Nuclear Equation of State: from Laboratory to Heavens

Pawel Danielewicz

Natl Superconducting Cyclotron Lab, USA

Institute for Particle and Nuclear Physics
Wigner Research Centre for Physics
Hungarian Academy of Sciences

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Equation of State of Nuclear Matter

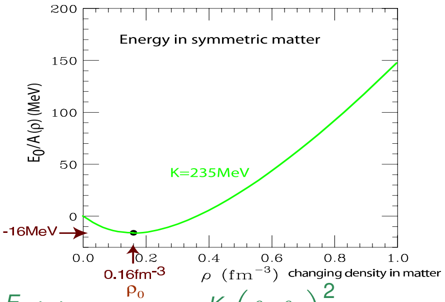
EOS: any nontrivial relation between thermodynamic vbles characterizing the matter, e.g. $p(\rho, T)$ or $\frac{E}{A}(\rho_p, \rho_n, T)$

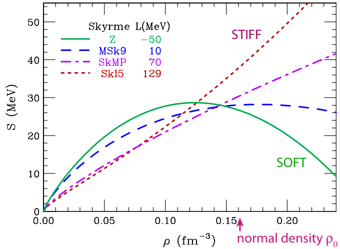
Central Reactions & *n*-Stars: ρ changed due to compression

Nuclear Structure: ρ changes in surface & dynamic oscillations

Energy breakdown in uniform matter, due to charge symmetry:

$$\frac{E}{A}(\rho_n, \rho_p) = \frac{E_0}{A}(\rho) + S(\rho) \left(\frac{\rho_n - \rho_p}{\rho} \right)^2$$





$$\frac{E_0}{A}(\rho) = -a_V + \frac{K}{18} \left(\frac{\rho - \rho_0}{\rho_0} \right)^2 + \dots$$

$$S(\rho) \simeq a_a^V + \frac{L}{3} \frac{\rho - \rho_0}{\rho_0}$$

$a_a^V = ?$ $L = ?$

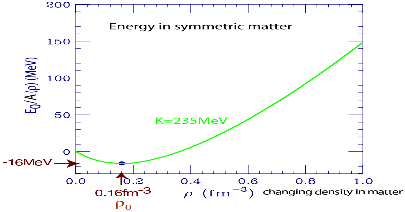
Pressure and Energy

In cold matter, at $T = 0$: $$P = \rho^2 \frac{\mathrm{d}}{\mathrm{d}\rho} \frac{E}{A} \qquad \rho = \rho_n + \rho_p$$

Further $$\frac{E}{A}(\rho_n, \rho_p) \approx \frac{E_0}{A}(\rho) + S(\rho) \left(\frac{\rho_n - \rho_p}{\rho}\right)^2$$

In neutron matter $\rho_p \ll \rho_n$: $$\frac{E}{A}(\rho) \approx \frac{E_0}{A}(\rho) + S(\rho)$$

With $S(\rho) = a_a^V + \frac{L(\rho - \rho_0)}{3\rho_0} + \ldots,$ $$P \simeq \rho^2 \frac{\mathrm{d}S}{\mathrm{d}\rho} \simeq \frac{L}{3\rho_0}\rho^2$$



While symmetry-energy effects are strong in neutron matter, in nuclei $\frac{\rho_n - \rho_p}{\rho} \lesssim 0.3$, so $S(\rho)\left(\frac{\rho_n - \rho_p}{\rho}\right)^2 \ll \left|\frac{E_0}{A}(\rho)\right|$ & symmetry-energy effects are weak

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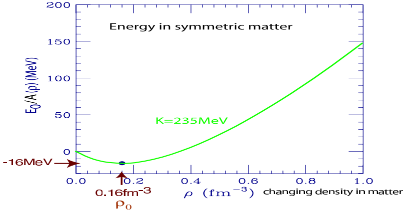

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Central Reactions in Terms of Boltzmann Eq

