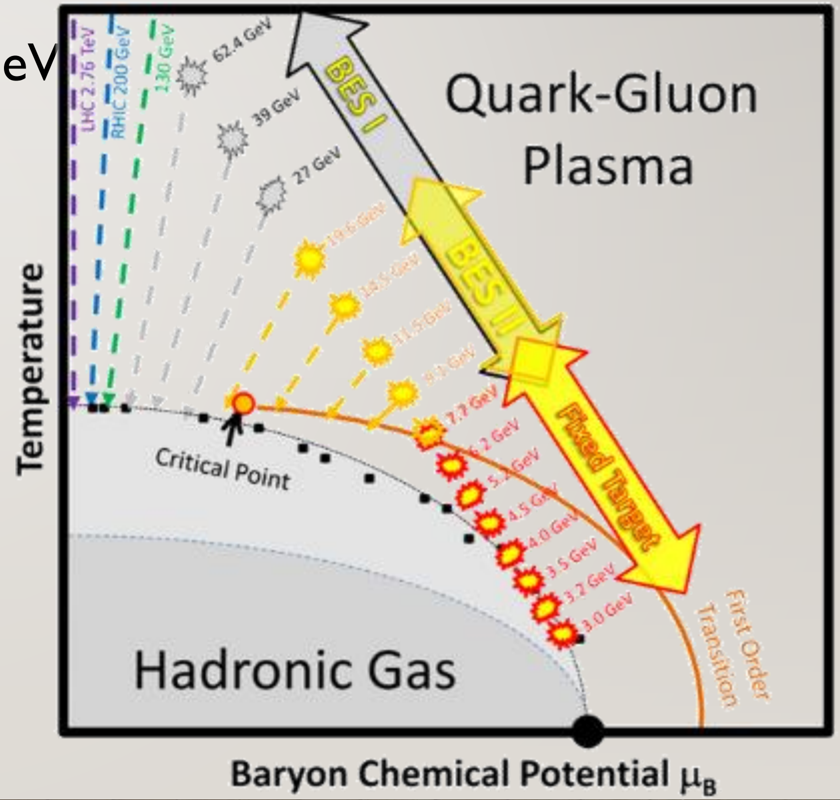
THE RELATIVISTIC HEAVY ION COLLIDER

- At the Brookhaven National Laboratory, Long Island, New York, USA
- Collisions of: $\vec{p}$, d, ^{3}He, Al, Cu, Au, U
- Accelerator energies: 7.7-200 GeV/nucleon, even 0.51 TeV for $\vec{p}$
- Experiments: STAR; future: sPHENIX; past: BRAHMS & PHOBOS & PHENIX

THE RHIC BEAM ENERGY SCAN

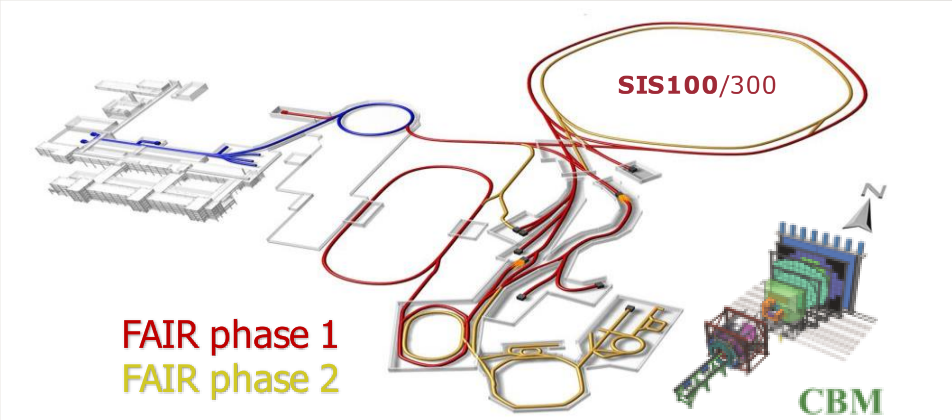
- Collision experiments: acceptance independent of energy
- **BES-I**: 7.7-200 GeV; **BES-II**: 7.7-19.9 GeV, increased luminosity
- Small system scan: x+Au, 19.6-200 GeV
- STAR **fixed target** mode: down to 3 GeV

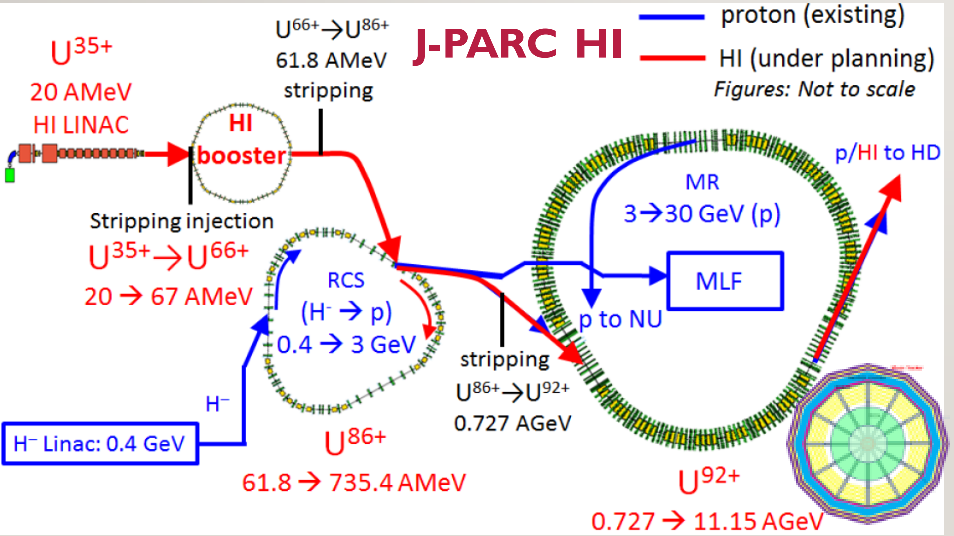
$\sqrt{s_{NN}}$ [GeV]	STAR Au+Au events [10^6]	PHENIX Au+Au events [10^6]	Year
200.0	2000	7000	2010
62.4	67	830	2010
54.4	1300	-	2017
39.0	130	385	2010
27.0	70	220	2011
19.6	36	88	2011
14.5	20	247	2014
11.5	12	-	2010
7.7	4	1.4	2010



FUTURE FACILITIES: NICA, FAIR, J-PARC HI

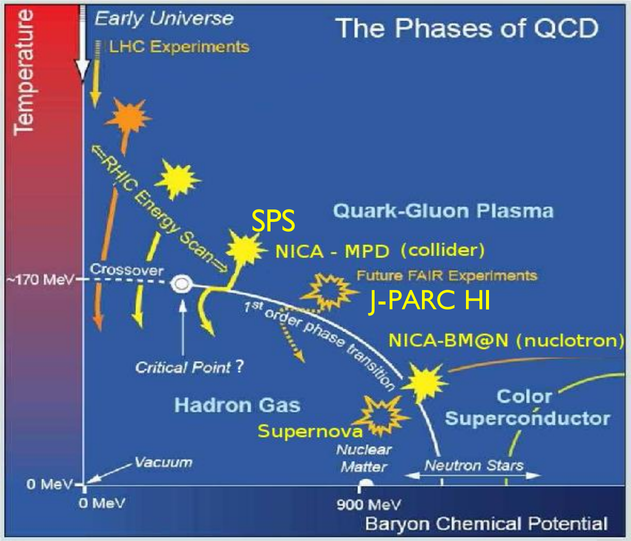
- New facilities planned/built
- NICA: 2020, MPD&BM@N
- FAIR: 2022, CBM
- J-PARC HI: 2025, JHITS





(FUTURE) FACILITIES COMPARISON

- Many future facilities and experiments, SPS and RHIC already running
- RHIC, NICA: Collider and fixed target
- SPS, FAIR, J-PARC: fixed target
- Energy ranges from 2 to 20 GeV in $\sqrt{s_{NN}}$

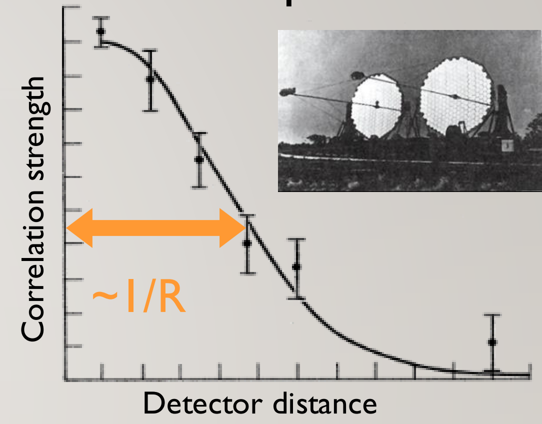


Compilation from Daniel Cebra and Olga Evkidomiv:

Facility	RHIC BES-II & Fixed Target	SPS	NICA	FAIR	J-PARC HI
Experiment	STAR	NA61	MPD & BM@N	CBM	JHITS
Start	2019	2009	2020-23	2025	2025
Energy ($\sqrt{s_{NN}}$, GeV)	2.9-19.6 GeV	4.9-17.3	2.0-11	2.7-8.2	2.0-6.2
Rate	100-1000 Hz	100 Hz	10 kHz	10 MHz	10-100 MHz
Physics	Critical Point Onset of Deconf.	Critical Point Onset of Deconf.	Onset of Deconfinement Compr. Hadronic Matter	Onset of Deconfinement Compr. Hadronic Matter	Onset of Deconfinement Compr. Hadronic Matter

THE IDEA OF FEMTOSCOPY

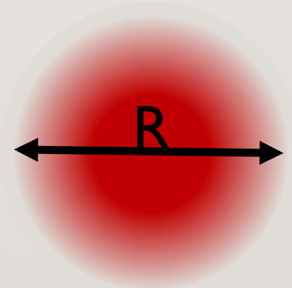
- R. Hanbury Brown, R. Q. Twiss - observing Sirius with radio telescopes
  - Intensity correlations vs detector distance ⇒ source size
  - Measure the sizes of apparently point-like sources!



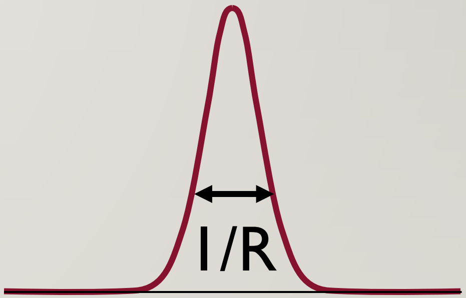
- Goldhaber et al: applicable in high energy physics
  - Momentum correlation $C(q)$ related to source $S(r)$
  - $C(q) \cong 1 + \left|\int S(r) e^{iqr}\, dr\right|^2$ (under some assumptions)

  or the distance distribution $D(r)$

  $C(q) \cong 1 + \int D(r) e^{iqr}\, dr$



source function S(r)

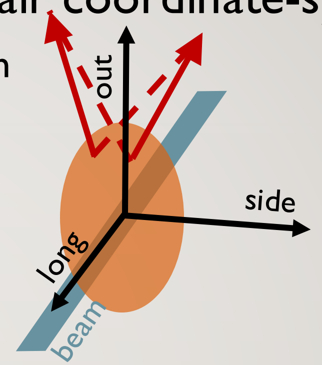


correlation function C(q)

- Measure C(q): map out source space-time geometry on femtometer scale!

9/28 HBT AND THE PHASE TRANSITION

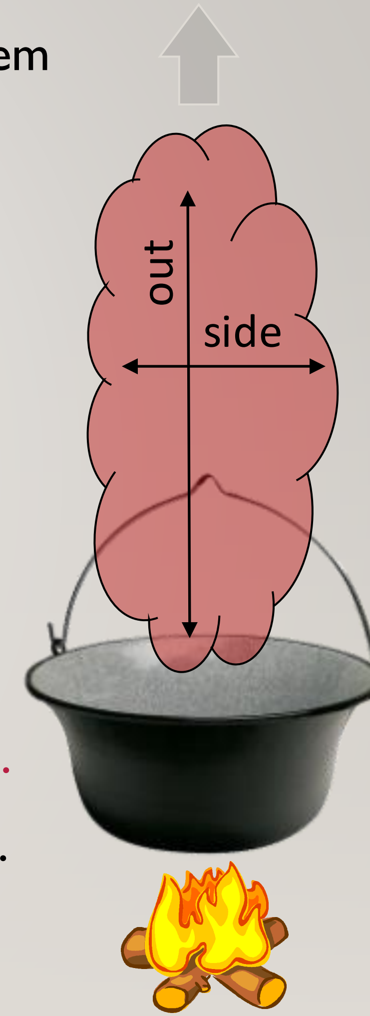
- C(q) usually measured in the Bertsch-Pratt pair coordinate-system
  - out: direction of the average transverse momentum
  - long: beam direction
  - side: orthogonal to the latter two



- R_{out}, R_{side}, R_{long} : HBT radii
- Δτ emission duration, i.e. $S(r,\tau) \sim e^{-\frac{(\tau-\tau_0)^2}{2\Delta\tau^2}}$
- From a simple hydro calculation:

$$R^2_{\text{out}} = \frac{R^2}{1+u_T^2 m_T/T_0} + \beta_T^2 \Delta\tau^2, \qquad R^2_{\text{side}} = \frac{R^2}{1+u_T^2 m_T/T_0}$$

- RHIC, 200 GeV: $R_{\text{out}} \approx R_{\text{side}} \rightarrow$ no strong 1st order phase trans.
- Plus lots of other details: pre-equilibrium flow, initial state, EoS, ...



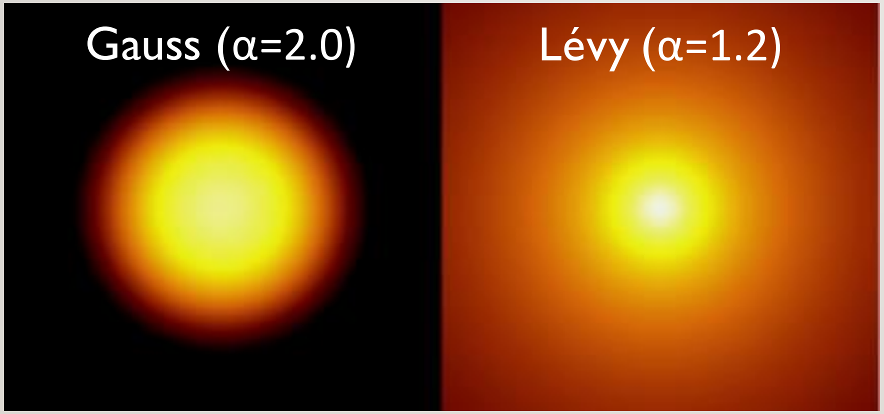
S. Chapman, P. Scotto, U. Heinz, Phys.Rev.Lett. 74 (1995) 4400
T. Csörgő and B. Lörstad, Phys.Rev. C54 (1996) 1390
S. Pratt, Nucl.Phys.A830 (2009) 51C

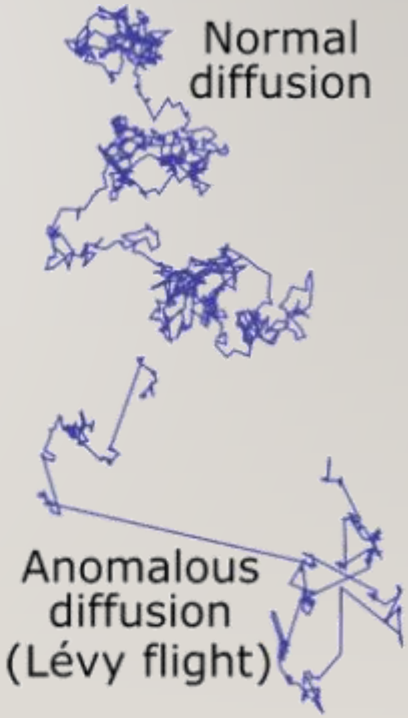
SECOND ORDER PHASE TRANSITION?

