Some aspects of gravitation: from thermodynamics to Eötvös balance

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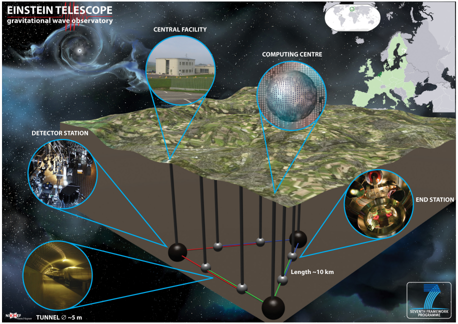
T. Biró, V. Czinner, H. Iguchi and VP, PLB **782** (2018) 228.
VP and S. Abe, arXiv:1905.10631
L. Völgyesi, et al., Magyar Geofizika, **59**/4, (2018) 165. (in Hungarian)

Content

Introduction – motivations

- Weak nonlocality and nonequilibrium thermodynamics
- Einstein Telescope → geophysics and gravitation
- Relativistic fluids → dissipation is local
- Volume and mass → extensivity



Extensivity

Additivity and extensivity

The so-called nonextensive thermostatistics is not extensive, because it has nothing to do with extensivity. (C. Tsallis: Introduction to Nonextensive Statistical Mechanics, 2009)

Additivity

- Link between statistical physics and thermodynamics. The origin of logarithmic entropy.
- For probabilistic theories of independent thermodynamic bodies: $S(f_1) + S(f_2) = S(f_1 f_2)$
- Boltzmann-Gibbs-Shannon, Tsallis, Rényi, Fisher, etc. entropies

Extensive thermodynamic quantities

- Measures - σ-additive set functions, generalization of area, volume, etc.
- $V_1 + V_2 = V_{12}$: interaction?

Extensivity : body and continuum

$X_A = (X_1, X_2, ..., X_n)$ extensive physical quantities
Gibbs relation :

$$dS = Y_1 dX_1 + Y_2 dX_2 + ... + Y_n dX_n.$$

Extensivity = Euler homogeneity

- $S(\lambda X_A) = \lambda S(X_A)\ \forall \lambda \in \mathbb{R}^+$, the entropy is a first order Euler homogeneous function ;
- $\exists s$, density :

$$S(X_A) = X_1 s\left(\frac{X_B}{X_1}\right), \qquad B = 2, ..., n;$$

- Gibbs-Duhem : $S = Y_1 X_1 + Y_2 X_2 + ... + Y_n X_n$.

Extensivity : a connection between continuum and homogeneous bodies.

E.g. elasticity : $de = Tds - \frac{P_{ij}}{\rho} d\epsilon^{ij}$

Thermodynamic limit : $\lim_{N\to\infty} \frac{S(N)}{N} < \infty$

Black hole thermodynamics

Thermodynamics of Schwarzschild black holes

Planck units: $M_P L_P = \hbar$, $L_P / M_P = G$, $c = 1$.
M is the invariant mass, R is the radius of the event horizon

Bekenstein–Hawking entropy

- $E := M = R/2$, internal energy;
- $S(E) = 4\pi E^2 = \pi R^2$, area of the horizon;
- $$\frac{dS}{dE} = \frac{1}{T} = 8\pi E, \qquad \frac{d^2 S}{dE^2} = -\frac{1}{T^2 C} = 8\pi > 0.$$
- $C = -2S < 0$.
- It is not extensive.

Background from general relativity:

- No hair theorems: there are no other variables.
- $\delta A \geq 0$ is increasing.
- $T = \kappa / 8\pi = T_{Hawking} = T_{Unruh}$. κ is the surface gravity.

Is there a volume of a black hole?

Is there an invariant volume?
Christodoulou-Rovelli (2015) + radiation: $V_{CR} \sim R^5$.