Central reactions of heavy nuclei described statistically in terms of Boltzmann equation for the Wigner function f - density of particles in space and momentum:

velocity force

$$\frac{\partial f}{\partial t} + \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{p}} \frac{\partial f}{\partial \mathbf{r}} - \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{r}} \frac{\partial f}{\partial \mathbf{p}} = \int d\mathbf{p}_2 \int d\Omega' \, v_{12} \, \frac{d\sigma}{d\Omega'}$$

$$\times \Big((1 - f_1)(1 - f_2) f_1' \, f_2' - (1 - f_1')(1 - f_2') f_1 \, f_2 \Big)$$

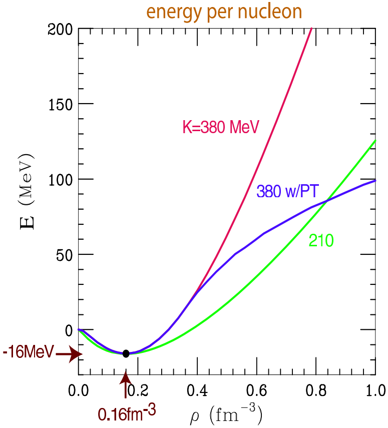
gain loss

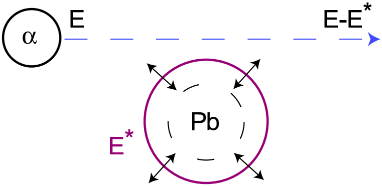
Here, ϵ_p - single-particle energy and $\frac{d\sigma}{d\Omega}$ - nucleon-nucleon scattering cross-section.

System energy specified in terms of the Wigner functions, allowing to consider nonequilibrium situations, while constraining the equilibrium, $E = E\{f\}$. Single-ptcle energy:

$$\epsilon(\mathbf{p}) = \frac{\delta E}{\delta f(\mathbf{p})}$$

Incompressibility from Vibrations?





$$E^* = \hbar\Omega = \hbar\sqrt{\frac{K}{m_N \langle r^2 \rangle_A}}$$

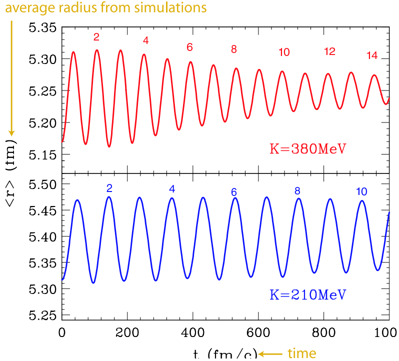
Problem: surface, Coulomb, isospin imbalance

$\Rightarrow$ all that in Boltzmann eq.

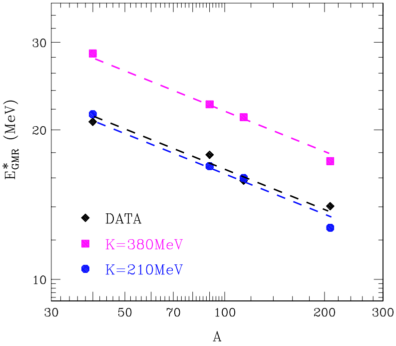
$$K = 9\,\rho_0^2 \frac{d^2}{d\rho^2}\left(\frac{E}{A}\right) = R^2 \frac{d^2}{dR^2}\left(\frac{E}{A}\right)$$

Monopole Oscillations

Pb Oscillations



$$E^*_{GMR} = \hbar\Omega$$

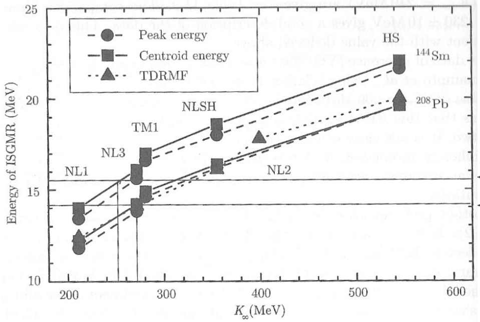


data Youngblood, Garg *et al*

$\Rightarrow \quad K \sim 235\,\mathrm{MeV}?$

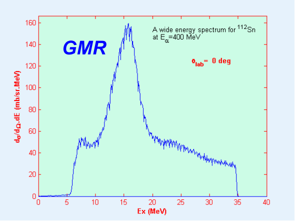
Some Model Dependence

Relativistic RPA for different Lagrangians (Van Giai *et al*)