- Second order phase transitions: critical exponents
  - Near the critical point
    - Specific heat ~ $((T-T_c)/T_c)^{-\alpha}$
    - Order parameter ~ $((T_c-T)/T_c)^{-\beta}$
    - Susceptibility/compressibility ~ $((T-T_c)/T_c)^{-\gamma}$
    - Correlation length ~ $((T-T_c)/T_c)^{-\nu}$
  - At the critical point
    - Order parameter ~ (source field)$^{1/\delta}$
    - Spatial correlation function ~ $r^{d+2-\eta}$
  - Ginzburg-Landau: α=0, β=0.5, γ=1, ν=0.5, δ=3, η=0
- QCD $\leftrightarrow$ 3D Ising modell
- Can we measure the η power-law exponent?
- Depends on spatial distribution: measurable with femtoscopy!
- What distribution has a power-law exponent? Levy!

LÉVY DISTRIBUTIONS IN HEAVY ION PHYSICS

- Expanding medium, increasing mean free path: anomalous diffusion
  Metzler, Klafter, Physics Reports 339 (2000) 1-77, Csanad, Csörgő, Nagy, Braz.J.Phys. 37 (2007) 1002
- Lévy-stable distribution: $\mathcal{L}(\alpha, R; r) = \frac{1}{(2\pi)^3} \int d^3q e^{iqr} e^{-\frac{1}{2}|qR|^\alpha}$
  - From generalized central limit theorem, power-law tail ~ $r^{-(1+\alpha)}$
  - Special cases: α = 2 Gaussian, α = 1 Cauchy





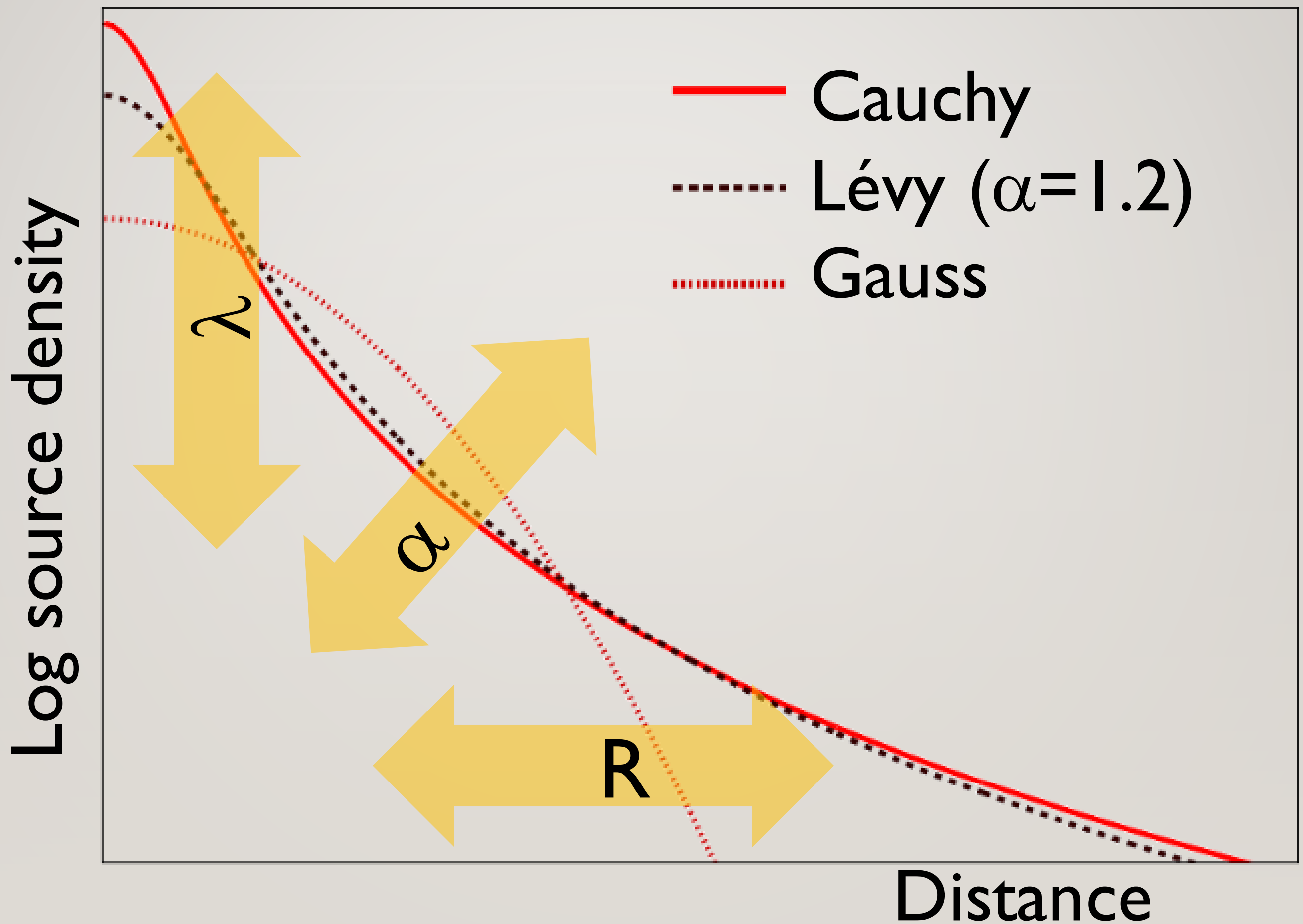
- Shape of the correlation functions with Levy source:

$$C_2(q) = 1 + \lambda \cdot e^{-|qR|^\alpha}$$

$\alpha = 2$: Gaussian
$\alpha = 1$: Exponential

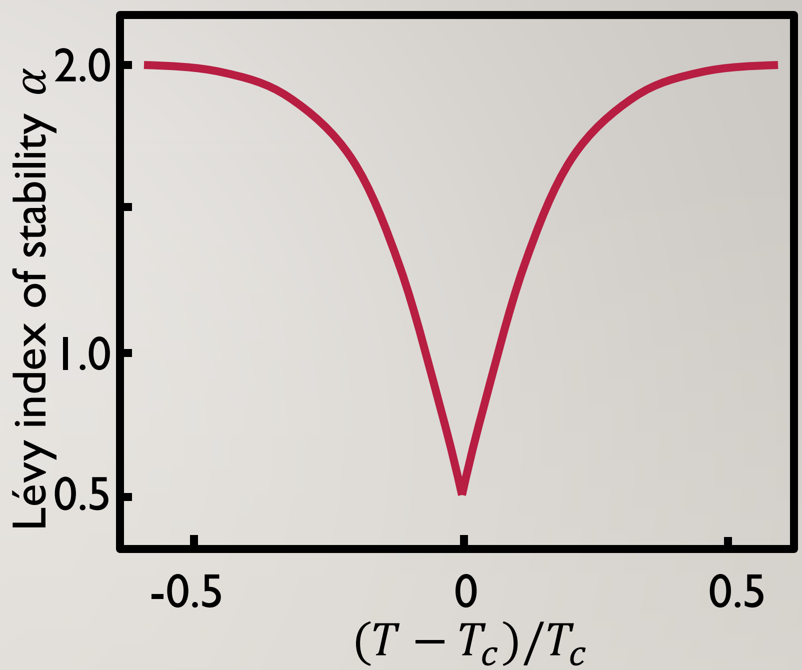
12/28 LÉVY VERSUS GAUSS VERSUS EXPONENTIAL

- No tail if $\alpha = 2$, power law if $\alpha < 2$; correlation between α and R,λ



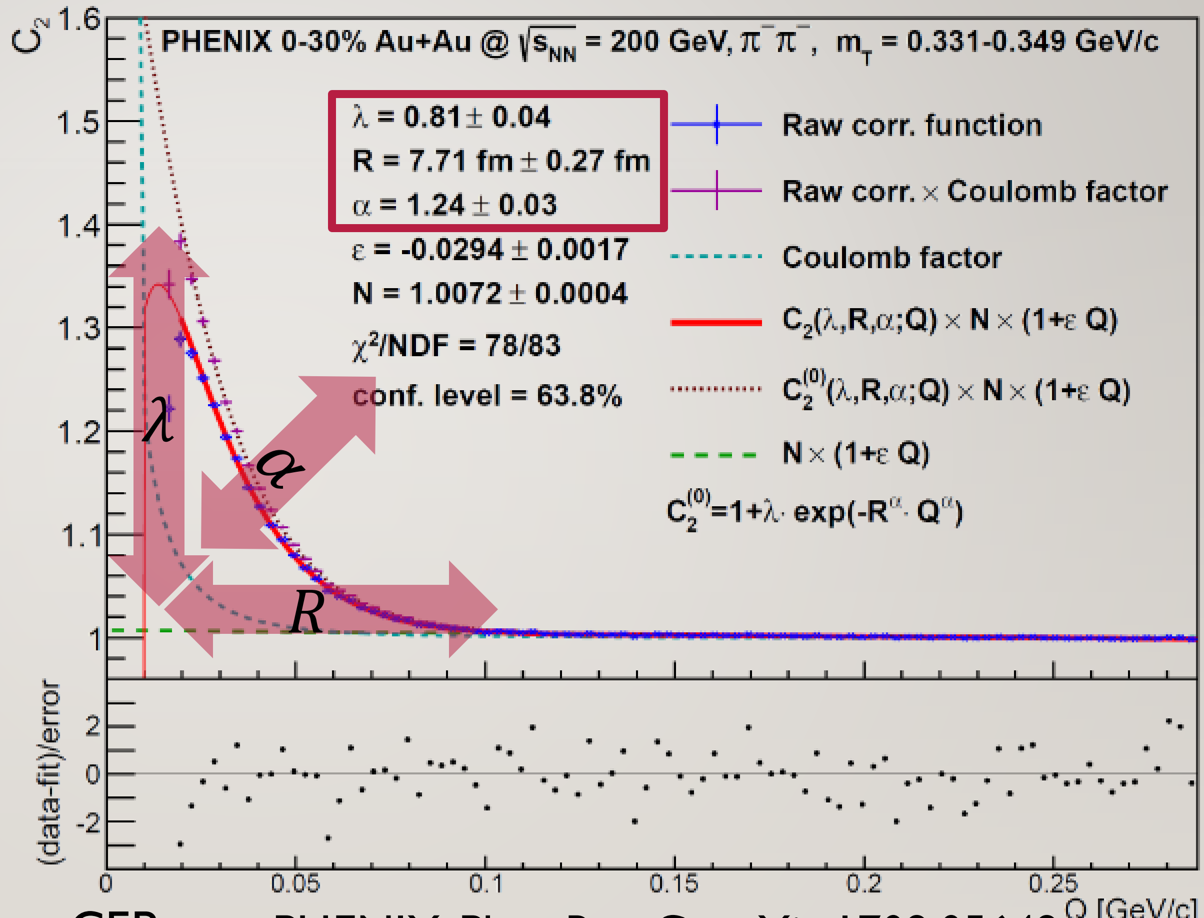
LÉVY INDEX AS A CRITICAL EXPONENT?

- Critical spatial correlation: $\sim r^{-(d-2+\eta)}$;
  Lévy source: $\sim r^{-(1+\alpha)}$; $\alpha \Leftrightarrow \eta$?
  Csörgő, Hegyi, Zajc, Eur.Phys.J. C36 (2004) 67,
- QCD universality class ↔ 3D Ising
  Halasz et al., Phys.Rev.D58 (1998) 096007
  Stephanov et al., Phys.Rev.Lett.81 (1998) 4816
- At the critical point:
  - Random field 3D Ising: η = 0.50±0.05
    Rieger, Phys.Rev.B52 (1995) 6659
  - 3D Ising: η = 0.03631(3)
    El-Showk et al., J.Stat.Phys.157 (4-5):869
- Motivation for precise Lévy HBT!
- Change in α_{Levy} proximity of CEP?
- Modulo finite size/time and non-equilibrium effects
- Other possible reasons for Lévy distributions: anomalous diffusion, QCD jets, …



14/28 EXAMPLE $C_2(Q_{LCMS})$ CORRELATION FUNCTION

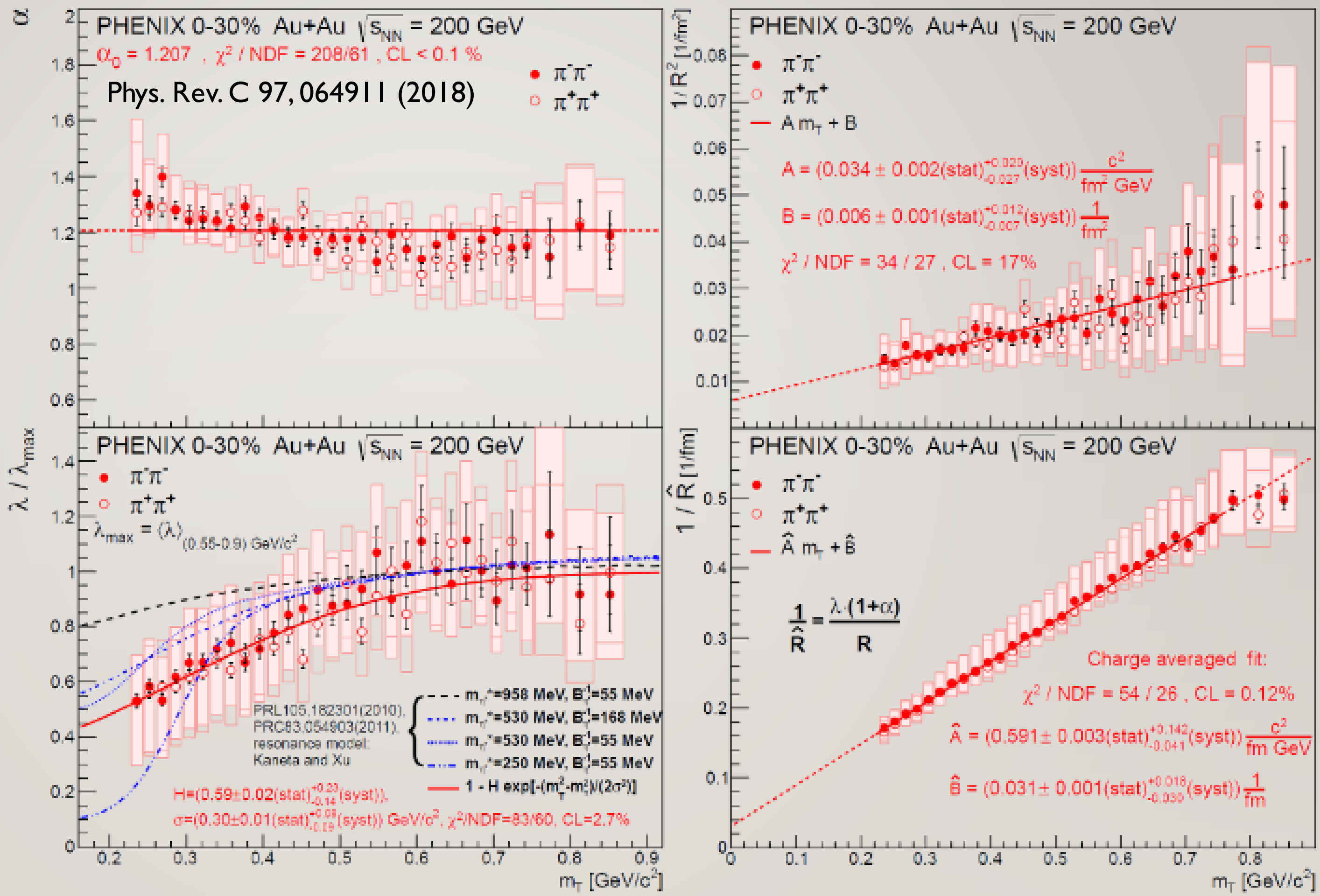
- Measured in m_T bins
- Fitted with Coulomb-incorporated function
- Coulomb-factor displayed separately
- All fits converged, good confidence levels
- χ values scatter around 0 properly
- Physical parameters: R, λ, α measured versus pair m_T
- Recall α: Lévy index, 0.5 at CEP