Thermodynamic of black holes with volume

- $V(M = R/2)$ and $E(M = R/2)$, because it is bald;
- $S(E, V) = \zeta E^\alpha V^\beta$, $V \sim R^{\gamma+3}$, the parameters are: α, β, γ.
- $$\frac{\partial S}{\partial E} = \frac{1}{T} = \alpha \frac{S}{E}, \qquad \frac{\partial S}{\partial V} = \frac{p}{T} = \beta \frac{S}{V}.$$
- $$\frac{\partial^2 S}{\partial E^2} = -\frac{1}{T^2 C} = \alpha(\alpha - 1)\frac{S}{E^2} \quad \Rightarrow \quad \boxed{C = \frac{\alpha}{1-\alpha} S}$$

$C > 0$, ha $0 < \alpha < 1$.

Extensive thermodynamics of black holes

1. Gravitation theory: $S = \lambda 4\pi R^2$, ha $\lambda = 1/4$, then it is Bekenstein-Hawking. $\longrightarrow$ $\boxed{2 = \alpha + \beta(\gamma + 3)}$
2. Hawking temperature, $T_H = 1/(4\pi R)$: $\longrightarrow$ $\boxed{\lambda = 1/(2\alpha)}$
3. Euler homogeneity ($S = \zeta E^\alpha V^\beta$). $\longrightarrow$ $\boxed{\alpha + \beta = 1}$
4. Stefan-Boltzmann radiation (3D : $p = e/3$). $\longrightarrow$ $\boxed{\alpha = 3\beta}$

$$\gamma = 2, \quad \alpha = 3/4, \quad \beta = 1/4, \quad \lambda = 2/3$$

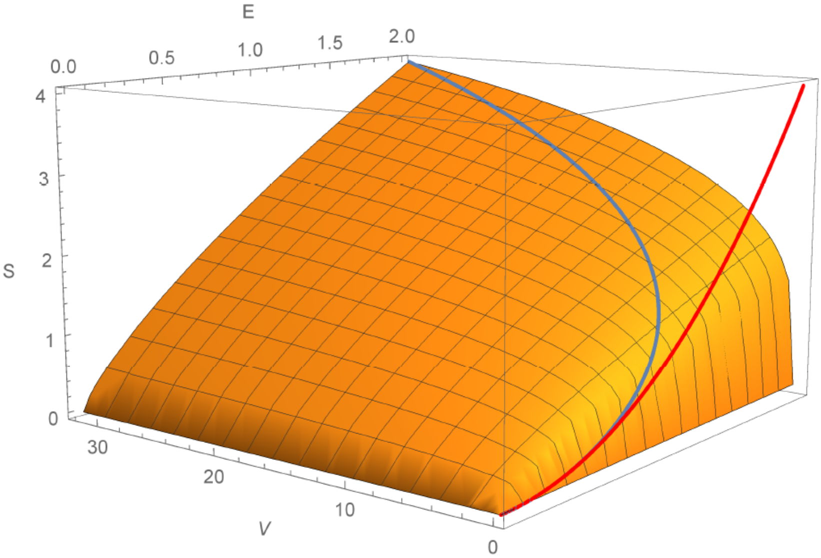
Consequences

- Christodoulou–Rovelli volume,
- 3rd law, because

$$\frac{C}{V} \sim (\gamma + 1)\, T^{\gamma+1},$$

- thermodynamic stability, positive specific heat, $C = 3S$.

Convex and concave



Negative heat capacity of condensed matter

Two arguments

Condensed matter: attraction, without a container.

Schrödinger - canonical ensemble

$$\langle E\rangle = \frac{\sum_i E_i e^{-\beta E_i}}{\sum_i e^{-\beta E_i}}.$$

$$C = \frac{d\langle E\rangle}{dT} = -\beta^2 \frac{d\langle E\rangle}{d\beta} = \beta^2 \langle (E_i - \langle E\rangle)^2\rangle > 0$$

Astronomers - virial theorem

$$2K + nV = 3p_e V$$

$K = \frac{3}{2}NkT$ – kinetic energy; V - potential energy;
$n = 1$ - for gravity; $p_e = 0$ - isolation
$E = K + V$ -total energy

$$C = \frac{dE}{dT} = -\frac{dK}{dT} = -\frac{3}{2}Nk < 0$$

Continuum theory I

- Microcanonical approaches, long range forces. Nonextensive?
- Thirring (1970), internal singularity, external confinement. Spatial distribution: not canonical, local thermometers.
- Chandrasekar: equilibrium, spherical.