data (Garg *et al*)

exp



$$\Rightarrow \quad K = (230 - 285)\,\text{MeV}$$

Central Reactions

Reaction plane: plane in which the centers of initial nuclei lie

Spectators: nucleons in the reaction periphery, little disturbed by the reaction

Participants: nucleons that dive into compressed excited matter

Nuclear EOS deduced from the features of collective flow in reactions of heavy nuclei

Collective flow: motion characterized by significant space-momentum correlations, deduced from momentum distributions of particles emitted in the reactions

Euler eq. in $\vec{v} = 0$ frame:

$$m_N \, \rho \, \frac{\partial}{\partial t} \, \vec{v} = -\vec{\nabla} p$$

EOS and Flow Anisotropies

EOS assessed through reaction plane anisotropies characterizing particle collective motion

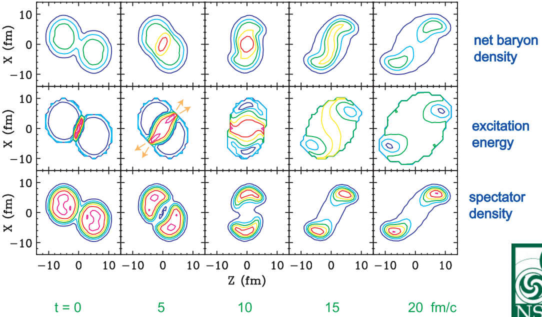
Hydro? Euler eq. in $\vec{v} = 0$ frame: $m_N \, \rho \, \frac{\partial}{\partial t} \, \vec{v} = -\vec{\nabla} p$

where p - pressure From features of v, knowing Δt, we may learn about p in relation to ρ Δt fixed by spectator motion

For high p, expansion rapid and much affected by spectators

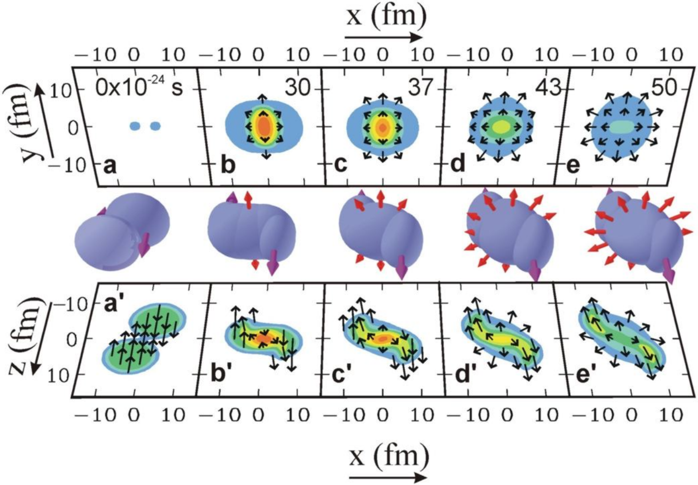
For low p, expansion sluggish and completes after spectators gone

Simulation by L. Shi



Medium-Energy Collisions of Heavy Nuclei

Thermalized matter at high baryon density! 2 GeV/u Au+Au



Top panels: pressure ⊥ to beam axis (up to 90 MeV/fm^3) + flow

Bottom panels: density (up to $3\rho_0$) in reaction plane + flow

Sideward Flow Systematics

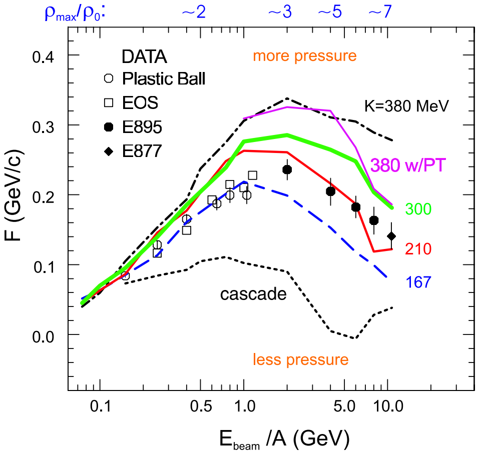
Deflection of forwards and backwards moving particles away from the beam axis, within the reaction plane