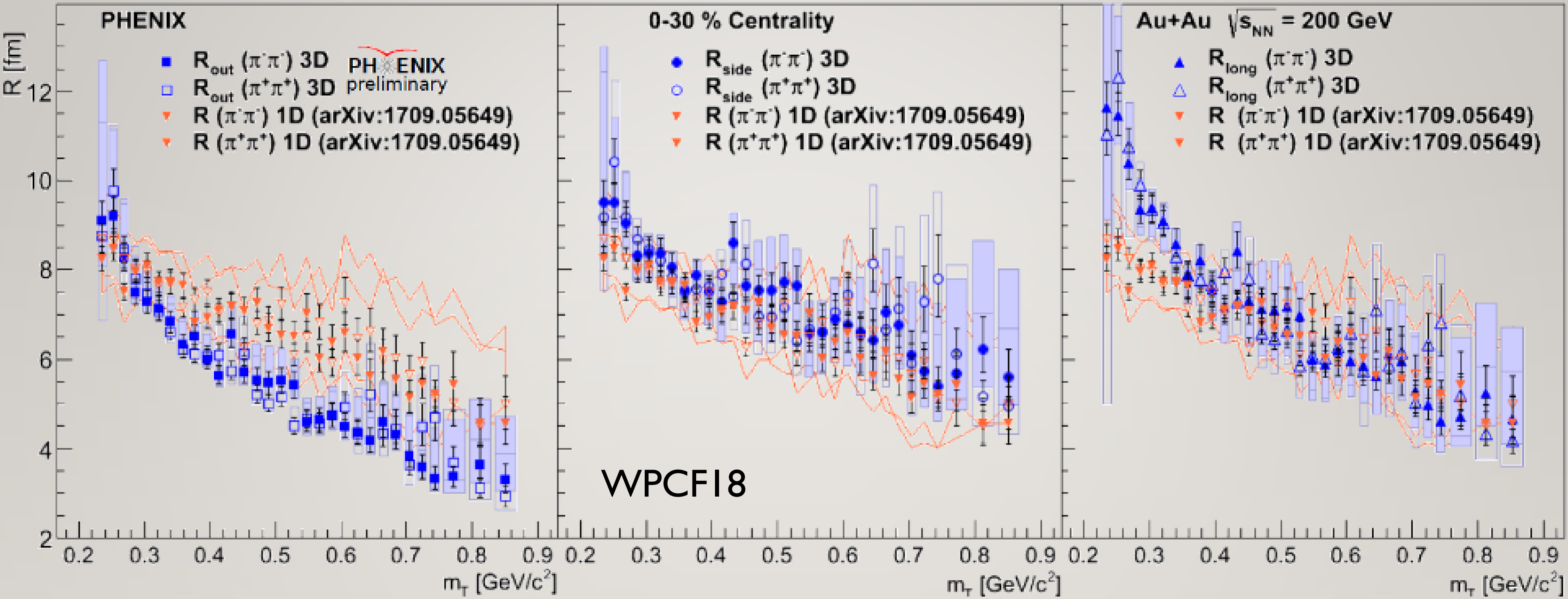
PHENIX, Phys. Rev. C., arXiv:1709.05649

200 GEV 1D LÉVY HBT RESULTS

- α: not 0.5 and not 2.0
- R: hydro scaling
- λ: „hole", compatible with mass modification
- $\hat{R}$: new scaling variable



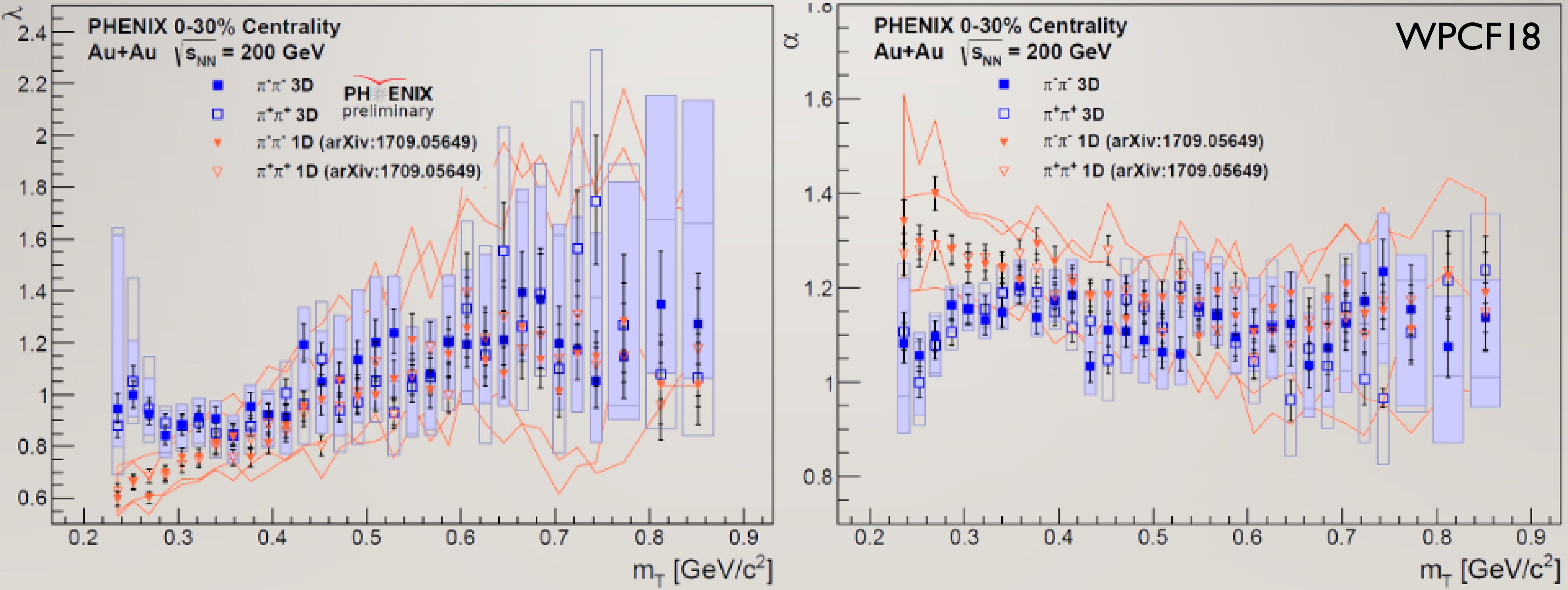
16/28 CROSS-CHECK WITH 3D VERSUS 1D



- Compatibility with 1D Lévy analysis
- Similar decreasing trend as Gaussian HBT radii, but it is not an RMS radius!
  - There is no 2nd moment (variance or root mean square) for Lévy distributions with $\alpha<2$!
- Asymmetric source for small mT, validity of Coulomb-approximation?

17/28 3D VERSUS 1D: STRENGTH λ AND SHAPE α

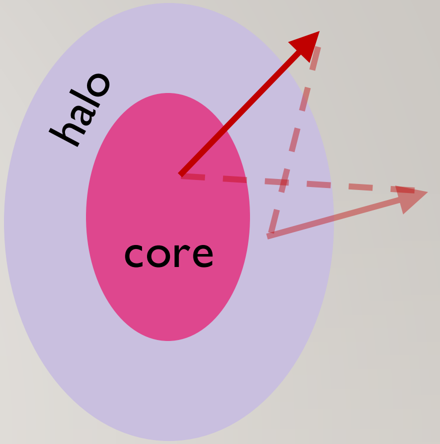
- Compatible with 1D (Q_{LCMS}) measurement of Phys. Rev. C 97, 064911 (2018)
- Small discrepancy at small m_T: due to large R_{long} at small mT?

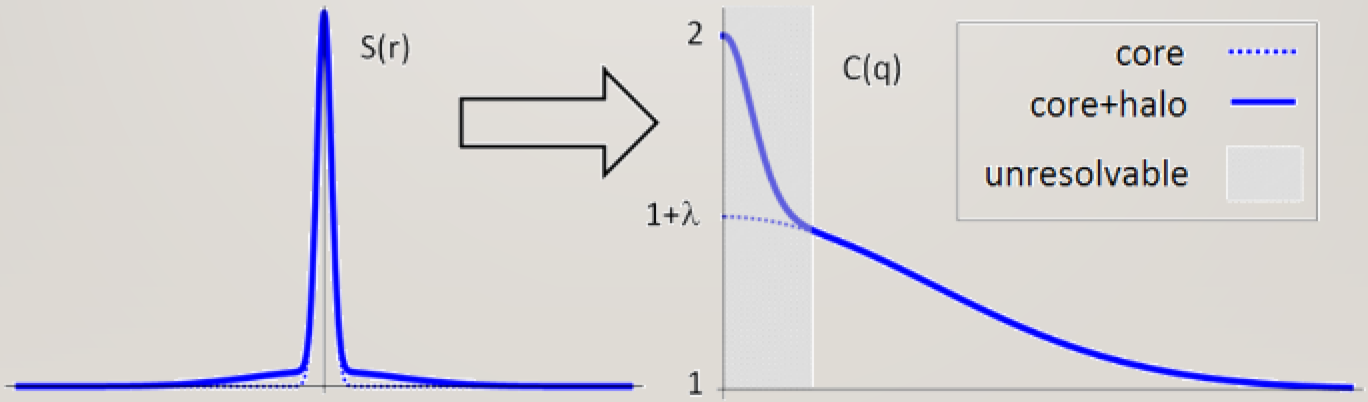


18/28 CORRELATION STRENGTH λ: CORE FRACTION

- Two-component source
  - Core: hydrodynamically expanding, thermal medium
  - Halo: long lived resonances (≳ 10 fm/c, ω,η,η',K_0^S,...), unresolvable experimentally
  - Define $f_C = N_{\mathrm{core}}/N_{\mathrm{total}}$
- True $q \to 0$ limit: $C(0) = 2$
- Apparently $C(q \to 0) \to 1 + \lambda$
- $\lambda(m_{\mathrm{T}}) = f_C^2(m_{\mathrm{T}})$

Bolz et al, Phys.Rev. D47 (1993) 3860-3870
Csörgő, Lörstad, Zimányi, Z.Phys. C71 (1996) 491-497



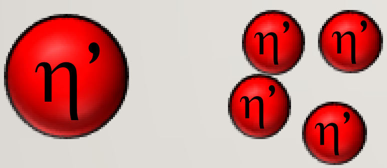


19/28 CORRELATION STRENGTH λ: IN-MEDIUM MASS?

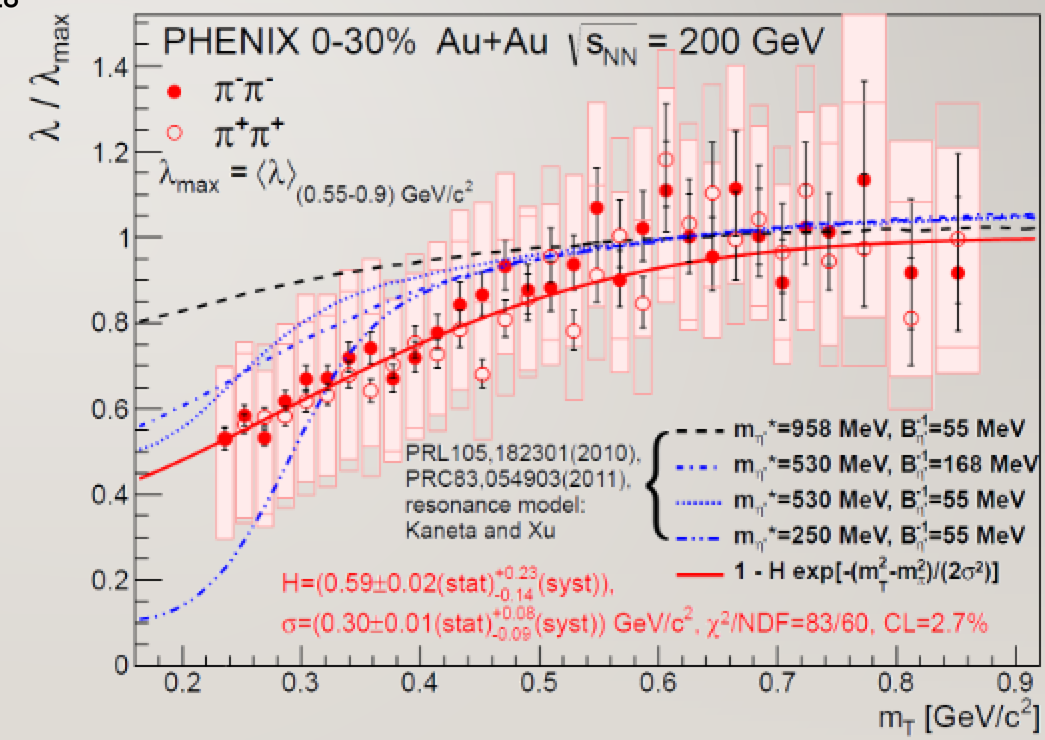
- Connection to chiral restoration
  - Decreased η' mass → η'enhancement → halo enhancement
  - Kinematics: η' → ππππ with low m_T → decreased $\lambda(m_T)$ at low m_T
  - Dependence on in-medium η' mass?

Kapusta, Kharzeev, McLerran, PRD53 (1996) 5028
Vance, Csörgő, Kharzeev, PRL 81 (1998) 2205
Csörgő, Vértesi, Sziklai, PRL105 (2010) 182301

  - 3D results compatible with 1D



$$T < T^*$$
$$m_{\eta'} > m^*_{\eta'}$$
$$N_{\eta'} < N^*_{\eta'}$$
$$\lambda > \lambda^*$$



- Not seen at SPS before!