Balances of mass, momentum and internal energy

$$\dot{\rho} + \rho \partial_i v^i = 0,$$

$$\rho \dot{v}^i + \partial_j P^{ij} = \boxed{\rho f^i},$$

$$\rho \dot{e} + \partial_i q^i = -P^{ij} \partial_i v_j - \boxed{\rho f_i v^i}.$$

Thermo: statics and dynamics

$$de = T ds + \frac{p}{\rho^2} d\rho, \qquad J^i = \frac{q^i}{T},$$

$$\rho \dot{s}(e - \phi, \rho) + \partial_i J^i = q^i \partial_i \frac{1}{T} - \left(P^{ij} - p\delta^{ij}\right) \frac{\partial_i v_j}{T} + \frac{\rho \dot{\phi}}{T} + \boxed{\frac{\rho}{T} f_i v^i} \geq 0$$

Continuum theory II

Balances of mass, momentum and energy

$$\dot{\rho} + \rho \partial_i v^i = 0,$$
$$\rho \dot{v}^i + \partial_j P^{ij} = 0,$$
$$\rho \dot{e} + \partial_i q^i = -P^{ij} \partial_i v_j.$$

Thermo: statics and dynamics

$$du = Tds + \frac{p}{\rho^2} d\rho = d\left(e - \phi - \frac{\partial_i \phi \partial^i \phi}{8\pi G \rho}\right),$$
$$\rho \dot{s}(u, \rho) = \ldots$$

Method: separation of divergences according to Eckard and de Groot and Mazur or Maugin. Coleman-Noll or Liu procedures.

Continuum theory III

Entropy balance:

$$\rho\dot{s} + \partial_i \left[\frac{1}{T} \left(q^i + \frac{1}{4\pi G} \dot{\varphi} \partial^i \varphi \right) \right] =$$

$$\left(q^i + \frac{1}{4\pi G} \varphi \partial^i \varphi \right) \cdot \partial_i \left(\frac{1}{T} \right) +$$

$$\frac{\dot{\varphi}}{4\pi G T} \left(\partial_i^i \varphi - 4\pi G \rho \right) -$$

$$\left[P^{ij} - p\delta^{ij} - \boxed{\frac{1}{4\pi G} \left(\partial^i \varphi \partial^j \varphi - \frac{1}{2} \partial_k \varphi \partial^k \varphi \delta^{ij} \right)} \right] \frac{\partial_i v_j}{T} \geq 0,$$

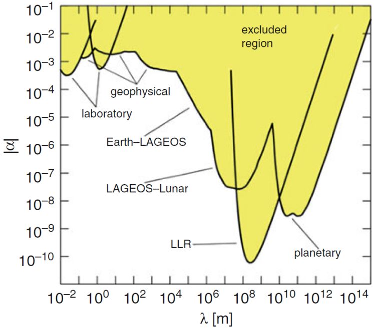
Ideal fluid:

$$\partial_i^i \varphi - 4\pi G \rho = 0$$

$$\partial_j P^{ij}_{GRAV} = \rho \partial_i \phi$$

Fields are important

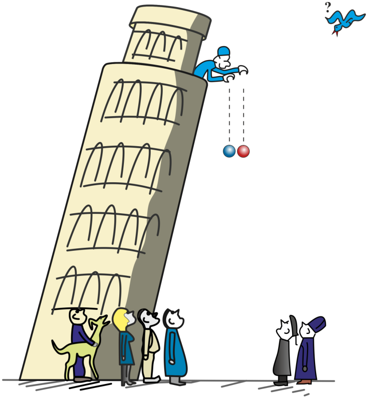
- Field energy: $\frac{\partial_i \phi \partial^i \phi}{8\pi G \rho}$ – Lagrangian
- Astronomical argument fails
- Boundary conditions
- Long range vs local and extensive
- Dissipative gravitation ?