Au + Au Flow
Excitation Function

Note: *K* used as a label

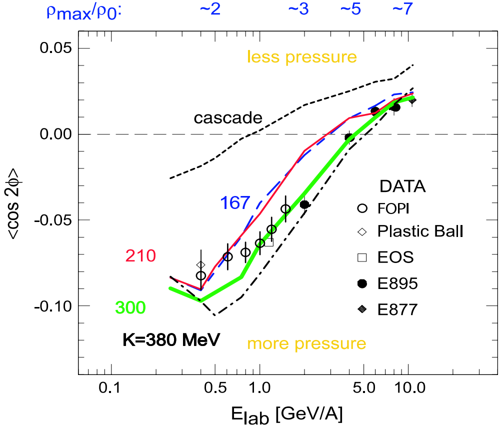
PD, Lacey & Lynch

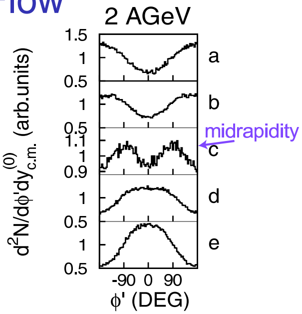
The sideward-flow observable results from dynamics that spans a ρ-range varying with the incident energy



2^{nd}-Order or Elliptic Flow

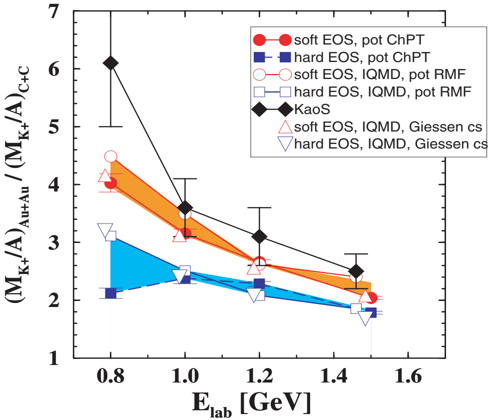
Another anisotropy, studied at midrapidity:
$v_2 = \langle \cos 2\phi \rangle$, where ϕ is azimuthal angle relative to reaction plane





Au+Au v_2
Excitation Function

Subthreshold Meson (K/π) Production



Ratio of kaons per participant nucleon in Au+Au collisions to kaons in C+C collisions vs beam energy

filled diamonds: KaoS data

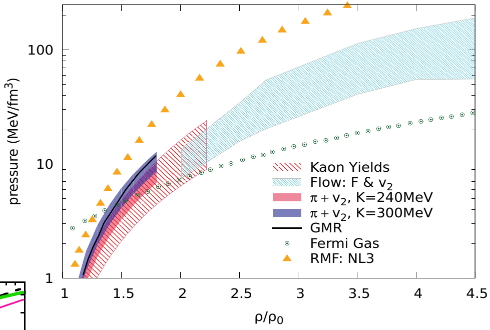
open symbols: theory Fuchs *et al*

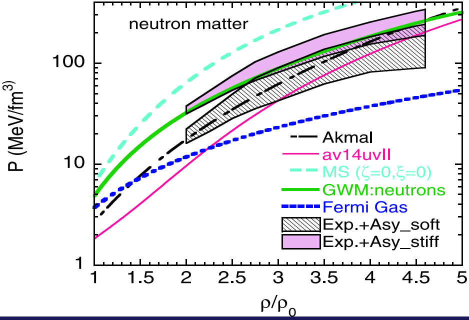
Kaon yield sensitive to EOS because multiple interactions needed for production, testing density

The data suggest a relatively soft EOS

Constraints from Flow on EOS

Au+Au flow anisotropies:
$\rho \simeq (2 - 4.6)\rho_0$
No one EOS yields both flows right. Discrepancies: inaccuracy of theory
Most extreme models for EOS can be eliminated





PD, Lacey & Lynch
+ Fuchs + Hong + others

Neutron Matter:

Uncertainty in symmetry energy

Nuclear Mass Formula & Charge Invariance

Symmetry-energy details in nuclear mass-formula intertwined with details of other terms: Coulomb, Wigner & pairing + even those asymmetry-independent, due to $(N-Z)/A$ - A correlations along stability line (PD)!