CROSS-CHECK WITH MULTI-PION LÉVY HBT

- Recall: two particle correlation strength $\lambda = f_C^2$ where $f_C = N_{\text{core}}/N_{\text{total}}$
- Generalization for higher order correlations: $\lambda_2 = f_C^2, \lambda_3 = 2f_C^3 + 3f_C^2$
- If there is partial coherence (p_C):

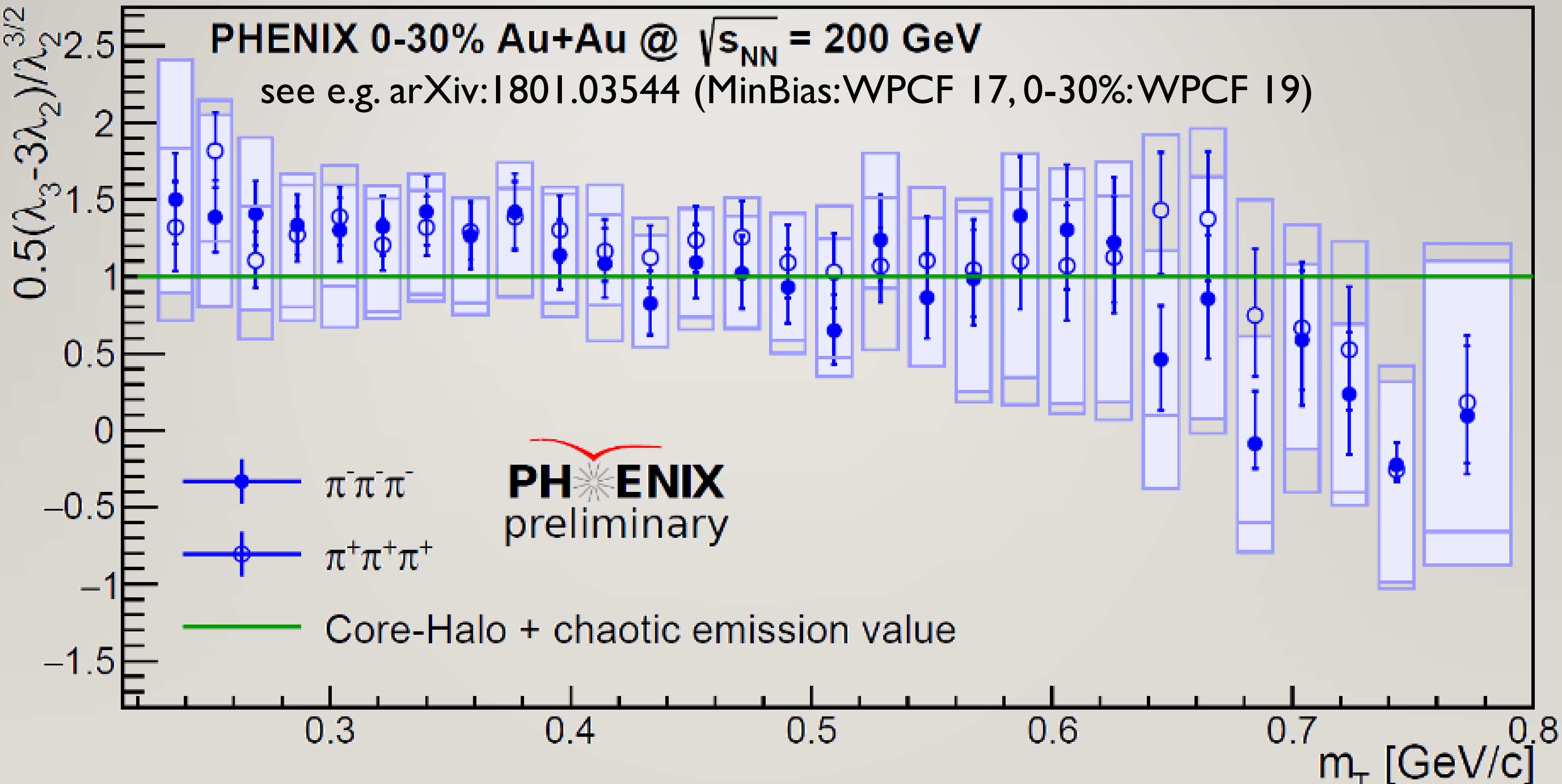
$$\lambda_2 = f_C^2[(1-p_C)^2 + 2p_C(1-p_C)]$$
$$\lambda_3 = 2f_C^3[(1-p_C)^3 + 3p_C(1-p_C)^2] + 3f_C^2[(1-p_C)^2 + 2p_C(1-p_C)]$$

- Introduce core-halo independent parameter $\kappa_3 = \frac{\lambda_3 - 3\lambda_2}{2\sqrt{\lambda_2}^3}$
  - does not depend on f_C
  - $\kappa_3 = 1$ if no coherence
- Finite meson sizes?

  Gavrilik, SIGMA 2 (2006) 074 [hep-ph/0512357]
- Phase shift (a la Aharonov-Bohm) in hadron gas?
  - Random fields create random phase shift, on average distorts Bose-Einstein correlations

TEST OF CORE-HALO MODEL / COHERENCE

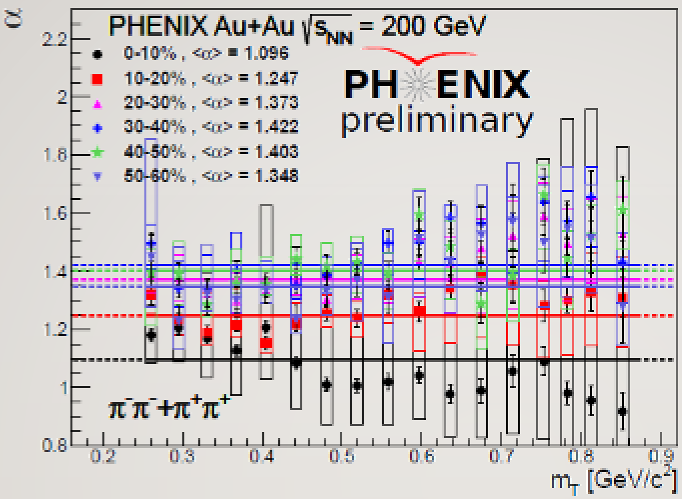
- Recall: $\kappa_3 = 1$ in pure core-halo model, $\kappa_3 \neq 1$ if coherence

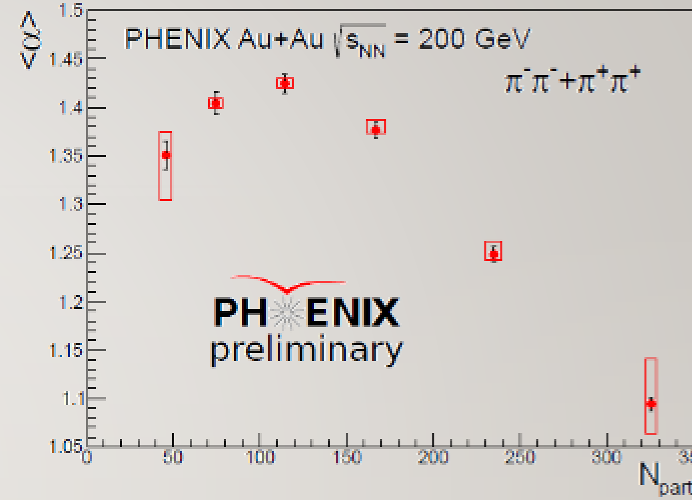


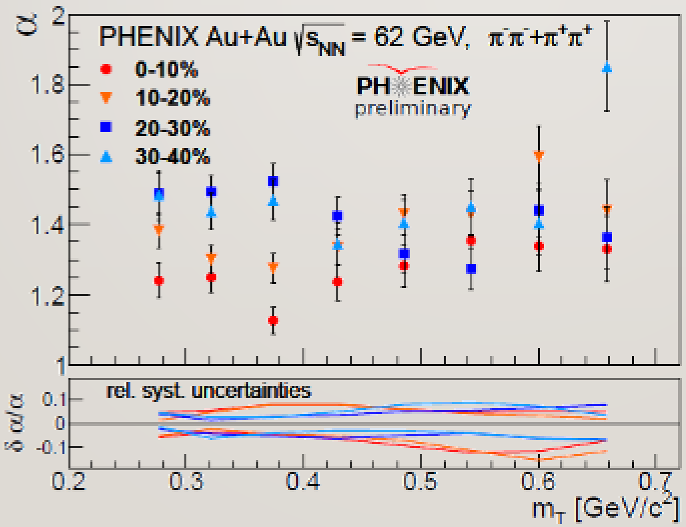
CENTRALITY DEPENDENCE

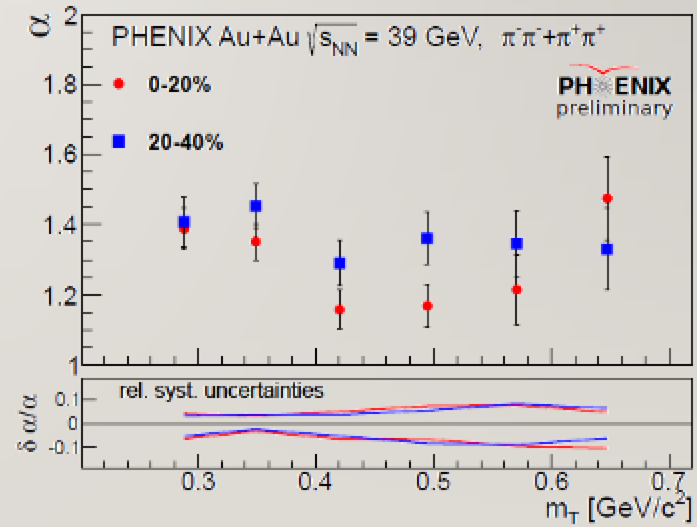
- Slightly non-monotonic vs m_T
- Non-monotonic vs N_{part}
- No significant change vs $\sqrt{s_{NN}}$

QM17,QM18,
WPCF17,WPCF18



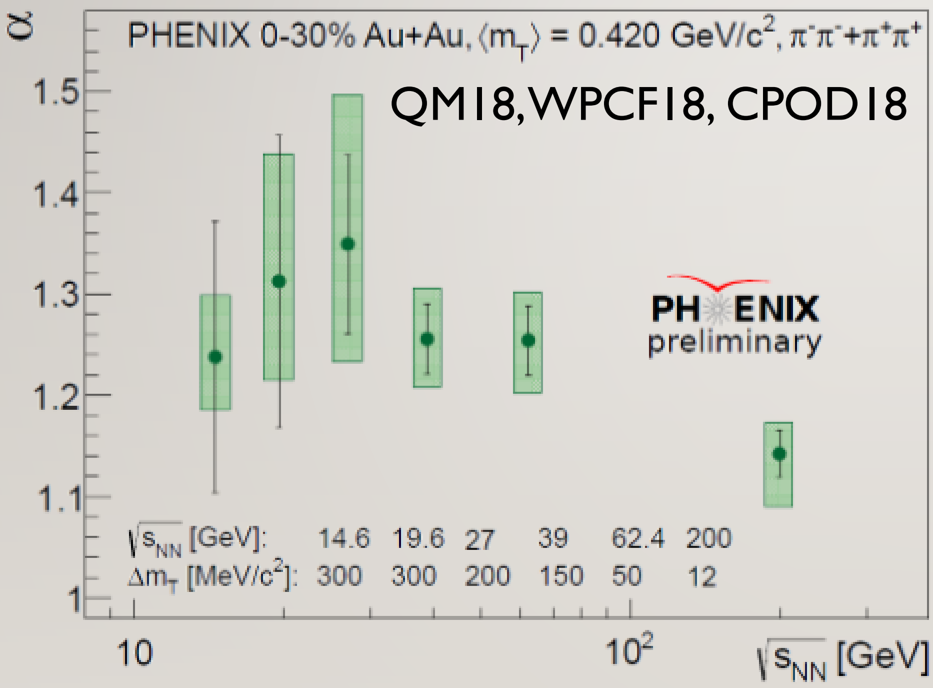


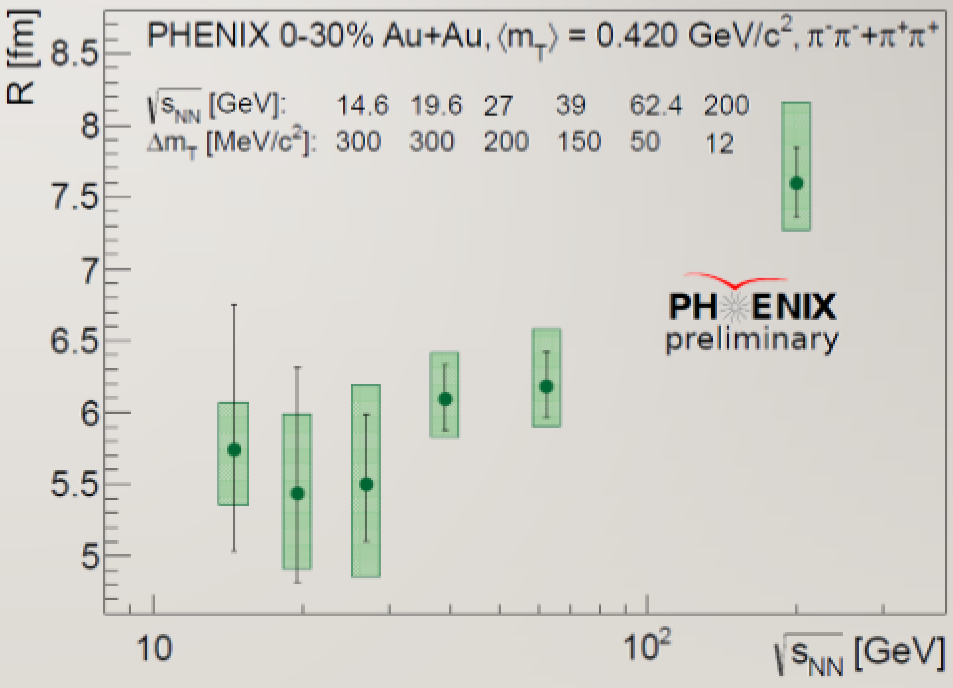




COLLISION ENERGY DEPENDENCE

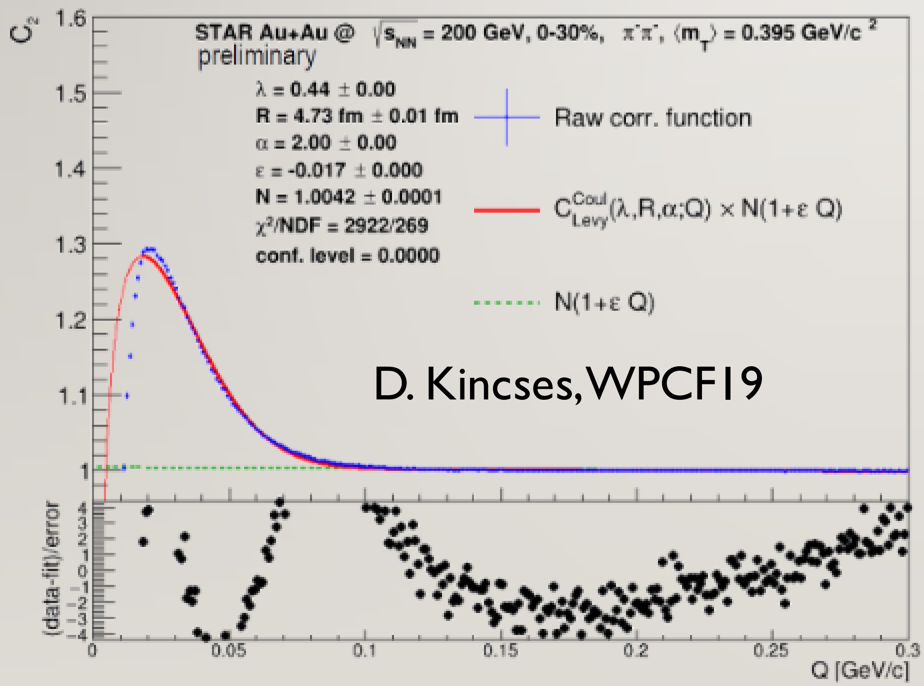
- Lévy exponent α still far from conjectured limit, interesting trends in R
- Very much mT bin width dependenting, wait for final results…