Eötvös experiment

Fifth force

$$m_I a = m_G g = m_G G \frac{M}{r^2}$$



$$\boxed{ma = m(1 + \Delta\kappa)g}$$

Newton's law vs extra charge (force):

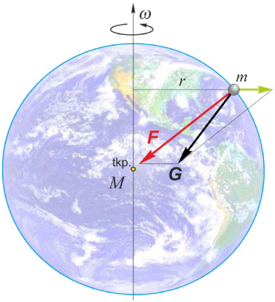
$$\phi(r) = G\frac{mM}{r}\left(1 + \alpha e^{-r/\lambda}\right)$$

$$m_G = m_I + \eta_A \frac{E_A}{c^2}$$

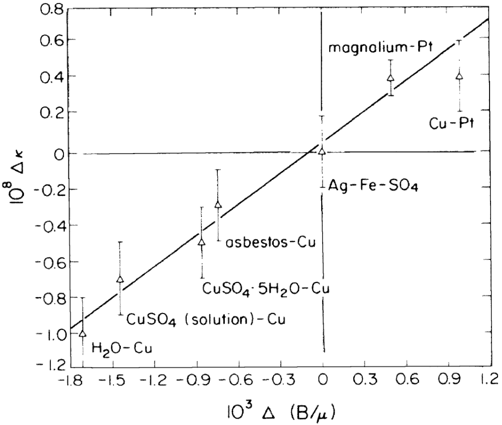
Eötvös experiment

$$ma = m(1 + \Delta\kappa)g = m(1 + \Delta\kappa)G\frac{M}{r^2}$$

- Free fall
- Pendulum
- More exact in equilibrium:

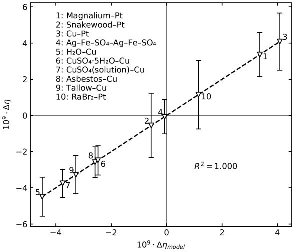


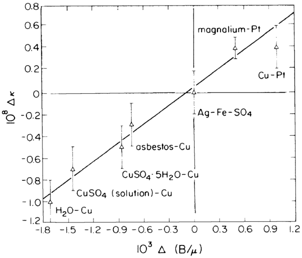
Mysterious deviation

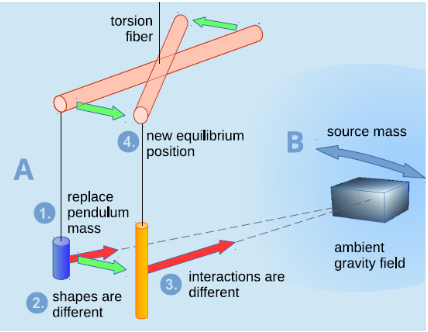


- New measurements: no fifth force
- Modell independence
- What about Eötvös, Pekár and Fekete?

Shape effect ?







Tóth (2018)

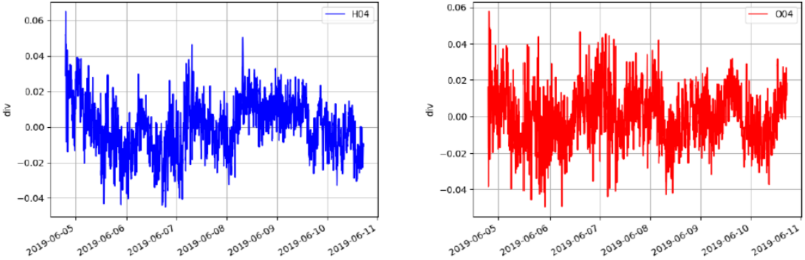
Eötvös balances: AutERBal, small original Eötvös