Best would be to study the symmetry energy in isolation from the rest of mass-formula! Absurd?!

Charge invariance to rescue: lowest nuclear states characterized by different isospin values (T, T_z), $T_z = (Z-N)/2$. Nuclear energy scalar in isospin space:

sym energy $$E_a = a_a(A)\,\frac{(N-Z)^2}{A} = 4\,a_a(A)\,\frac{T_z^2}{A}$$

$$\rightarrow\; E_a = 4\,a_a(A)\,\frac{T^2}{A} = 4\,a_a(A)\,\frac{T(T+1)}{A}$$

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Symmetry Coefficient Nucleus-by-Nucleus

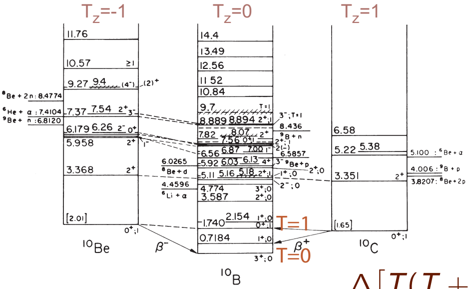
Mass formula generalized to the lowest state of a given T:

$$E(A,T,T_z) = E_0(A) + 4a_a(A)\,\frac{T(T+1)}{A} + E_{\rm mic} + E_{\rm Coul}$$

In the ground state T takes on the lowest possible value

$T = |T_z| = |N-Z|/2$. Through '+1' most of the Wigner term absorbed.

?Lowest state of a given T: isobaric analogue state (IAS) of some neighboring nucleus ground-state.

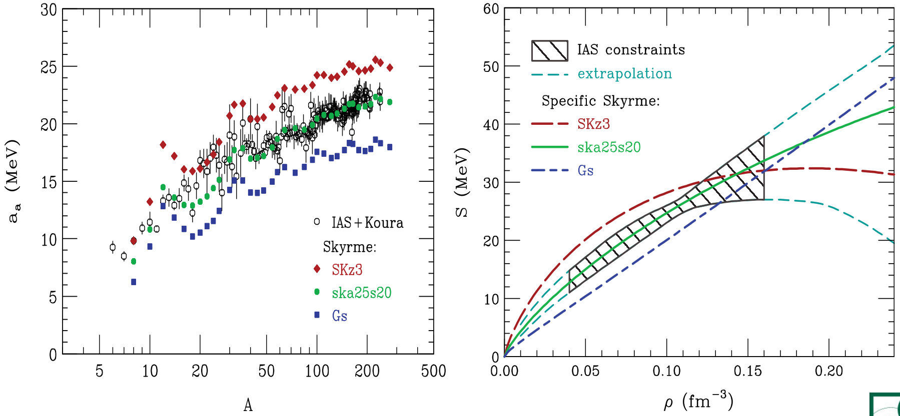


Study of changes in the symmetry term possible nucleus by nucleus

$$E^*_{\rm IAS} = \Delta E = a_a\,\frac{\Delta\left[T(T+1)\right]}{A} + \Delta E_{\rm mic}$$