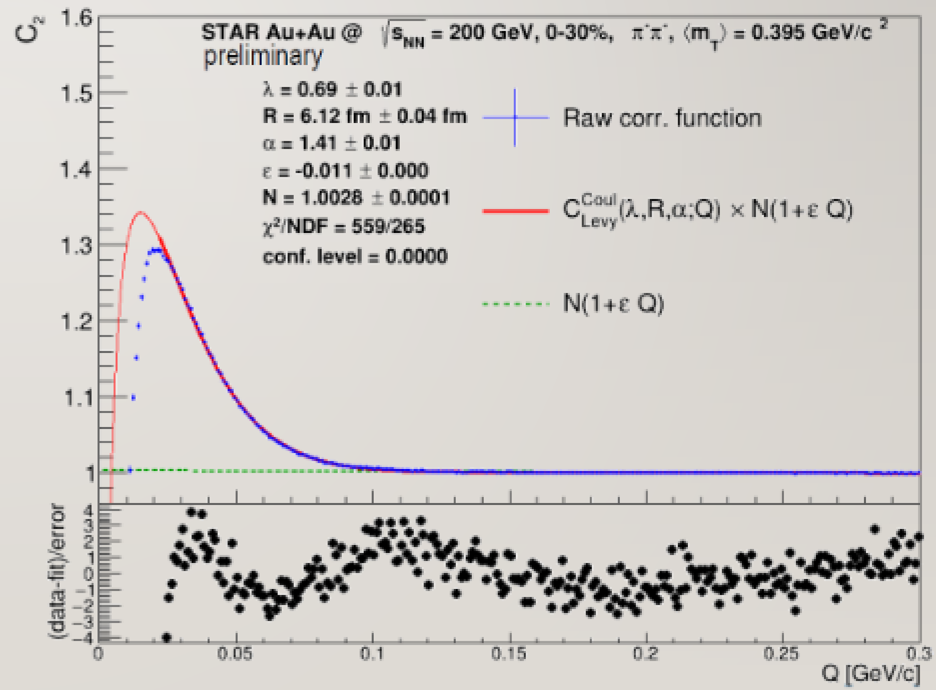




LÉVY FEMTOSCOPY AT STAR

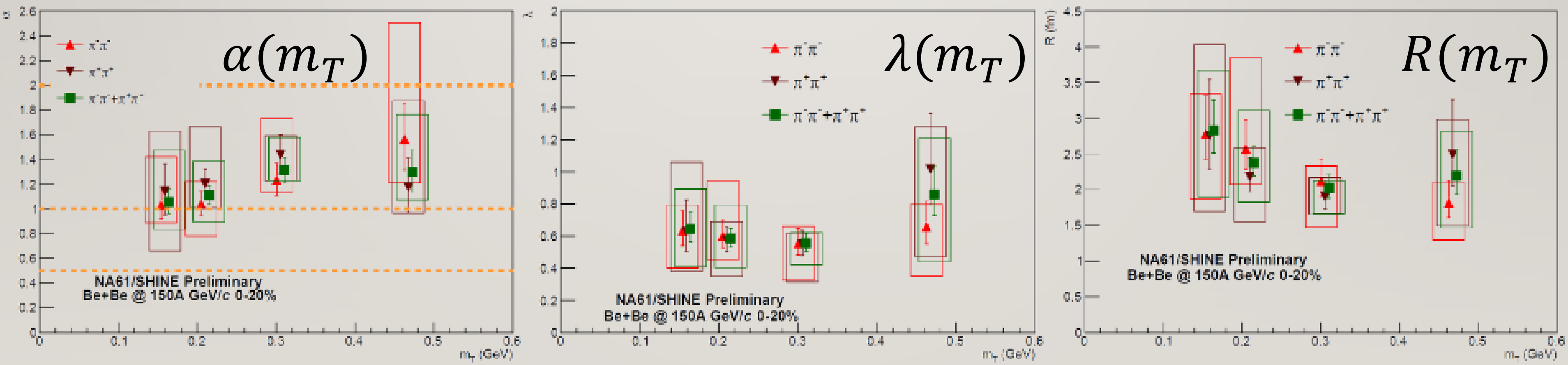
- First 1D Lévy fits tested, (very) preliminary results (presented at WPCF19)
- Q>20 MeV region describable by Lévy fits, Gaussian fits fail
- Low Q behavior still unclear





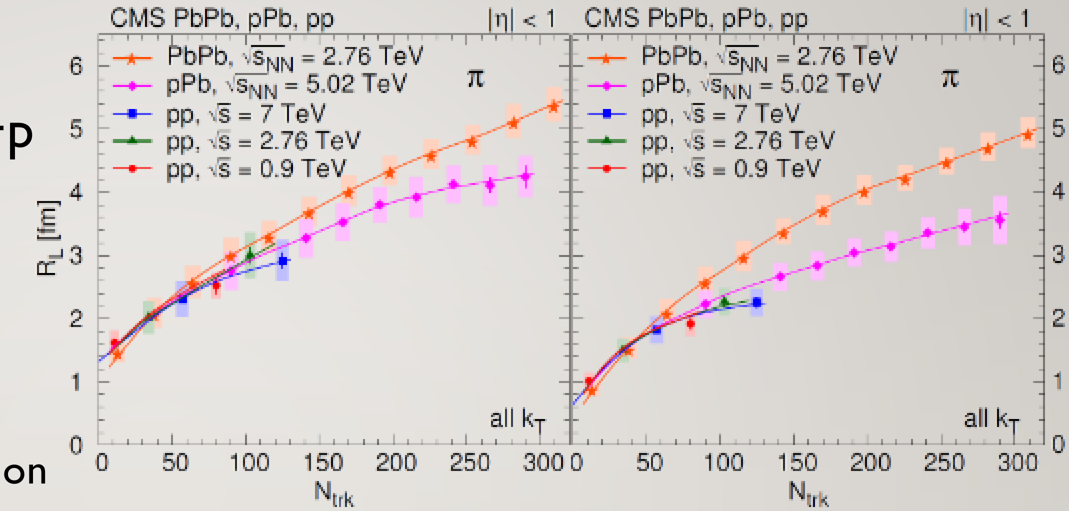
25/28 LÉVY FEMTOSCOPY AT NA61/SHINE

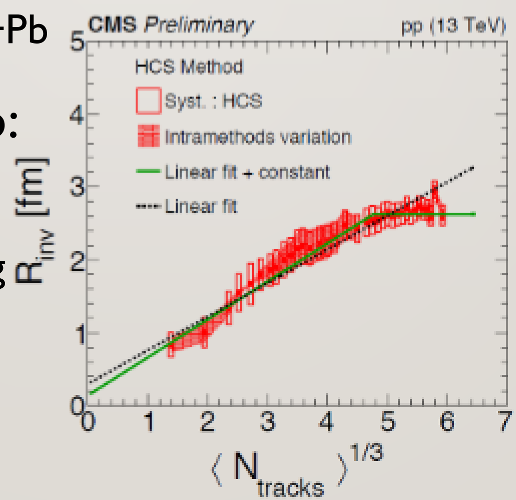
- NA61/SHINE at SPS, 150A GeV/c Be+Be analysis
- Lévy fits statistically acceptable
- $R(m_T)$: Decreasing trend, transverse flow?
- $\lambda(m_T)$: Slight/no dependence with m_T, no „hole”
- $\alpha(m_T)$: Not Gaussian, nearly Cauchy, around 1.0-1.5

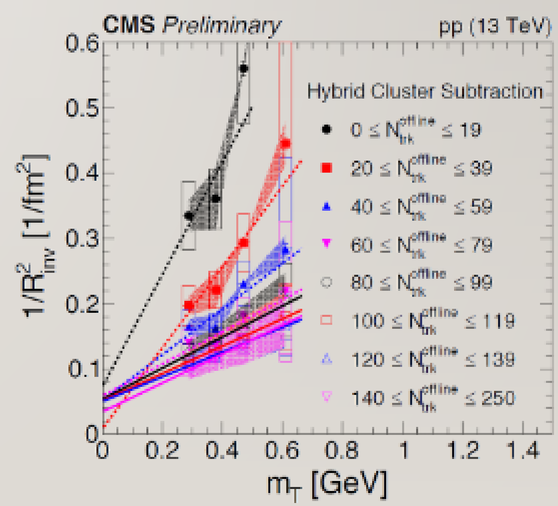


26/28 RESULTS AT CMS, TOUCHING DIFFERENT TOPICS

- Analysis performed at 0.9-13 TeV, Pb+Pb, p+Pb, p+p
- Using $\alpha = 1$ fixed
- 3D analysis for 0.9-7 TeV
  - Analysis: Wigner (F. Siklér)
  - Detailed geometry exploration
  - Elongated source: p+p and p+Pb
- High multiplicity 13 TeV p+p: similar results as ion-ion
  - Geometric multiplicity scaling
  - Hydro type of mT scaling?
  - Analysis: USP+ELTE

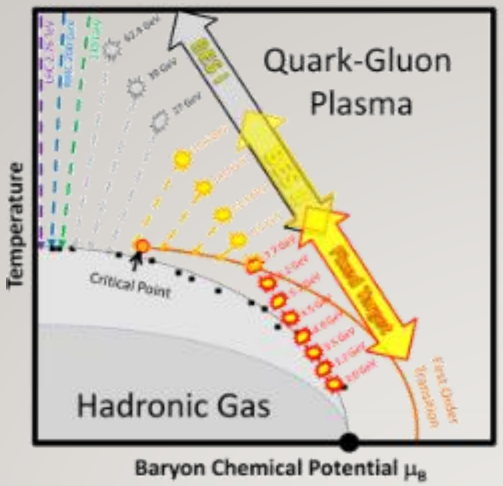


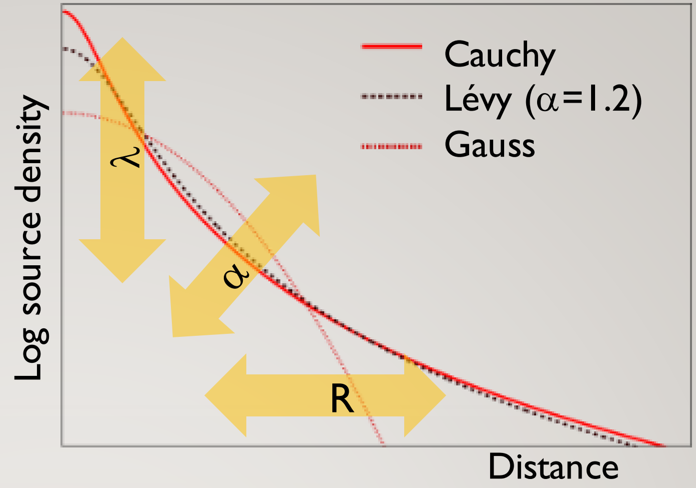




SUMMARY

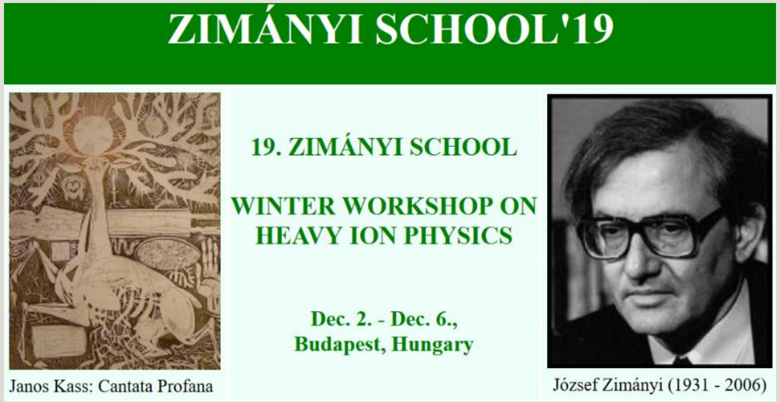
- Bose-Einstein correlations measured from 10 GeV to 10 TeV
- Levy fits yield acceptable description, parameters R, λ, α measured
  - Stability parameter α<2 ↔ anomalous diffusion/CEP/QCD jets?
  - Linear scaling of $1/R^2$ vs m_T ↔ hydro (but non-Gaussian source!)
  - Low-m_T decrease in $\lambda(m_T)$ ↔ core-halo model, in-medium η’ mass?
- Many open questions
  - Non-monotonicity in $\alpha\left(\sqrt{s_{NN}}\right)$ or $\alpha(\text{centrality})$?
  - Hole in $\lambda(m_T)$ at low $\sqrt{s_{NN}}$? Really due to η'?
  - What is the Lévy exponent for kaons?
  - Correlation strenght versus core-halo picture: are there other effects?





THANK YOU FOR YOUR ATTENTION

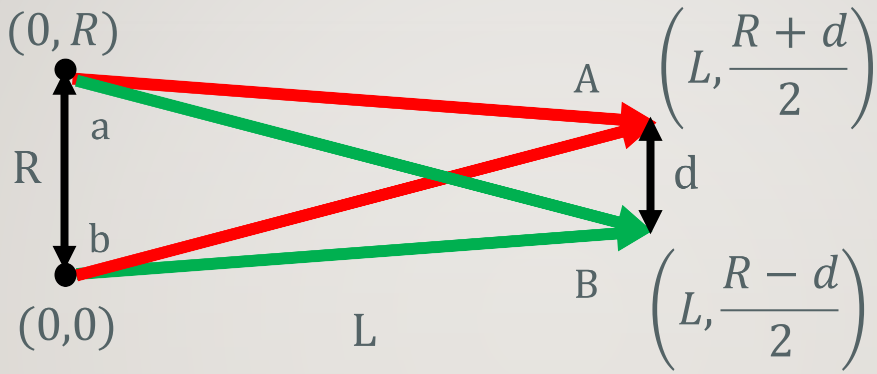
If you are interested in these subjects, come to our
Zimányi School 2019
December 2-6., Budapest, Hungary



http://zimanyischool.kfki.hu/19

BACKUP SLIDES

A CLASSICAL DESCRIPTION OF THE HBT EFFECT



$$\vec{r}_{aA} = \left(L, \frac{-R-d}{2}\right)$$

$$\vec{r}_{aB} = \left(L, \frac{-R+d}{2}\right)$$

$$\vec{r}_{bA} = \left(L, \frac{+R+d}{2}\right)$$

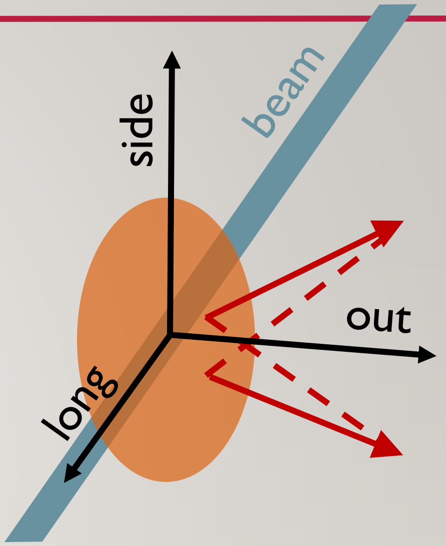
$$\vec{r}_{bB} = \left(L, \frac{+R-d}{2}\right)$$

$$r_{aA} - r_{bA} + r_{aB} - r_{bB} = 2\sqrt{L^2 + \frac{(R+d)^2}{4}} - 2\sqrt{L^2 + \frac{(R-d)^2}{4}} \approx \frac{Rd}{L}$$

$$C_{AB}(\Delta) = \frac{\langle I_A I_B \rangle}{\langle I_A \rangle \langle I_B \rangle} = 1 + \frac{1}{2}\cos\frac{kRd}{L}$$

$$R_{AB}(\Delta) = \frac{C_{AB}(\Delta) - 1}{C_{AB}(0) - 1} = \cos\frac{kRd}{L}$$