sensitivity: $\Delta\kappa = 10^{-9} - 10^{-11}$

automatic readout, automatic rotation, underground laboratory

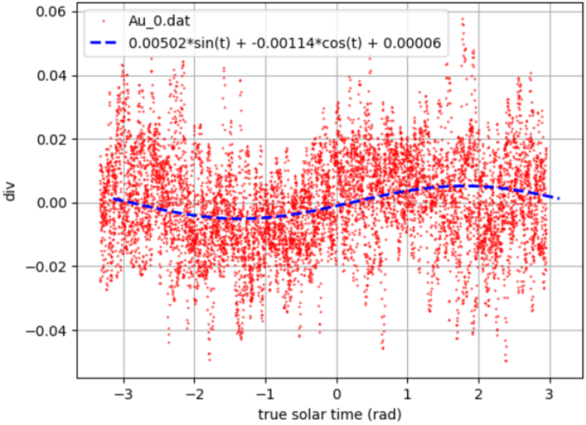
Test of the solar gravity field

- Sun's gravity was next tested on Al-Au pair
  - 7+ days in 0° azimuth with Al-Au pair
  - quadratic drift was removed
  - data were downsampled to 1 min



Data processing

- Method by Roll, Krotkov, Dicke (1964)
  - least-squares fitting of readings n as function of true local solar time t ($t = 0$ at noon):
  - $n(t) = S\sin(t) + C\cos(t) + K$
  - calculation of η from amplitude S



Preliminary results

- Estimated errors are at level $2 \cdot 10^{-9}$ (from non-null results on Au-Au)
  - low-frequency noise limits precision
  - must correct for pressure effects
  - analyze possible thermal effects

measurement period	**material**	η
05.15-05.20.	Cu	$0.60 \cdot 10^{-9}$
05.15-05.20.	Au	$1.15 \cdot 10^{-9}$
05.21-06.04.	Cu	$-1.85 \cdot 10^{-9}$
05.21-06.04.	Au	$-1.01 \cdot 10^{-9}$
06.04-06.11.	Al	$1.50 \cdot 10^{-9}$
06.04-06.11.	Au	$1.86 \cdot 10^{-9}$

Summary

- Extensivity is not additivity
- Black hole thermodynamics is normal with volume
- Extensive emergent Newtonian gravity
- Improved Eötvös balance:
  - Eötvös year
  - Fifth force mystery
  - Modified gravitation
  - Geophysics for Einstein Telescope
  - Dissipative relativistic fluids


Az Eötvös-kísérlet újramérésében résztvevők:
Völgyesi Lajos
Szondy György
Tóth Gyula
Ván Péter
UNESCO
Egyesült Nemzetek Nevelésügyi, Tudományos és Kulturális Szervezete
Eötvös Loránd (1848-1919) fizikus, geofizikus és a felsőoktatás megújítójának 100. évfordulója
Az UNESCO-val közösen emlékezve
Fenyvesi Edit
Kiss Bálint
Péter Gábor
Harangozó Péter
Gróf Gyula
Lévai Péter
Barnaföldi Gergely
Deák László
Égető Csaba
Somlai László

Thank you for your attention !

The history of Hawking radiation

Hawking radiaiton was not measured yet.

The black side

- Princeton: Wheeler (black hole, no hair), Dicke
- Black holes are radiating with pair production (Zeldovics és Sztarobinszkij 1971)
- The surface (mass) cannot decrease (Christodoulou, PhD 1971)
- Thermodynamics? (Bekenstein, PhD 1972)
- Hawking does not believe it, because there is no interaction with the environment (Bardeen-Carter-Hawking 1973)
- Hawking visits Moscow, 1973
- After that he believes that. (Hawking 1974, not mentioning the Russians),
- Then he mentions them. (Hawking: The short story of time, 1988).