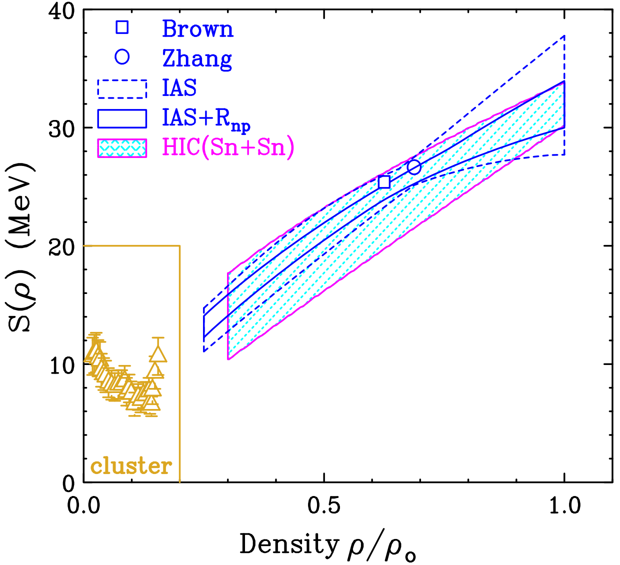
From $a_a(A)$ to $S(\rho)$

Strong $a_a(A)$ dependence (PD & Lee NPA922(14)1):
lower $A \Rightarrow$ more surface $\Rightarrow$ lower $\rho \Rightarrow$ lower S

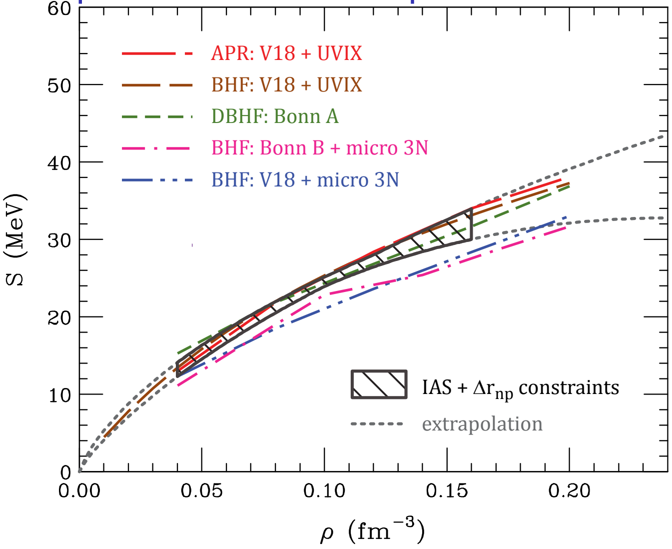


$a_a(A)$ from IAS give rise to constraints on $S(\rho)$ in Skyrme-Hartree-Fock calculations

Subnormal $S(\rho)$ from Different Data



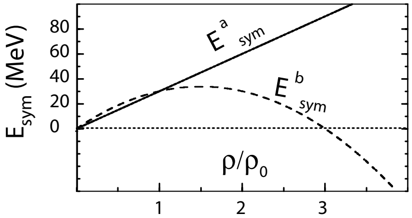
Comparison to Microscopic Calculations



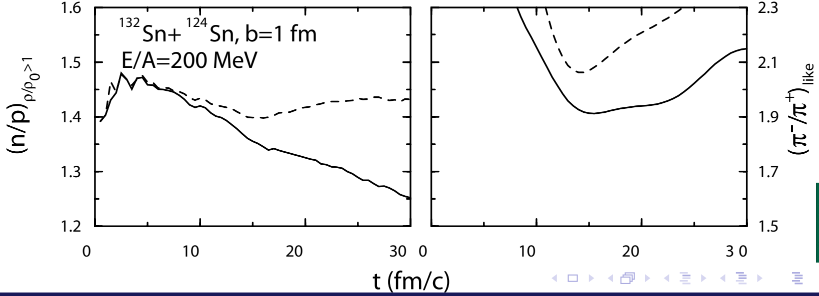
Microscopic results from Baldo *et al*

Pions as Probe of High-ρ Symmetry Energy

B-A Li PRL88(02)192701: $S(\rho > \rho_0) \Rightarrow n/p_{\rho>\rho_0} \Rightarrow \pi^-/\pi^+$