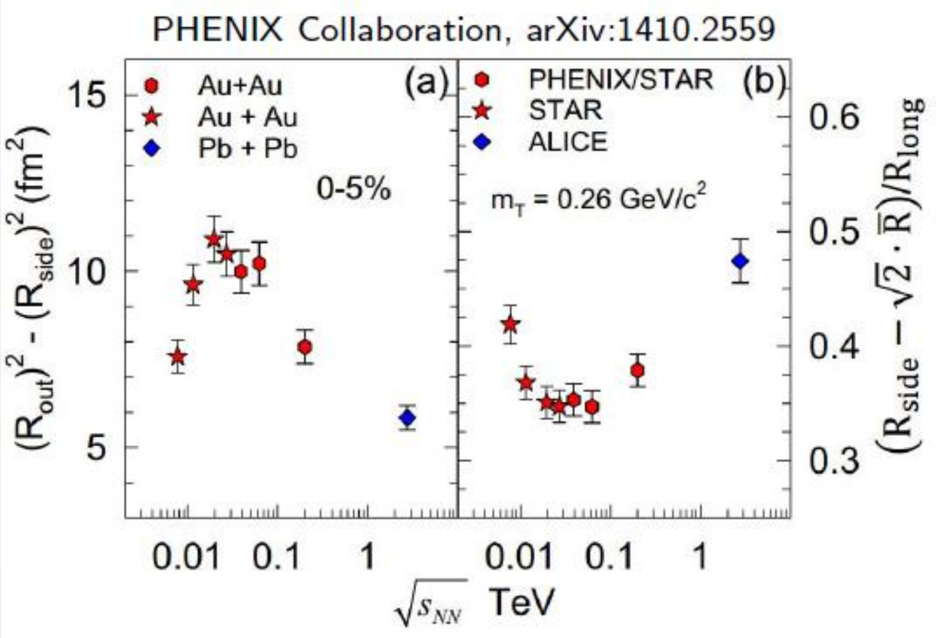
31/28 AZ OUT-SIDE-LONG RENDSZER, HBT SUGARAK



- 1D változóban többnyire: $q_{\mathrm{inv}} = \sqrt{-q^2} = \sqrt{-(k_1 - k_2)^2}$
- 3 vagy 4D információ kinyerhető?
- Általánosságban: $C_2(q) = 1 + \lambda e^{-R^2_{\mu\nu} q^\mu q^\nu}$
- Pár-koordinátarendszer!
  - Out: a pár átlagos transzverz imp. iránya
  - Long: nyaláb-irány
  - Side: mindkettőre merőleges
- Ekkor az átlagos „side" impulzus nulla, $K_{\mathrm{side}} = 0$
- Tipikusan LCMS-ben (longitudinally comoving system)
  - Nulla átlagos long. impulzus, i.e. $K^\mu = (M_t, K_t, 0,0)$
  - Ekkor: $q_0 = \frac{m^2_{1t} - m^2_{2t}}{2M_t}, q_{\mathrm{out}} = \frac{p^2_{1t} - p^2_{2t}}{2K_t}, q_{\mathrm{side}} = \frac{p_{2x}p_{1y} - p_{1x}p_{2y}}{K_t}, q_0 = \frac{E_2 p_{1z} - E_1 p_{2z}}{M_t}$
  - Tömeghéjfeltétel: $q^\mu K_\mu = 0 \;\Rightarrow\; q_0 = \frac{K_t}{M_t} q_{\mathrm{out}} = \beta_t q_{\mathrm{out}}$
- Az $R^2_{\mu\nu}$ mátrixból $R_{\mathrm{out}}, R_{\mathrm{side}}, R_{\mathrm{long}}$ nem nulla: HBT sugarak
- Szögfüggés vizsgálata: R_{os} is megjelenik

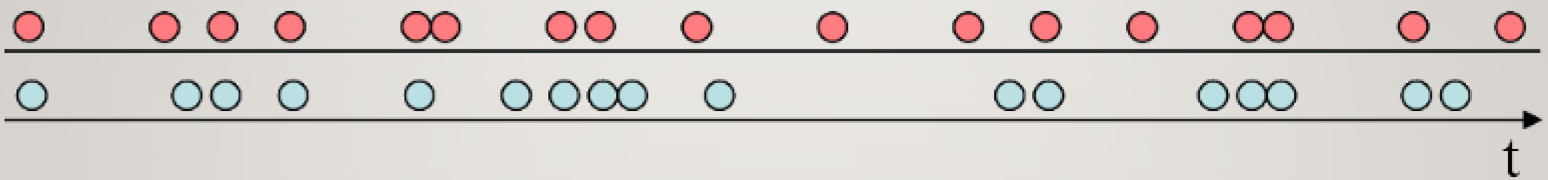
HBT RADII AND THE SEARCH FOR THE CEP

- Signals of QCD CEP: softest point, long emission
- $R_{\text{out}}^2 - R_{\text{side}}^2$: related to emission duration
- $(R_{\text{side}} - \sqrt{2}\bar{R})/R_{\text{long}}$: related to expansion velocity, $\bar{R}$: initial transverse size
- Non-monotonic patterns
- Indication of the CEP?
- Further details in Roy Lacey, arXiv:1606.08071 & arXiv:1411.7931 (PRL114)
- How about finite size scaling?

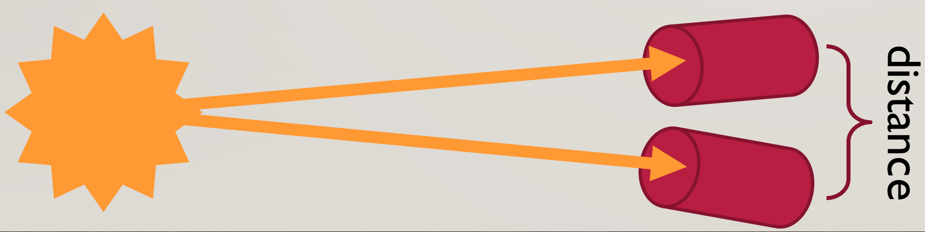


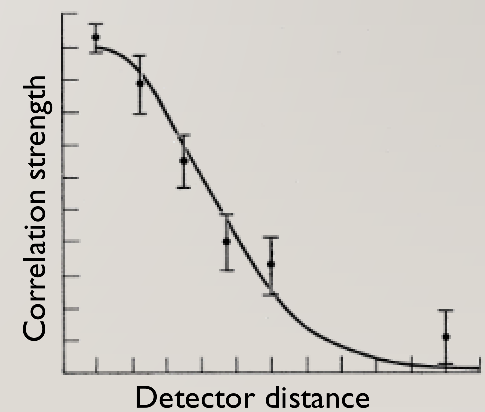
THE HBT CORRELATION

- Observation of Hanbury Brown & Twiss: at small detector distances, large correlation between the two detectors
- Joint intensity „too frequent”: $I(A, B) > I(A)I(B)$



- What is the reason for it? Interference?
- „Interference between different photons never occurs” P.A. M. Dirac, Quantum Mechanics
- Why does the correlation reduce with distance?

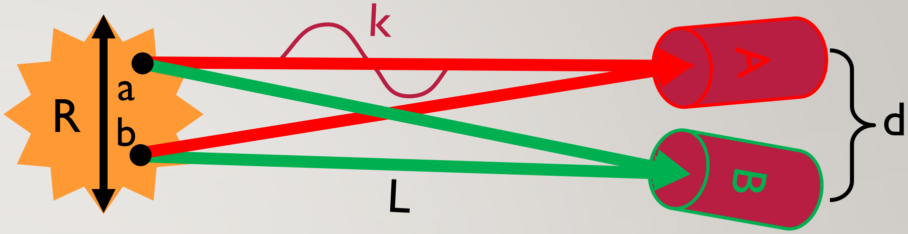




A CLASSICAL DESCRIPTION OF THE HBT EFFECT

- Spherical waves from pointlike sources:

$$A_a(r) = \frac{1}{|r - r_a|} \alpha e^{ik|r-r_a|+i\Phi_a}$$



- Total wave in detector ‚A':

$$A(r_A) = \frac{1}{L}\left(\alpha e^{ikr_{aA}+i\Phi_a} + \beta e^{ikr_{bA}+i\Phi_b}\right)$$

- Intensity in detector ‚A':

$$I_A = |A(r_A)|^2 = \frac{1}{L^2}\left(|\alpha|^2 + |\beta|^2 + \alpha^* \beta e^{ik(r_{bA}-r_{aA})+i(\Phi_b-\Phi_a)} + c.c.\right)$$

- In the time average, for chaotic (random phase) source, phases disappear:

$$\langle I_A \rangle = \langle I_B \rangle = \frac{1}{L^2}(|\alpha|^2 + |\beta|^2)$$

A CLASSICAL DESCRIPTION OF THE HBT EFFECT

- Let us calculate the time average of the multiplied intensity:

$$\langle I_A I_B \rangle = \langle |A(r_A)|^2 |A(r_B)|^2 \rangle$$

- Here

$$|A(r_A)|^2 = \frac{1}{L^2}\left(|\alpha|^2 + |\beta|^2 + \alpha^* \beta e^{ik(r_{bA}-r_{aA})+i(\Phi_b-\Phi_a)} + c.c.\right)$$

and hence phases disappear in some terms

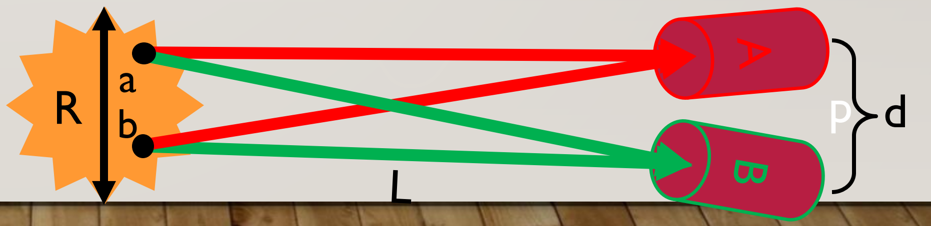
- Finally, one gets:

$$\langle I_A I_B \rangle = \frac{1}{L^4}(|\alpha|^2 + |\beta|^2)^2 + \frac{2}{L^4}|\alpha|^2|\beta|^2 \cos k(r_{aA} - r_{bA} + r_{aB} - r_{bB})$$

- And then for $\alpha = \beta$ and $d, R \ll L$, the result is:

$$C_{AB}(\Delta) = \frac{\langle I_A I_B \rangle}{\langle I_A \rangle \langle I_B \rangle} = 1 + \frac{1}{2}\cos\frac{kRd}{L}$$

$$R_{AB}(\Delta) = \frac{C_{AB}(\Delta)-1}{C_{AB}(0)-1} = \cos\frac{kRd}{L}$$



HBT EFFECT WITH TWO QUANTUM PARTICLES

- Single particle wave functions
  $\Psi_B^b = e^{ikr_{bB}+i\Phi_b}$ és $\Psi_A^a = e^{ikr_{aA}+i\Phi_a}$

- Two particle wave function:

$$\Psi_{AB} = \frac{1}{\sqrt{2}}\left(\Psi_A^a \Psi_B^b + \Psi_B^a \Psi_A^b\right)$$

because of indistinguishable nature of particles

- From here the correlation function is:

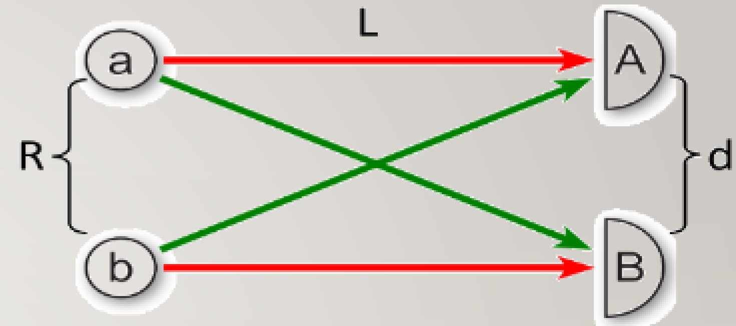
$$C_{AB} = \frac{P(A,B)}{P(A)P(B)} = \frac{1}{2}\left\langle \left| \left(\Psi_A^a \Psi_B^b + \Psi_B^a \Psi_A^b\right) \right|^2 \right\rangle$$

where $\langle \quad \rangle$ is the thermal average (over the phases)

- From there the result is, similarly to the classical case:

$$C_{AB} = 1 + \cos k(r_{aA} - r_{bA} + r_{aB} - r_{bB}) \approx 1 + \cos k \frac{Rd}{L}$$

ha $d \ll R \ll L$



QUANTUM FIELD THEORY FORMALISM

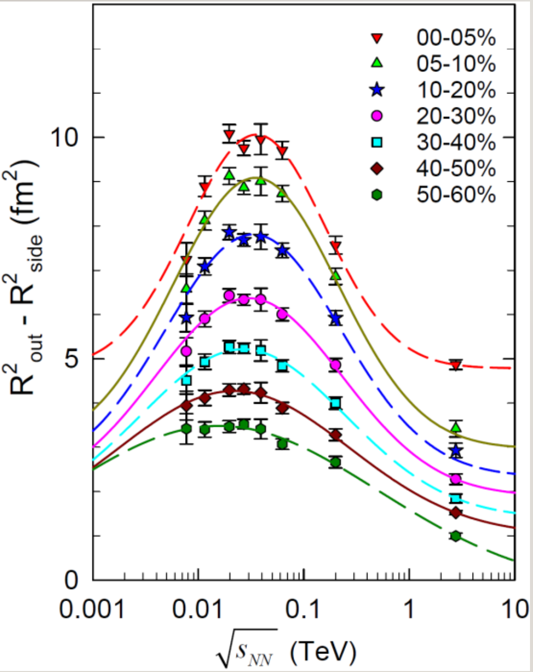
- Single particle distributions: $N_1(k) = \mathrm{Tr}\left[\rho a(k)^\dagger a(k)\right] = \langle n(k)\rangle$
- Pair distributions: $N_1(k_1, k_2) = \mathrm{Tr}\left[\rho a(k_1)^\dagger a(k_2)^\dagger a(k_1) a(k_2)\right]$
- ρ: final state density matrix; expressable in initial state with transfer-matrix
- Wigner function: Fourier-transform of density matrix at given time

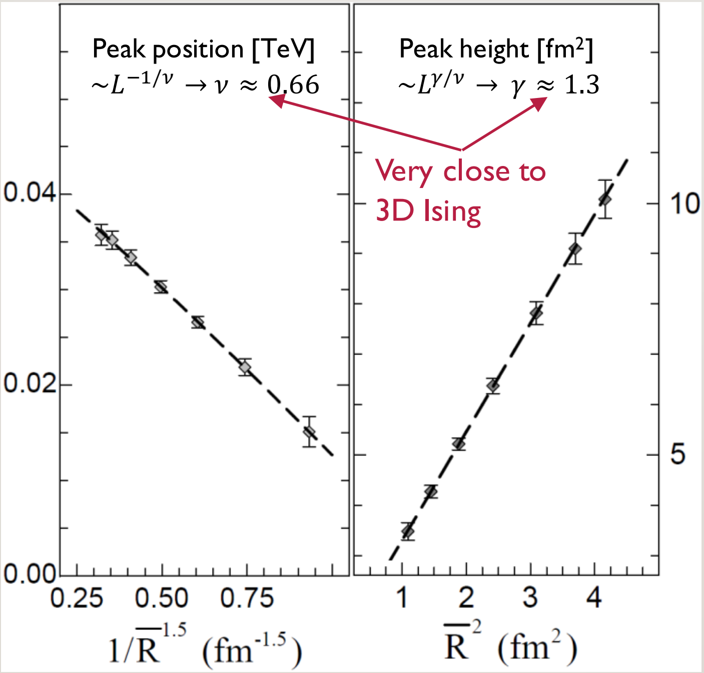
$$\rho(k, k') = \int dx e^{-iqx} W(x, K)$$