CR volume

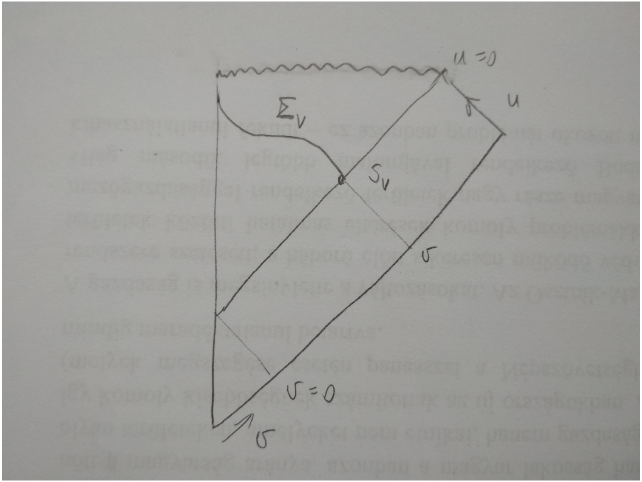
Is there a volume of a black hole?

- This is an old problem. Rovelli, Gibbons
- Coordinate dependence. Schwarzschild: inside it is not stationary. Inside can be different. Rindler: symmetry is violated.
- Parikh (2006): Killing (t = constant.): $V_P \approx M^3$

Invariant definition

- Special relativity: Σ is a volume inside a two-volume S. The largest spherical symmetric spacelike surface with the boundary S. Σ is synchronized with S.
- Increasing spheres, elongating cylinder, asymptotic formula in (Eddington-Finkelstein coordinates): $\lim_{\nu \to \infty} V(\nu) = 3\sqrt{3}\pi M^2 \nu$
- Schwarzschild: $\nu = t + r + 2M \ln |r - 2M|$
- Black hole remnants. Together with radiation, for a static black hole: $V_{CR} \approx M^5$.

CR volume, Penrose diagram



$u = t - r$, $v = t + r$. $u = 0$ is the event horizon. Above $u = 0$ is the internal part of the black hole.

Gibbs-Duhem in general relativity: Smarr relation

Schwarzschild:

$$S(E) = 4\pi E^2, \quad dE = TdS, \quad \boxed{E = 2TS}$$

Kerr-Newman-(Reissner-Nordström)

Charged, rotating black holes, Smarr formula (1973):

$$A = 4\pi \left[2M^2 + 2\sqrt{M^4 - L^2 - M^2Q^2} - Q^2\right]$$

$$dM = TdA + \Omega dL + \Phi dQ, \qquad \boxed{M = 2TA + 2\Omega L + \Phi Q}.$$

(1; 2,2,1) Euler homogeneous.

- The surface as independent thermodynamic body. York (1986) and Martinez (1996)
- The generalization of extensivity: geometrothermodynamics
- Nonadditive: Tsallis-Cirto (2013), Biró-Czinner (2013,2016) and Rényi.

Here there is a CR volume, too.

Further remarks

What kind of homogeneity?

Sűrűségekkel is (v1): $\epsilon(r) = R^a e(r)$, $\mathfrak{s}(r) = R^b s(x)$, $h(r) = R^c g(r)$

$$E = \int_0^R 4\pi r^2\, \epsilon(r)\, h(r)\, dr, \quad S = \int_0^R 4\pi r^2\, \mathfrak{s}(r)\, h(r)\, dr, \quad V = \int_0^R 4\pi r^2\, h(r)\, dr$$

Evaporation of black holes

Instability is not a problem is the body is insulated. Hawking radiation is the method of evaporation. Only temperature is necessary.
Pressure is meaningful/expected from a physical point of view and it is necessary for evaporation.

Holographic principle, state: a loss of information?

Susskind: *The Black Hole War: My Battle with Stephen Hawking to Make the World Safe for Quantum Mechanics (2008)*,
Thorne-Preskill-Hawking bet, ...
Brown,....Susskind,... (2016): the holographic complexity is the dual of spatial

További kérdések n: gravotermo instabilitás

Kanonikus v. mikrokanonikus

Minden vonzó, gravitációval kötött rendszernek, sőt, minden kötött rendszernek negatív a hőkapacitása. Magyarázat: Thirring (1970).