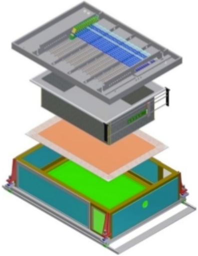
Pions originate from high ρ



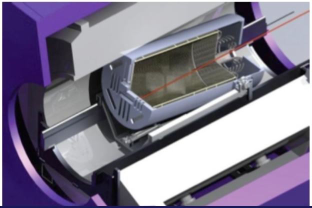
Dedicated Experimental Efforts

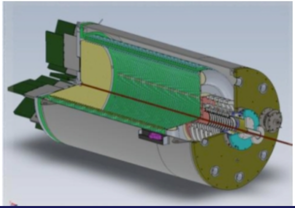
SAMURAI-TPC Collaboration (8 countries and 43 researchers): comparisons of near-threshold π^- and π^+ and also *n*-*p* spectra and flows at RIKEN, Japan.

NSCL/MSU, Texas A&M U
Western Michigan U, U of Notre Dame
GSI, Daresbury Lab, INFN/LNS
U of Budapest, SUBATECH, GANIL
China IAE, Brazil, RIKEN, Rikkyo U
Tohoku U, Kyoto U



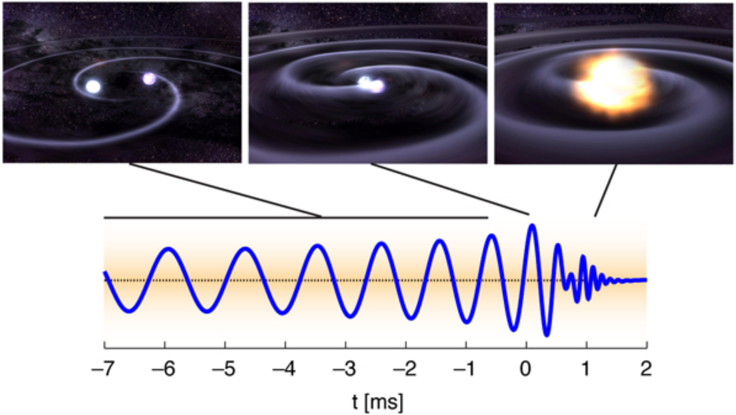
AT-TPC Collaboration (US & France)





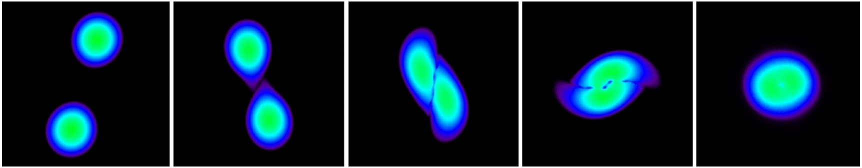
Neutron-Star Merger

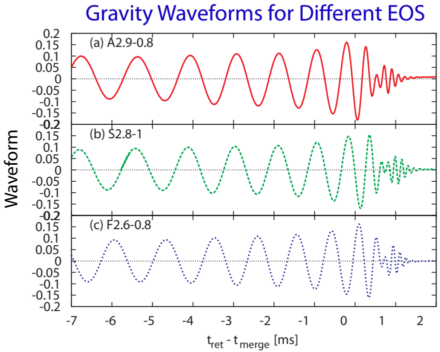
Gravitational-wave signal informs on history of merger



Spero, Physics 3(10)29; Kiuchi *et al.* PRL104(10)141101

Deformation Changes Quadrupole Moment



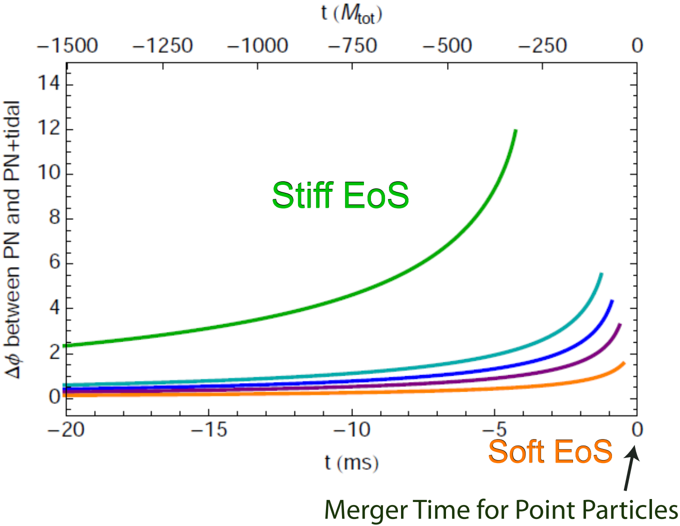


Deformation depends on star size and latter on equation of state (EOS)