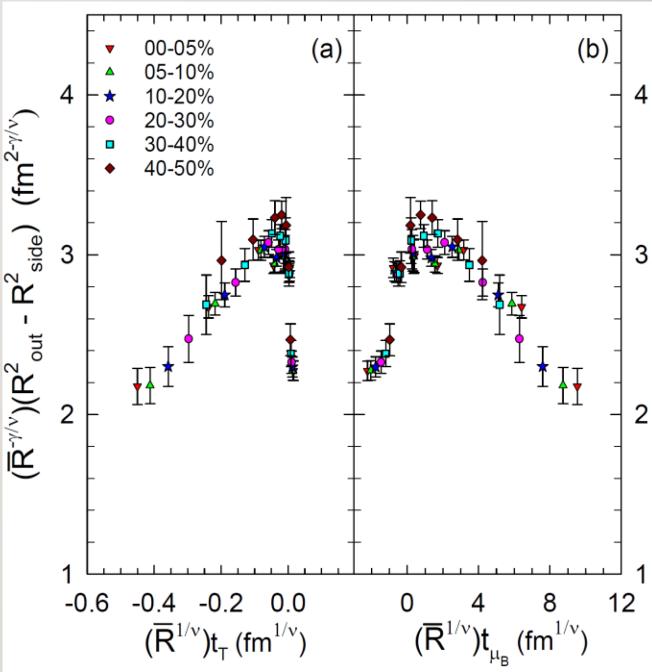
- All details can be worked out with $W \leftrightarrow S$
- Wigner function ~identical to source distribution
- All this is important when coherence effects or squeezed states play a role
- Otherwise we can stick to the QM treatment

38/28 FINITE SIZE EFFECTS TAKEN INTO ACCOUNT

- Finite size scaling analysis with $L = \bar{R}$, peak position and height tell ν and γ
- Critical EndPoint location for $L \to \infty$: $\sqrt{s_{NN}} \approx 47.5$ GeV
- Scaling vs $t_T = \frac{T - T_c}{T_c}$, $t_{\mu_B} = \frac{\mu_B - \mu_{B,c}}{\mu_{B,c}}$: $T_c = 165$ MeV & $\mu_B = 95$ MeV







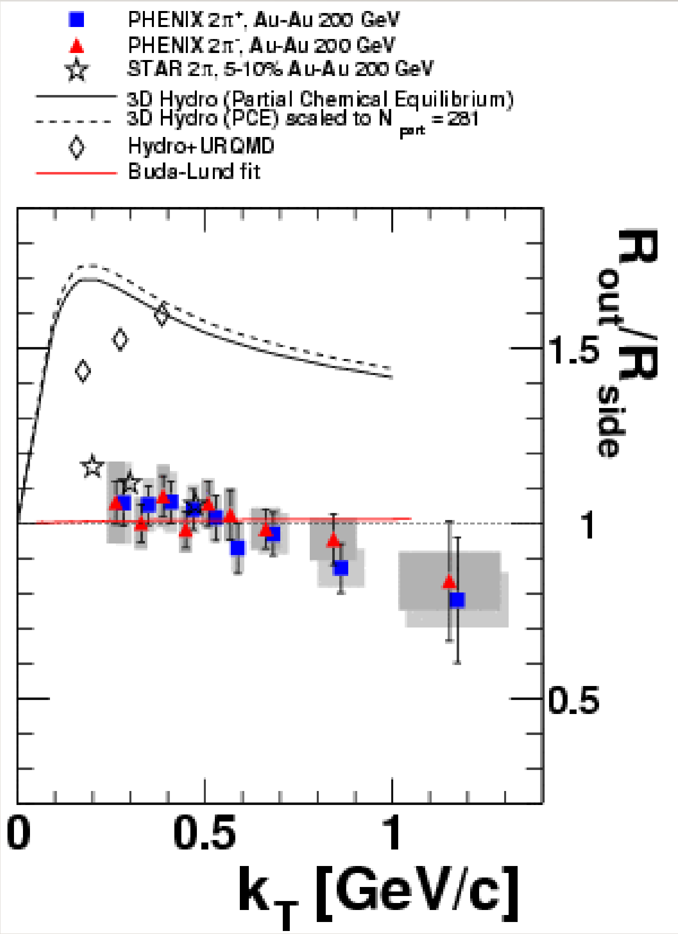
Roy Lacey, PRL114 (2015),142301

39/28 NO FIRST ORDER PHASE TRANSITION!

- Out-side különbség: pionkeletkezés időtartama
- Elsőrendű fázisátalakulás:

  Out » Side
- Hidrodinamikai jóslat:

  Out ≈ Side
- ~50 modell rossz:
  „HBT rejtély"
- Kísérlet: Out ≈ Side
- „Azonnali kifagyás"



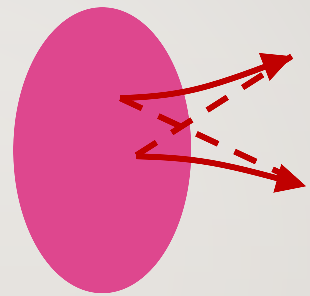
INTERACTIONS: THE COULOMB-EFFECT

- Plane-wave result, based on $\left|\Psi_2^{(0)}(r)\right|^2 = 1 + e^{iqr}$:

$$C_2(q,K) \cong \int D(r,K)\left|\Psi_2^{(0)}(r)\right|^2 dr = 1 + \int D(r,K) e^{iqr} dr$$

- If interaction:

$$\Psi_2^{(0)}(r) \to \Psi_2^{(\mathrm{int})}(r_1, r_2)$$



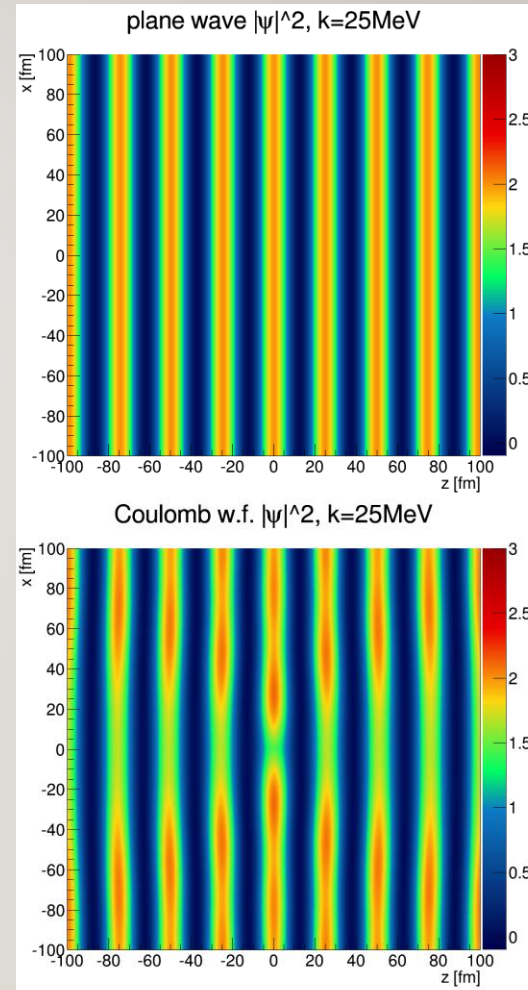
- For Coulomb:

$$\left|\Psi_2^{(C)}(r)\right|^2 = \frac{\pi\eta}{e^{2\pi\eta}-1} \cdot (\text{complicated hypergeometric expression})$$

- Direct fit with this, or the usual iterative Coulomb-correction:

$$C_{\mathrm{Bose-Einstein}}(q)K(q), \text{where } K(q) = \frac{\int D(r,K)\left|\Psi_2^{(C)}(r)\right|^2 dr}{\int D(r,K)\left|\Psi_2^{(0)}(r)\right|^2 dr}$$

- In this analyis: assuming spherical source

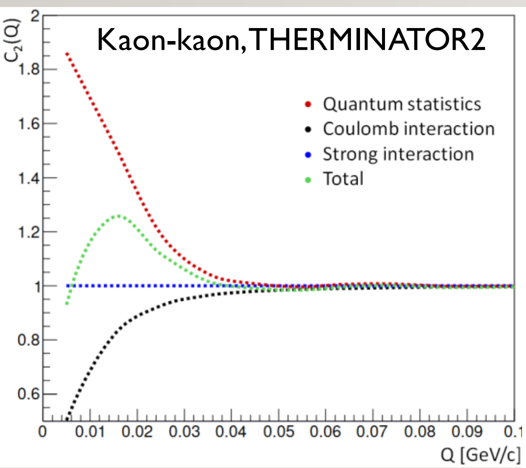


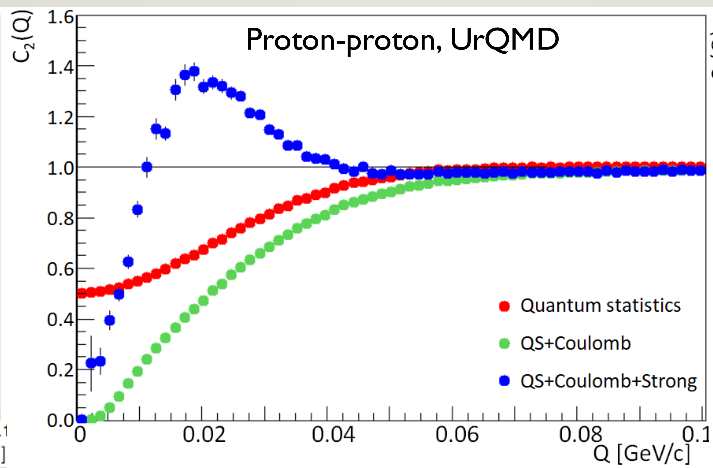
FERMIONS, NON-IDENTICAL CORRELATIONS

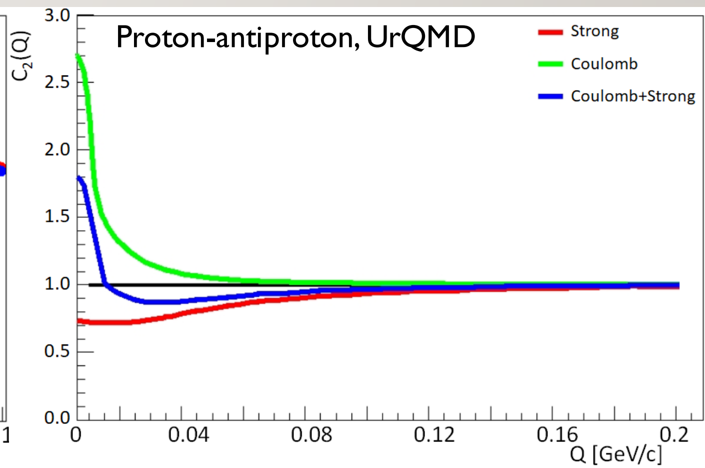
- Even if not identical bosons the formula still works:

$$C_2(q,K) \cong \int D(r,K)|\Psi_2(r)|^2 dr$$

- Pair wave function determines $D \leftrightarrow C_2$ connection
- Fermions: anticorrelation
- Non-identical pairs: interaction modifies wave function

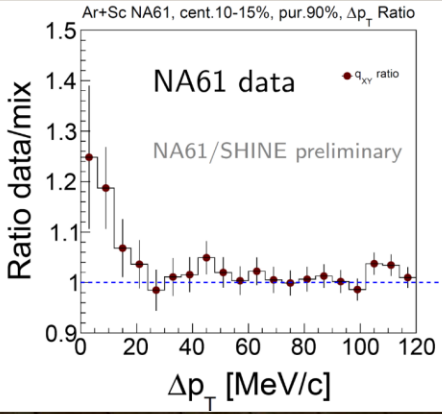






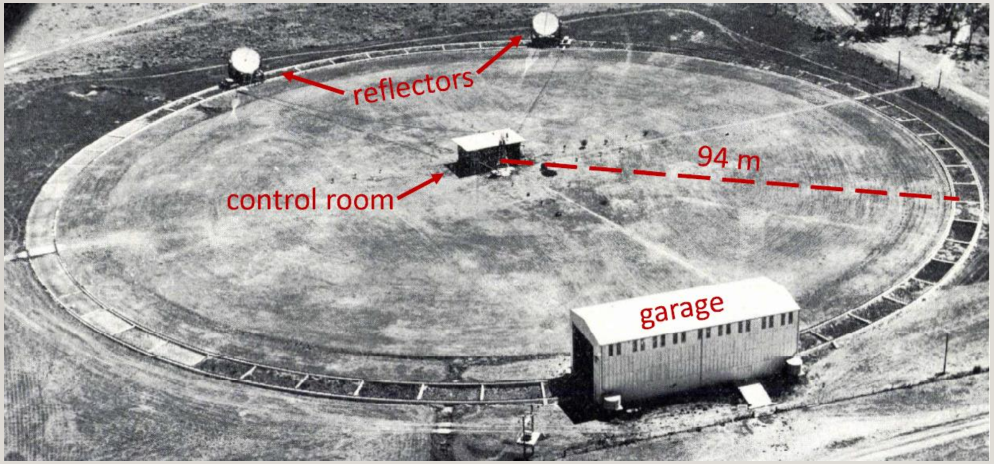
INTERMITTENCY VS FEMTOSCOPY?

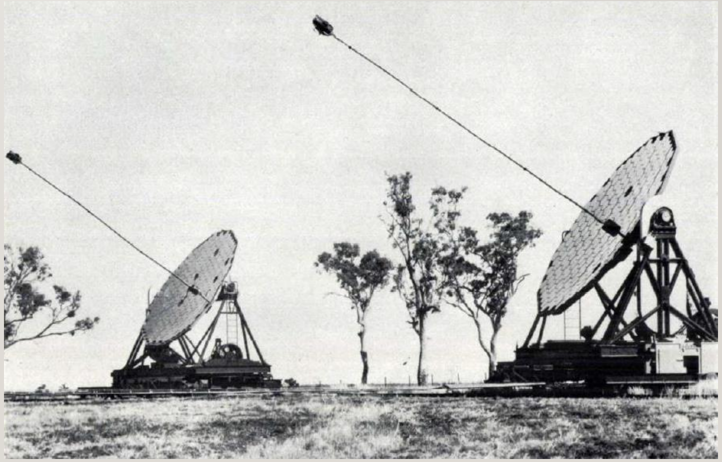
- When one writes $\rho_2(p_1,p_2) = \rho(p_1)\rho(p_2)[1 + R_2(p_1,p_2)]$ then $1 + R_2 = C_2$, i.e. this is the same as the femtoscopy type of definition
- Neglecting interactions, using $\widetilde{D}$, the Fourier-transformed of the pair source:
  - For bosons, $R_2 = \widetilde{D}$
  - For fermions, $R_2 = -\widetilde{D}$
  - For non-identical particles, $R_2 = 0$
- Note: in femtoscopy, $R_2(q = 0) < 1$, except for exotics like squeezed states
- Where does the intermittency effect come from?
- Is it on top of the quantum statistics and interactions?
- This plot looks exactly like femtoscopic (Bose-Einstein) correlations, what about PID?