Schrödinger – kanonikus:

$$\langle E \rangle = \frac{\sum_i E_i exp(-\beta E_i)}{\sum_i exp(-\beta E_i)}$$

$$C = \frac{d\langle E \rangle}{dT} = -k\beta^2 \frac{d\langle E \rangle}{d\beta} = k\beta^2 \langle (E_i - \langle E \rangle)^2 \rangle$$

Csillagászok – mikrokanonikus:

$$H = K + V, \quad 2K + V = 3pV, \quad \langle K \rangle = \frac{3}{2} NT$$

$$E = \langle H \rangle = \langle K \rangle + \langle V \rangle = -\langle K \rangle = -\frac{3}{2} NT$$

$$\frac{dE}{dT} = -\frac{3}{2} N < 0.$$

Thirring: mesterséges csillag 1

Fajhő $\neq$ hőkapacitás. Kontinuum és homogén.

$$\theta_{V_0}(x) := \begin{cases} 1 & ha \quad x \in V_0 \\ 0 & ha\ nem. \end{cases}$$

Nem lokális potenciál V_0 kölcsönhatási térfogathoz:
$\phi(x,y) = -2k\theta_{V_0}(x)\theta_{V_0}(y)$. N_0 részecske a kölcsönhatási térfogatban, potenciális energia: $V = -kN_0^2$.
Fázistérfogat:

$$g(E_0, N_0) = \frac{1}{N!}\int d^{3N}p\, d^{3N}x\, \delta(E_0 - \sum_i p_i^2 + kN_0^2)$$

Boltzmann-entrópia (Gibbs ua., negatív hőmérséklet?!):

$$S_0(E_0, V_0, N_0) = \ln g = N_0\left(\frac{3}{2}\ln(E_0 + kN_0^2) - \frac{5}{2}\ln N_0 + \ln V_0\right).$$

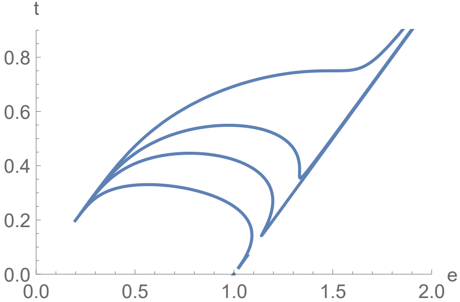
Egyensúlyban ideális gázzal:

$$S(E, V, N) = N\left(\frac{3}{2}\ln E - \frac{5}{2}\ln N + \ln V\right).$$

Teljes entrópia $S_T = S_0 + S$ feltételes maximuma, $E_T = E_0 + E$, $V_T = V_0 + V$, $N_T = N_0 + N$ állandók:

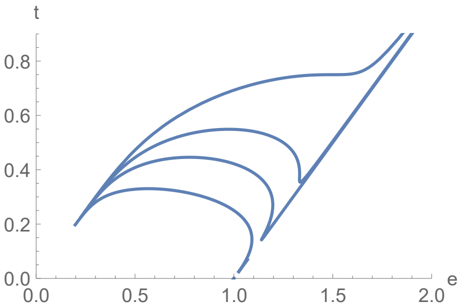
$$t = e - 2n + n^2, \quad e = 2n - n^2 + \frac{3(1-n)}{F + \ln(1-n)/n}$$

ahol
$F = \ln(V/V_0)$, $n = 1 - N_0/N_T$, $e = 1 + E/(kN_0^2)$ és $t = 3T/2Nk$.



$F = (2, 3, 4, 6)$ felülről

Thirring: mesterséges csillag 3



$F = (2, 3, 4, 6)$ felülről

- F elég nagy: nincs szabad gáz.
- A Thirring-gáz nem extenzív, (-2;-4,1,-2) Euler-homogén:

$$\lambda^{-2} S_0(E_0, V_0, N_0) = S_0(\lambda^{-4} E_0, \lambda V_0, \lambda^{-2} N_0)$$

- Mikrokanonikus gravitáció és csillagfejlődési modellek: kvalitatívan hasonló. Gömbhalmaz. Mag/halo.