Andreas Bauswein;

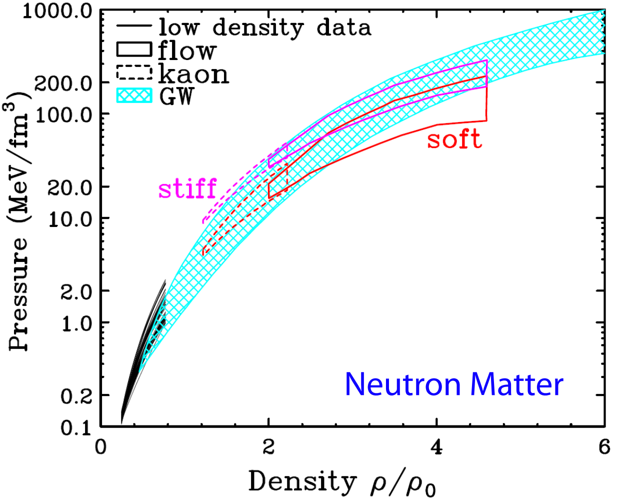
Kiuchi *et al.* PRL104(10)141101

Phase Shift Compared to Point Masses



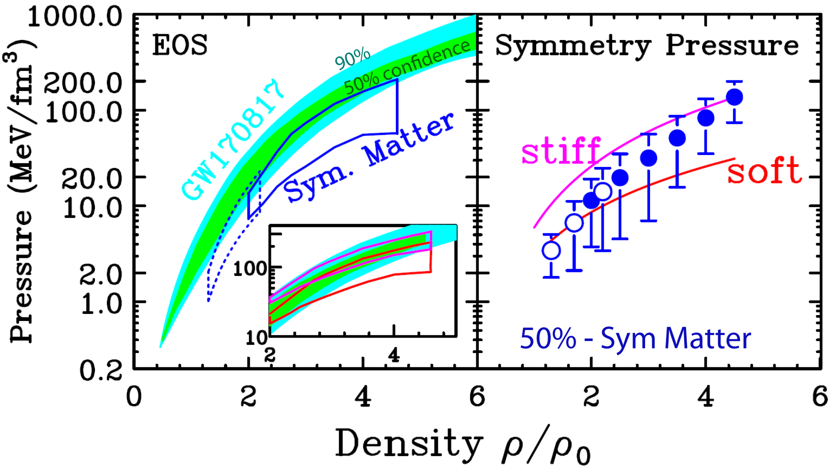
Andreas Bauswein

Pressure from Merger & Nuclear Collisions



Merger: Abbott *et al.* PRL121(18)161101
Collisions: PD *et al.* Science 298(02)1592 & arXiv:1807.06571

Merger-Collision Subtraction: Symmetry Pressure



Tsang *et al.* arXiv:1901.07673

Conclusions

- Excitations of giant collective resonances constrain incompressibility of symmetric matter to $K = (230\text{-}285)$ MeV
- Collective flow + threshold meson production in central heavy-ion reactions constrain nuclear pressure at densities $\rho = (1.2\text{-}4.5)\rho_0$. Most extreme model EOS eliminated
- Convergence on symmetry energy at $\rho \lesssim \rho_0$, from variety of data, isospin diffusion, isobaric analog states etc., and from microscopic calcs testing mostly 2-body ints
- Gravitational-wave data from neutron-star merger yield neutron-matter EOS consistent with inferences from nuclear collisions. Subtraction constrains symmetry-energy pressure at $\rho > \rho_0$.

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