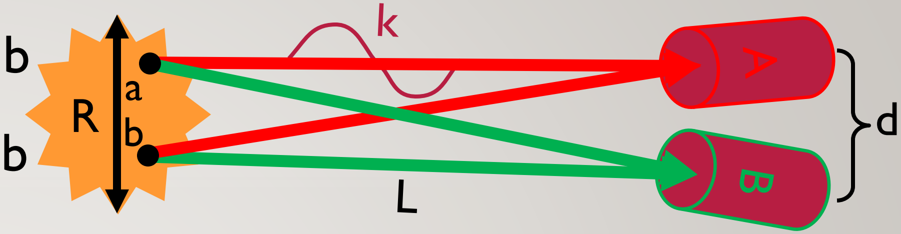
43/28 A SURPRISING DISCOVERY: HBT CORRELATIONS

- R. Hanbury Brown: found strange correlations in signals of telescope pairs
- Built dedicated tabletop experiment, explaind results with R. Q. Twiss
- Technique nowadays known as HBT interferometry
- Applicable in high energy heavy ion physics: G. Goldhaber et al.
- Clearly understood by R. J. Glauber in terms of quantum optics





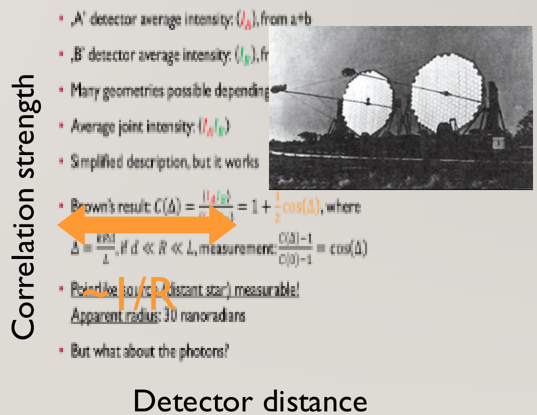
A CLASSICAL DESCRIPTION OF THE HBT EFFECT



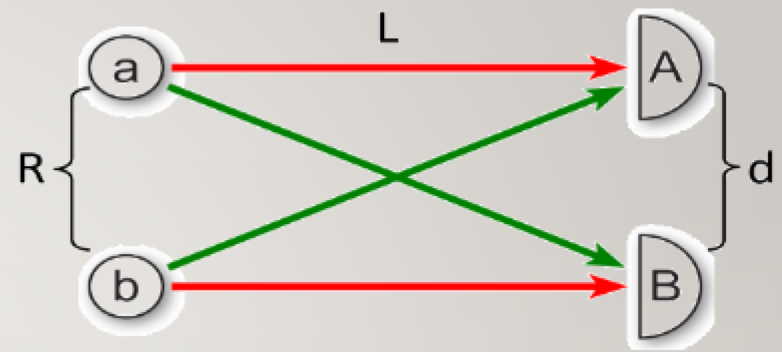
- ‚A' detector average intensity: $\langle I_A \rangle$, from a+b
- ‚B' detector average intensity: $\langle I_B \rangle$, from a+b
- Many geometries possible depending on the source
- Average joint intensity: $\langle I_A I_B \rangle$
- Simplified description, but it works
- Brown's result: $C(\Delta) = \frac{\langle I_A I_B \rangle}{\langle I_A \rangle \langle I_B \rangle} = 1 + \frac{1}{2}\cos(\Delta)$, where $\Delta = \frac{kRd}{L}$, if $d \ll R \ll L$, measurement: $\frac{C(\Delta)-1}{C(0)-1} = \cos(\Delta)$

correlation!

- Pointlike source (distant star) measurable! Apparent radius: 30 nanoradians
- But what about the photons?



45/28 A QUANTUM DESCRIPTION OF THE HBT EFFECT



- „Symmetrized wave function”: $a \to A$ and $b \to B$ and $a \to B$ és $b \to A$ are equivalent
- Photons „like to go in the same direction”
- Taking e^{ikx} plane waves one gets the joint intensity (probability) in the two detectors similarly to the classical result:

$$\frac{P(A,B)}{P(A)P(B)} = 1 + \cos k\frac{Rd}{L}$$

- Correlation width inverse of source size
- Reason: Bose-Einstein symmetrization
- Classical (scalar, vector) waves are of intrinsically bosonic nature!

HBT EFFECT FOR EXTENDED SOURCES

- What happens for an $S(r)$ source distribution?
- Similarly to the previous description:

$$\Psi(r) = e^{ikr}, \Psi_2(r_1, r_2) = \frac{1}{\sqrt{2}}\left(e^{ik_1 r_1} e^{ik_2 r_2} + e^{ik_1 r_2} e^{ik_2 r_1}\right)$$

$$N_1(k) = \int S(r,k)|\Psi(r)|^2 d^4 r$$

$$N_2(k_1, k_2) = \int S(r_1, k_1) S(r_2, k_2) |\Psi_2(r_1, r_2)|^2 d^4 r_1 d^4 r_2$$

$$C_2(k_1, k_2) = \frac{N_2(k_1, k_2)}{N_1(k_1) N_1(k_2)} \cong 1 + \left|\frac{\tilde{S}(q,K)}{\tilde{S}(0,K)}\right|^2$$

where $q = k_1 - k_2, K = (k_1 + k_2)/2$

- Simply $C(q) = 1 + \left|\tilde{S}(q)\right|^2$, where $\tilde{S}(q) = \int S(r) e^{iqr}$
- Invertable (sort of), $S(r)$ can be reconstructed from $C(q)$
- Approximations: no interaction, no multiparticle correlation, thermal emission …

SOURCE OR PAIR DISTRIBUTION?

- Under some circumstances (thermal emission, no interactions, …):

$$C_2(q,K) = \int S\left(r_1, K+\frac{q}{2}\right) S\left(r_2, K-\frac{q}{2}\right) |\Psi_2(r_1,r_2)|^2 dr_1 dr_2$$
$$\cong 1 + \left|\int S(r,K)e^{iqr}dr\right|^2$$

- Let us introduce the spatial pair distribution:

$$D(r,K) = \int S\left(\rho+\frac{r}{2},K\right) S\left(\rho-\frac{r}{2},K\right) d\rho$$

- Then the Bose-Einstein correlation function becomes:

$$C_2(q,K) \cong \int D(r,K)|\Psi_2(r)|^2 dr = 1 + \int D(r,K)e^{iqr}dr$$

- **Bose-Einstein correlations measure spatial pair distributions!**

A REALISTIC SOURCE FUNCTION

- Let us take a $v(r)$ velocity field, $T(r)$ temperature distribution and $n(r)$ density
- Then the source distribution becomes:

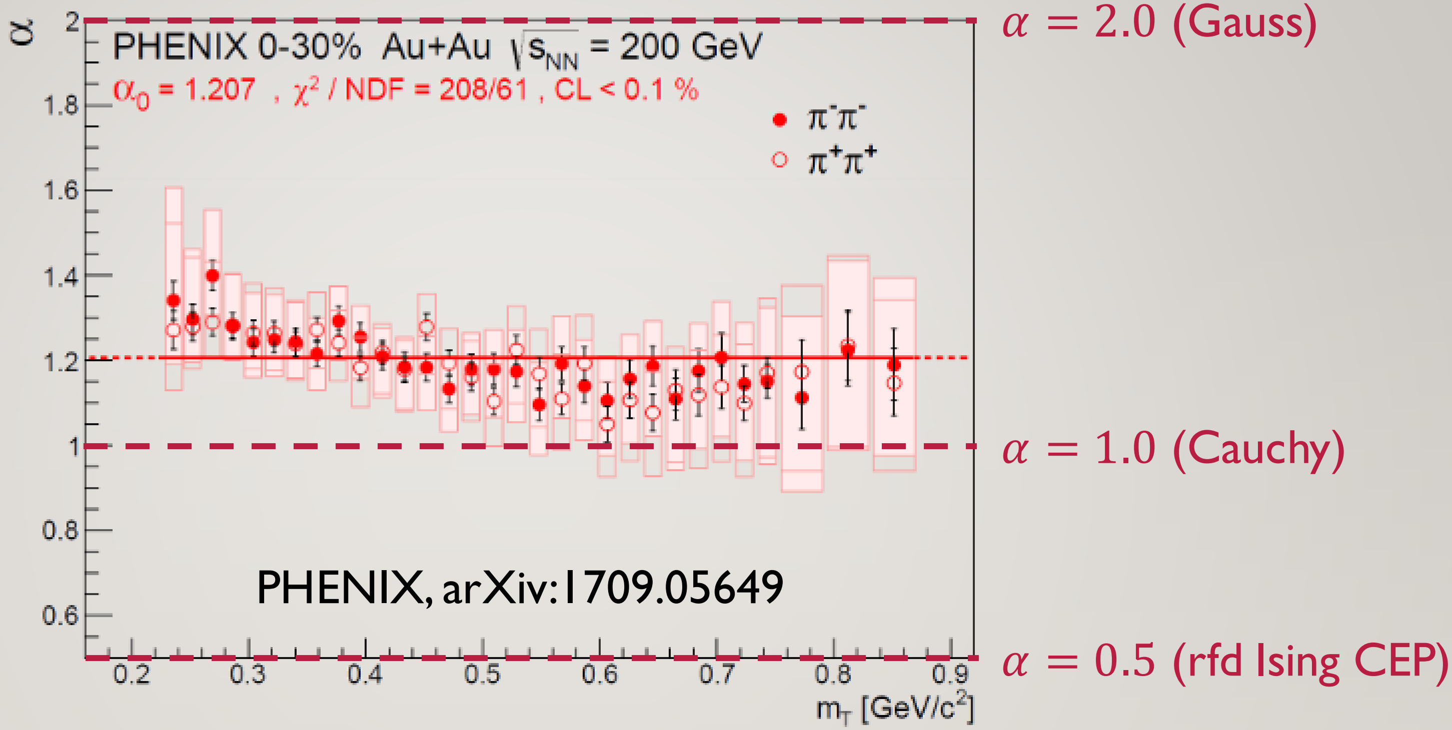
$$S(r) \sim n(r) e^{-\frac{(mv(r)-p)^2}{2mT(r)}}$$

- Let there be Hubble-flow: $v_i = \frac{\dot{R}_i}{R_i} r_i, n = n_0 e^{-\frac{r_x^2}{2R_x^2}-\frac{r_y^2}{2R_y^2}-\frac{r_z^2}{2R_z^2}}, T = T_0 \left(\frac{R_x^0 R_y^0 R_z^0}{R_x R_y R_z}\right)^{1/\kappa}$
- Then $C(k) = 1 + e^{-k_x^2 \tilde{R}_x^2 - k_y^2 \tilde{R}_y^2 - k_z^2 \tilde{R}_z^2}$, where $\tilde{R}_i$ are the HBT sizes (in 3D)

$$\frac{1}{\tilde{R}_i^2} = \frac{1}{R_i^2} + \frac{m}{T_0} \frac{\dot{R}_i^2}{R_i^2} \quad \text{(i.e. HBT size } \tilde{R}_i \neq R_i \text{ geometrical size)}$$

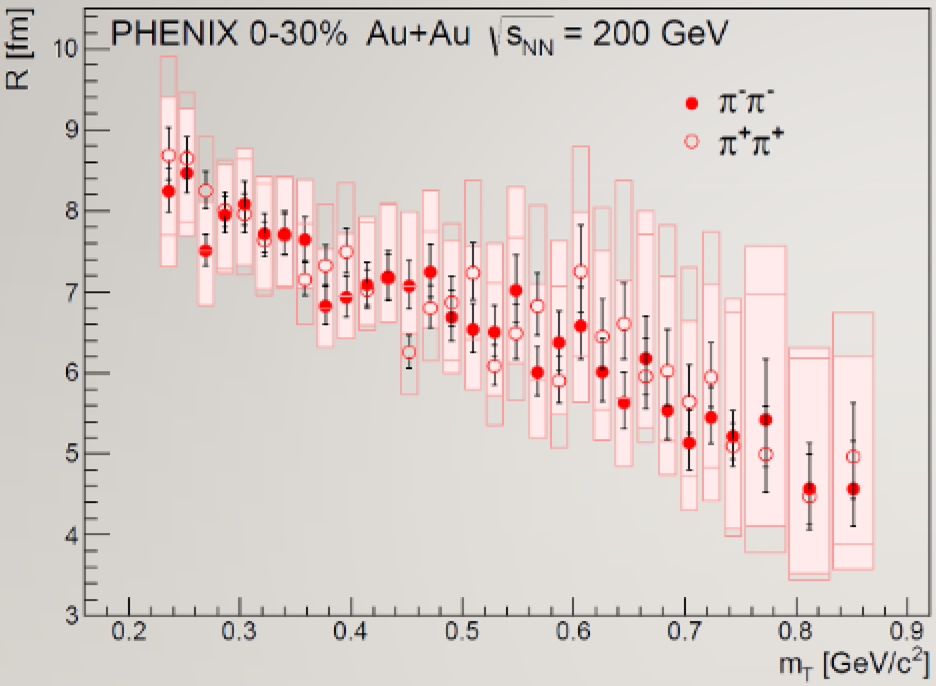
- What we really measure: homogeneity length

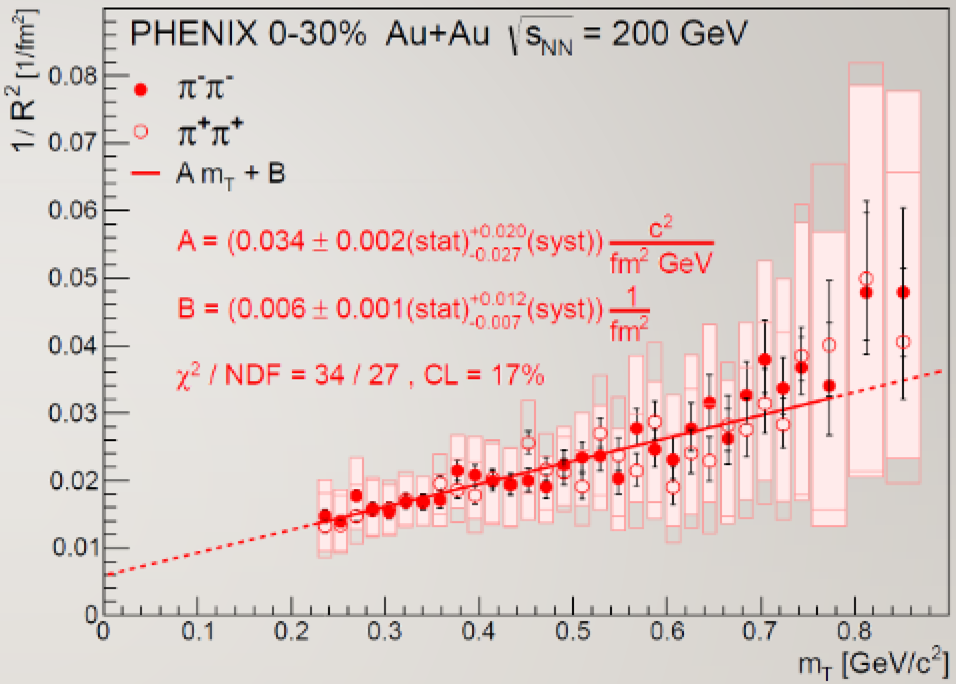
LÉVY EXPONENT (SHAPE PARAMETER) α



- Measured value far from Gaussian (α = 2), inconsistent with expo. (α = 1)
- Also far from the random field 3D Ising value at CEP (α = 0.5)
- More or less constant (at least within systematic uncertainties)
- What do models and calculations say?

LÉVY SCALE PARAMETER R





- Similar decreasing trend as Gaussian HBT radii, but it is not an RMS!
- Hydro behavior not invalid
- The linear scaling of $1/R^2$, breaks for high m_T?

PHENIX, arXiv:1709.05649

A CROSS-CHECK: 3D LÉVY FEMTOSCOPY

- Femtoscopy done in 3D: Bertsch-Pratt pair frame (out/side/long coordinates)
- Physical parameters: $R_{out/side/long}$ λ, α measured versus pair m_T
- Fit in this case: modified log-likelihood (small statistics in peak range)